Pakistan Journal of Applied Sciences 2(4):526-529, 2002

Explanation of Integer Quantum Hall Effect in Impurity Systems by Gauge Invariance

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Abstract: It is shown that the quantization of the Hall conductivity of two- dimensional samples, Which has been observed previously by Von Klitzing *et al.*, 1980 and by Tsui *et al.*, 1982, is a consequence of gauge invariance and the existence of a mobility gap. It is clear that extended and localized levels have important role in explaining of QHE. Within our work, the gauge invariance has been used in the case of impurity potentials. Other possible sources of deviation are briefly examined.

Key words: Integer Quantum, Impurity Systems, Gauge Invariance

Introduction

One of the most interesting properties of the two-dimensional electron gas which can occur at a semiconductor interface is quantization of the Hall resistance (Chakraborty and Pietiloinen, 1995). At very low temperature T and high magnetic field strength B, the Hall resistance Rxy(=PH=-ρxy) is characterized by flat steps at integral multiples of the fundamental value $\frac{h}{e^2}(R_{xy}=\frac{h}{ne^2}$; n integer). In those regions in which ρxy has the quantized value, ρxx is essentially equal to zero; it would presumably be zero at zero temperature, (Fig. 1).

This result suggests very strongly that if we think of the density of states associated with a particular Landau level as broadened by impurity scattering, extended states exist only very close to the center of Landau level and localized states exist in any other regions Establishing the validity of this picture from microscopic theory remains a fundamental unsolved problem. Work in this direction has been carried out by several authors(Giuliani *et al.*, 1983).

Laughlin has presented an elegant argument in which he explains that the quantization is due to the long range phase - rigidity characteristic of a super current, and that it can be derived from gauge invariance and the existence of a mobility gap (Laughlin, 1981). He has shown this by considering the response of a two dimensional metallic ribbon to a chang in the flux threading the ribbon (Fig. 2). Because changing of flux threading the ribbon is certainly not a simple gauge transformation, the terminology of Laughlin's argument is not suitable one. Furthermore, his argument assume that the only consequence of adding an integral number of flux quanta $\frac{hc}{e}$ is to repopulate the current carrying states. This assumption is obviously valid for the ideal system, but it is not so obvious in the presence of disorder when localized states can exist at the Fermi level.

In this paper we investigate the response of extended and localized levels due to changing of the magnetic flux passing through a ribbon system. In this word we use gauge invariance for explanation of IQHE and espesially in the case of impurity potentials.

Metallc Ribbon with Impurities: In the case of noninteraction electrons, Hamiltonian is:

$$H=\frac{1}{2m}(\vec{p}+\frac{e}{c}\vec{A})^2+eEx$$

Where E is the electric field applied in x-direction. Then the wave function is given by

$$\Psi_{nx}(x,y)=\pi^{-\frac{1}{4}}\,L_y^{-\frac{1}{2}}\,\exp\left(i\frac{X}{l_0^2}y\right)\Phi_n(x-X) \qquad (2)$$

Where Φ_n is the n^th harmonic oscillator wave function.

By definition, $X=l_o^2\,k$ k, $l_o=\left(\frac{hc}{eB}\right)^{\frac{1}{2}}$, $w_e=\frac{eB}{m^*c}$ and in this case energy will be

$$E_{nx}=(n+\frac{1}{2})hw_c+ux, u=\frac{cE_x}{B} \qquad (3)$$

When impurity is applied, we will introduce a new index (or set of indices) a in order to lable the wave function Ψ_a and eigenvalues E_a of the system. Then we have two forms

$$\Psi_{ae}=\sum_{nx}C_{nx}\Psi_{nx} \qquad (4)$$

for extended states, and

$$\Psi_{a1}=\sum_{nx}C_{nx}\Psi_{nx} \qquad (5)$$

for localized states. Here, localized wave functions tend to zero for position values far away from center, while the extended states have non zero value within $^0 \le y \le 1y$ (Fig. 3)

Since localized states contain no flux, they will not show any response to changing the flux. But extended states vary linearly. Variation of magnetic flux by $\pm\varnothing_o$ Value, results in sublevels map extend in to each other (Fig. 4).

