

Calculation of the Effective g-factor in Two Dimensional Systems

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Abstract: The effective Landé-g factor of two dimensional systems is calculated on the basis of the effective parameters approximation. By a simple diagram method, it is shown that the effective g-factor only takes even integer values. The calculations are carried out both in the absence and presence of the Landau level broadening.

Keywords: Effective Landé-G Factor, Landau Level Broadening, Effective Parameters Approximation

Introduction

Since the discovery of Quantum Hall Effect (Klitzing *et al.*, 1980) the question of effective Landé-g factor, g^* , in two dimensional (2D) systems has become more important. The enhancement of the g-factor in 2D systems was first proposed by Janak (Janak, 1969) to explain the measurement made by Fang and Stiles (1968) in which the effective g-factor of silicon metal-oxide semiconductors (MOS) systems was found considerably larger than the free electron value $g=2$. Suzuki and Kawamoto (1973) applied Landau's theory of Fermi liquid to calculate the effective g-factor of interacting electrons in silicon inversion layers. Ando and Uemura (1974) calculated oscillatory enhancement of the g-factor caused by the electronic exchange interaction. Lakhani and Stiles (1973) reported considerably large values of the g^* (~ 6). Nicholas *et al* (Bucholas and Brummel, 1982; Nicholas *et al.*, 1988) measured the effective g-factor in GaAs-GaAlAs systems. They reported g^* values such as 6.2 and 3.3. Although in bulk semiconductors side the effective g-factor depends strongly on the fundamental energy gap, E_g , and the spin-orbit splitting, Δ , of the top most valance band (Wilson and Feher, 1961; Duncan and Schneider, 1963; Pidgeon *et al.*, 1967; Hermann and Lampel, 1971; White *et al.*, 1972; Weisbuch and Hermann, 1977) the reason of the enhancement of g-factor in 2D systems must be more peculiar than the bulk semiconductors. 2D electron systems can be realized in several classes of systems. One example is the MOS systems, another example is the semiconductor heterostructures such as GaAs-Al_xGa_{1-x}As systems. Reviews of 2D systems are given (Ando *et al.*, 1982; Aoki, 1987). The common property of the 2D system is that the observation of a high mobility $\sim 10^4$ - 10^6 cm²/Vs (whereas the corresponding bulk value is about 10^3 cm²/Vs). So the collision time and the effective mass in 2D must be different from the corresponding bulk values. Because of the importance of the exchange and the correlation effects in 2D systems, the collision time and the effective mass will be different from the bulk semiconductors. Therefore the behavior of g^* must also be different from the bulk semiconductors. The theoretical calculations given above are based on the assumption that only the g^* changes in 2D and the effective mass is the same as the bulk value. In this study we consider the exchange

and the correlation effect as the mechanisms that alter the effective mass, the g-factor and the local magnetic field as well. We calculate g^* in 2D systems by a simple diagram method. It is shown that $|g^*|$ only takes even integer values such as $\pm 2, \pm 4, \pm 6, \dots$ and so on. Decimal values of g^* which have been observed experimentally are not allowed in an ideal 2D system. According to the conclusion of the present investigation an explanation of the observation of the decimal g^* values in 2D systems must be sought elsewhere such as the change of the effective mass from the bulk value, the change of the effective magnetic field (local magnetic field) or not achieving the ideal two dimensionality.

Materials and Methods

In the presence of an external magnetic field $\vec{B} = (0, 0, B)$ along the z direction, the one electron Hamiltonian of the 2D system in SI units is written as

$$H = H_1 + H' = H_0 + H_{Zeeman} + H_{e,l} + H_{e,e} \quad (2.1)$$

$$= \frac{p_x^2}{2m} + \frac{(p_y - eBx)^2}{2m} - g|\mu_B|B\sigma + V(r) + V_{ex} + V_H$$

where H_0 is the kinetic energy operator for a free electron without spin, H_{Zeeman} is the Zeeman potential, $H_{e,l}$ is the electron-lattice interaction which is mainly the potential $V(r)$ coming from the background impurities (Aoki, 1987) and $H_{e,e}$ is the electron-electron interaction term which covers the exchange potential V_{ex} and the Hartree electrostatic potential V_H . In the Zeeman potential, g is the Landé-g factor of the free

electron $\mu_B = \frac{eh}{2m}$ is the Bohr magneton and $\sigma = \pm \frac{1}{2}$ is the spin number.

We treat $H' (= H_{e,l} + H_{e,e})$ as a small perturbation.