Results and Discussion

We follow the Aoki method (Aoki, 1982). By changing the magmatic flux, the wave function, with no impurity, will be changed to

$$\psi \rightarrow \psi\,\exp(2\pi i\frac{\phi}{\phi_o});\phi_o=\frac{hc}{e}$$

and by using the periodic boundary conditions, we get

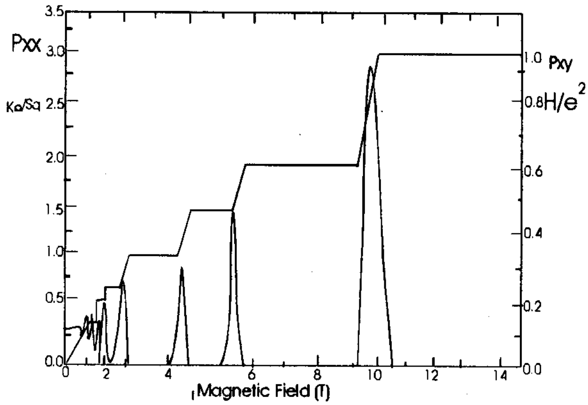

Fig. 1: The Hall Resistivity Pxy and Magnetoresistivity Pxx of Typical Sample. The Sample Congifuration is Shown as Inset

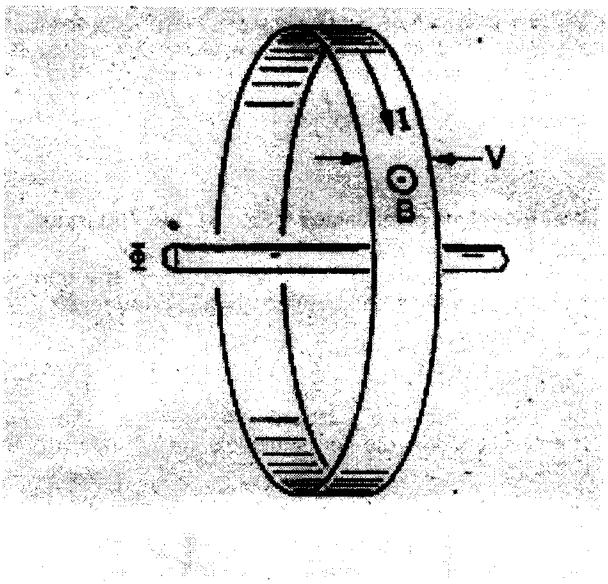

Fig. 2: Hall Effect in the Geometry of Laughlin's Gedankenexperiment

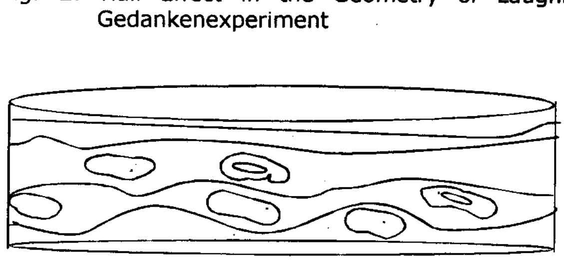

Fig. 3: Schematic of the Classical orbits for extended and Localized states in the Ribbon Geometry. Extended States are Associated with Classical Orbits which encircle any Flux Threading the Ribbon

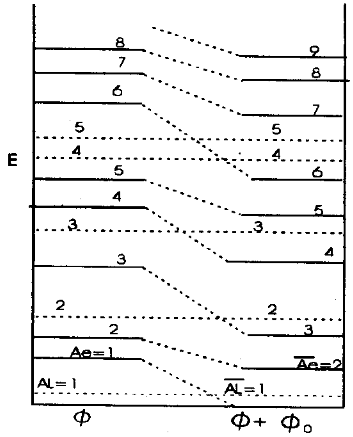

Fig. 4: Evalution of an Extended State Level E_{ae} and Adiabatic Change Δø of the Flux Threading, which Correspondes to Localized State Level E_{aL}

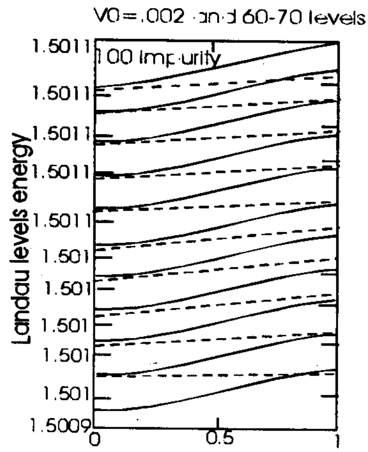

Fig. 5: Schematic Illustration of how the Spectrum of the System is Modified by an Adiabitic Chang of the Flux Threading the Ribbon. The Localized States are Unaffected by the Flux Chang