In the absence of H' , the eigenvalues corresponding to $H_1 (= H_0 + H_{Zeeman})$ is (Landau and Lifshitz, 1962),

$$E_l = (l + \frac{1}{2})\hbar\omega_c \pm \frac{g|\mu_B|B}{2} \quad (2.2)$$

$$= (l + \frac{1}{2})\hbar\omega_c \pm \frac{g}{4}\hbar\omega_c$$

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where $l = 0, 1, 2, \dots$ is the Landau index, $\omega_c = \frac{e\hbar}{m}$ is the cyclotron angular frequency, m is the free electron mass and g is the free electron Landé-g factor which is equal to 2. Here the (+) sign corresponds to spin-down electron and (-) sign to spin-up one. The energies in eq.(2.2) form a equidistant discrete energy spectrum with a level separation $\hbar\omega_c$. So for a free electron ($g=2$) the energies for spin up and spin down cases read the forms:

$$E_l(\uparrow) = l\hbar\omega_c \quad (2.3a)$$

$$E_l(\downarrow) = (l+1)\hbar\omega_c \quad (2.3b)$$

Next we consider the perturbing Hamiltonian in eq.(2.1):

$$H' = H_{e,l} + H_{e,e} = V(r) + V_{ex} + V_H \quad (2.4)$$

The effect of $V(r)$ is three-fold. First it causes a broadening (Aoki, 1987) in Landau levels given in eq.(2.2). Secondly it alters the mass of the electron and thirdly it may change the cyclotron radius of the electron (Shklovskii *et al.*, 1984). The third correction is almost negligible for shallow impurities. It can be shown that for an ideal 2D system the Landau level broadening, Γ is small compared to $\hbar\omega_c$ and independent of l . For each level, Γ is the same (Aoki, 1987) and given by,

$$\Gamma \approx \frac{\hbar\omega_c}{\sqrt{\omega_c \tau_0}} \quad (2.5)$$

where τ_0 is the scattering relaxation time in the absence of the magnetic field. Therefore for an ideal 2D system, for strong magnetic field and low temperatures $\omega_c \tau_0 \gg 1$, the Landau subbands are well separated (Aoki, 1987) and then the broadening effect of $V(r)$ can also be neglected.

The term $H_{e,e} = V_{ex} + V_H$ is the most important term which gives rise to the exchange and the correlation effects. In metals the energies corresponding to V_{ex} and V_H have been calculated in detail (Pines, 1963). It has been shown that exchange energy, $\varepsilon_{ex} \approx -(2-3) eV$ while the correlation energy is almost the same for all metal: $\varepsilon_{corr} \approx -0.258 eV$. So

for metals $\frac{\varepsilon_{corr}}{\varepsilon_{ex}} \approx 0.1$. Therefore ε_{corr} can be neglected when it is compare to ε_{ex} . We believe that this is also true for 2D systems. and the exchange energy is the dominant part of the energy E' corresponding to the perturbing potential H' :

$$E' \cong \varepsilon_{ex} \quad (2.6)$$

The exchange energy in a 2D system was first calculated by Stern (1972). He calculated the exchange energy in Si inversion layers. The exchange energy at the Fermi level was found to be at the order of ~ 10 meV. Therefore as will be discussed in section 3, for an ideal 2D system, for a high enough magnetic field it is possible to make ε_{ex} smaller than the Landau level separation $\hbar\omega_c$ and hence the energies given in

eq.(2.2). So the effect of H' in total Hamiltonian can be lumped in the effective mass, m^* ; the effective Landé-g factor, g^* ; and the effective magnetic field (local magnetic field), B^* . We will call this approach the effective parameters (m^* , g^* , B^*) approximation. In this approximation with an analogy to eq.(2.2) the eigenvalues corresponding to eq.(2.1) are

$$E_l(\uparrow) = (l + \frac{1}{2})\hbar\omega_c^* - \frac{g^*}{4}\hbar\omega_c^* \quad (2.7a)$$

$$E_l(\downarrow) = (l + \frac{1}{2})\hbar\omega_c^* + \frac{g^*}{4}\hbar\omega_c^* \quad (2.7b)$$

where $\omega_c^* = \frac{e\hbar}{m^*}$ is the effective cyclotron angular frequency.

Next we define the dimensionless energy, $\frac{E_l}{\hbar\omega_c^*}$ which is the conventional filling factor ν . From eq.(2.7) the plots of $\frac{E_l(\uparrow)}{\hbar\omega_c^*}$ and $\frac{E_l(\downarrow)}{\hbar\omega_c^*}$ for different l values against $|g^*|$ give us the diagram in Fig.1.