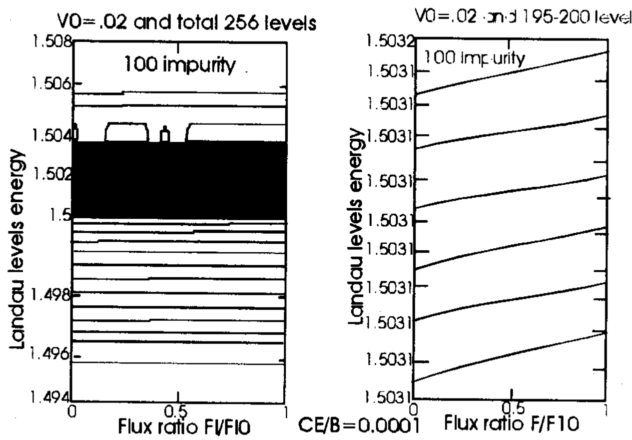

Fig. 6: Landau Levels energy Versus $\frac{\varnothing}{\varnothing_0}$ for V0=0.02

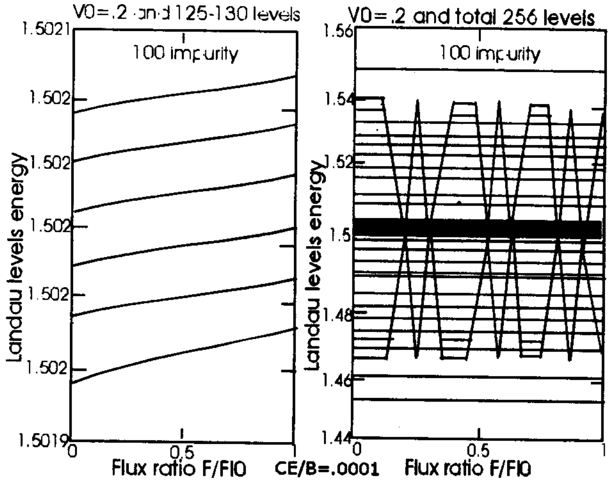

Fig. 7: Landau Levels energy Versus $\frac{Ø}{Ø_o}$ for V0=0.2

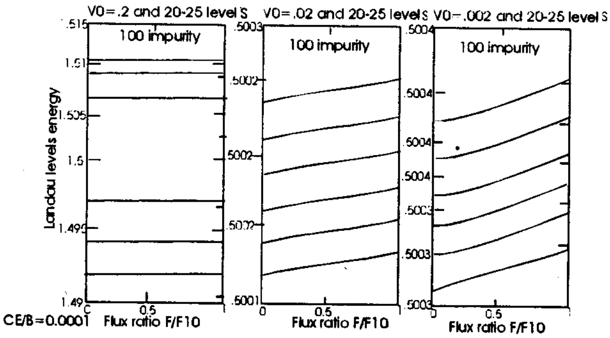

Fig. 8: Landau Levels energy Versus $\frac{Ø}{Ø_o}$ for V0=0.2, 0.02 and 0.002

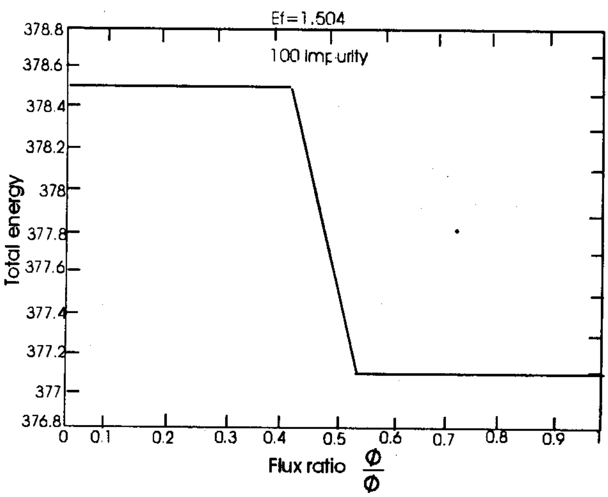

Fig. 9: Total Energy Versus $\frac{Ø}{Ø_o}$

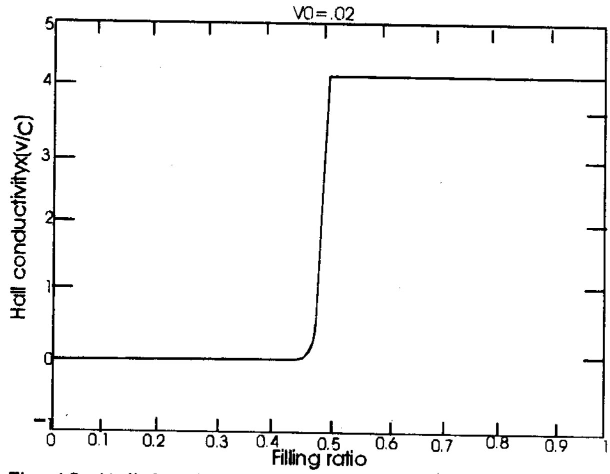

Fig. 10: Hall Conductivity Versus Filling Ratio for V0=0.02 and $\frac{Ø}{Ø_o} = \frac{6}{20}$

$$E_{nx} = (n+\frac{1}{2})hw_c + u(x+\frac{\phi}{\phi}\Delta x) \qquad (6)$$

We introduce an impurity potential with Delta function. We add 100 uniform impurities with repulsive and attractive potential in the form of

$$V(\vec{r}) = V_o\sum_{ij}\delta(x-x_i)\delta(y-y_j)-1)^{i+j} \qquad (7)$$

We measure eigenvalues (Eq. 6) by choosing $\frac{\phi}{\phi_o}$ within the interval [0, 1] and assuming the impurity potential is very small between two Landau levels. That is

$$\langle n,x|v(\vec{r})|n',x'\rangle = \alpha\delta_{nn'} \qquad (8)$$

In this case we should diagonalize the new Hamiltonian. We did for sublevels n=1, and with the conditions

$$L_x = Ly=(2\pi N_x)^{\frac{1}{2}}\ I_o\ =40.1\ I_o \qquad (9)$$

and number of sublevels is N_s =256. We also assume the electrical field such that $u=10^{-4}$ (Prange, 1981). The experiment was performed in three stages V0=0.002,
V0=0.02 and V0=0.2; note that

$Ø_1(x) = e^{\frac{x^2}{2}}(2x)$. Each Landau level will, however, be broadened by the effect of the random potential. In such a case the label can be resolved in to a Landau band index n and a generalized orbit - position quantum number x. With this notation we can write

$$E^{\phi}_{n}, X+\Delta X = E^{\phi+\phi o}_{n,x} \qquad (10)$$

Fig. 5 shows the impurity effect in sublevel due to Ø for V0=0.002 By increasing potential value, the level which don't follow Eq. (10), convert to localized levels. Fig. 6 and 7 show these levels for V0=0.02 and 0.2, respectively.

As we see extended levels show periodic response to magmatic flux variation, but localized sublevels do not show any response. Central black regions related to the extended levels. The strong responses in some periodic levels have no effect on the final result. These levels somehow related to charge exchange among localized levels which need more research.

By increasing the impurity potential, some extended levels vary gradually (Fig. 8). For V0=0.02, When Fermi level exceeds Ef=1.504 ev, a sharp degradation is observed in total energy versus $\frac{\phi}{\phi_o}$ (Fig. 9). With simple calculation, it can be seen that after extended levels fulfil at E=1.504 ev, more current transfer by them.

In Fig. 10 we measured Hall conductivit inerms of filling ratio. This calculation is for V0=0.02 and $\frac{\phi}{\phi_o}=\frac{6}{20}$ As seen there is a step up in Hall conductance as a function of magnetic field. For different values of $\frac{\phi}{\phi_o}$ the same steps with the same height will appear.

Conclusion

In this paper we have provided microscopic theory of the Hall conductance for a two - dimensional electron gas in the presence of an impurity Random potehtial.

We have analyzed the electronic level structure and have shown explicitly that extended for the states, when the trapped flux is adiabatically changed, the spectrum shifts in away. This behavior is similar to free electrons and maps in to itself when the chang in flux is precisely one quantum.

We concluded that when the number of impurities of sample is low, by increasing potential can get localized states. Extended states form in the central region of Landau Levels and they are periodic. Localized levels shift to upper gap and lower one of each Landau level and they make no current. The gauge invariance method can be used for studding impurity state. Localized levels have stronpeaks (Fig. 7) which probably related to electron transfer among localized sublevels.

There still exists an open question that what happen if we choose the other potential from such as Gaston, etc. or, instead of repulsive and attractive potential. We assume potentials with region distribution. These need more investigation.

References

D. C. Tsui, H. L. Stormer and A. C. Gossard, 1982. Phys. Rev. Lett. 48, 1559.

G. F. Giuliani, JJ. Quinnn and s. c. Ying, 1983. Physical Review B 28: 2964.

H. Aoki, 1982. Phys. Solid state phys 15L 1227.

K. Von Klitzing, G. Dorda and M. Pepper, 1980. Phys. Rev. lett. 45, 494.

R. B. Laughlin, 1981. Phys. Rev. B 23, 5632.

R. E. Prange, 1981. Phys. Rev. B23, 4802.

T. Chakraborty and P. Pietiloinen, 1995. "The Quanum Hall Effects", springer series in solid state sciences 85.