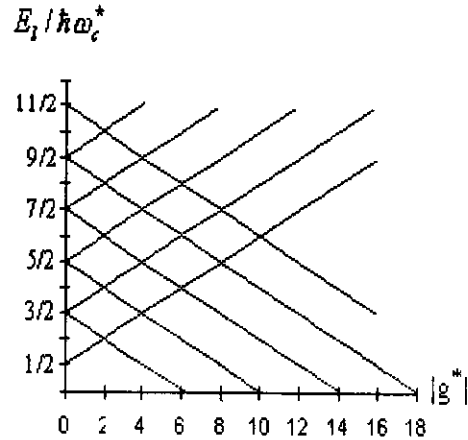


Fig.1: The plots of $\frac{E_l(\uparrow)}{\hbar\omega_c^*}$ and $\frac{E_l(\downarrow)}{\hbar\omega_c^*}$ against $|g^*|$

As it is seen from fig.1, for certain values of $|g^*|$ two different Landau levels $E_l(\uparrow)$ and $E_l(\downarrow)$ will have the same energy or will be correlated to each other. Namely, an electron with the energy $E_l(\uparrow)$ will be correlated to the electron with the energy $E_l(\downarrow)$. The correlation corresponds to the crossing points of two different Landau levels in the figure. At these crossing points $|g^*|$ only takes even integer values such as 0, 2, 4, 6, ... and so on. In addition the average $|g^*|$ values corresponding to the same energy will be an even integer also: $\langle g^* \rangle = 2n$ ($n=1,2,3,\dots$). In Table.1 we give the list of allowed $|g^*|$ values and associated $\langle g^* \rangle$

corresponding to same $\frac{E_l}{\hbar\omega_c^*} = \nu$.

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Table 1: The List of Allowed $|g^*|$ Values and Associated $\langle g^* \rangle$ Corresponding to Same $\nu = 5$

ν	$ g^* $								$\langle g^* \rangle$
1	2								2
3/2		4							4
2	2		6						4
5/2		4		8					4
3	2		6		10				6
7/2		4		8		12			6
4	2		6		10		14		8
9/2		4		8		12		16	8
5	2		6		10		14		10
							18		10

So far we have not considered Landau level broadening. (Nicholas *et al.*, 1988; Englart and von Klitzing, 1980) experimentally determined the width of the Landau levels, which were independent of Landau level indexes. The present study also assumes that each Landau level has the same broadening, Γ .

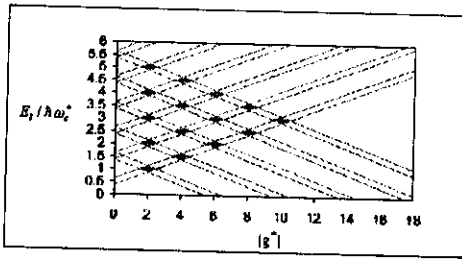


Fig 2: The plots of $\frac{E_i(\uparrow)}{h\omega_c}$ and $\frac{E_i(\downarrow)}{h\omega_c}$ for different l values against $|g^*|$ in the presence of the Landau level broadening ($\frac{\Gamma}{h\omega_c} = \frac{1}{8}$)

In Fig.2 the plots of $\frac{E_i(\uparrow)}{h\omega_c}$ and $\frac{E_i(\downarrow)}{h\omega_c}$ for different l values against $|g^*|$ are given on the basis of the assumption that $\frac{\Gamma}{h\omega_c} = \frac{1}{8}$.

As it is seen, the crossing of two broadened Landau levels occur in shaded diamond shaped (rhombus) areas whose centers correspond to the crossing points of figure 1. It is also noted that the different g^* values corresponding to the same energy are evenly distributed about the central points. They lie in range $2n - \Delta g^*$ and $2n + \Delta g^*$ where Δg^* is $\frac{4\Gamma}{h\omega_c}$. Then the

average of g^* values in each rhombus, g_{av}^* , will be equal to $2n$ again:

$$g_{av}^* = 2n \quad (n=1,2,3,\dots) \quad (2.8)$$

With a similar argument we discussed earlier, the average of g_{av}^* values corresponding to the same energy will be an even integer again. Therefore the Landau level broadening doesn't change the average g^* values which are even integers.

Results and Discussions

The effective Landé-g factor of two dimensional systems has been calculated on the basis of the effective parameters (m^* , g^* , B^*) approximation. The validity of this approximation depends on the smallness

of the Landau level broadening and the exchange energy compared to the Landau level separation $\hbar\omega_c$. From eq.(2.5), the ratio of Landau level broadening Γ to the spacing of the Landau levels, $\hbar\omega_c$, is expressed as $\frac{\Gamma}{h\omega_c} \approx \frac{1}{\sqrt{\omega_c \tau_0}}$. In Si MOS systems $m=0.2m_0$, $\tau_0=8$

ps. For $B=20$ T we get $\hbar\omega_c=11$ meV and $\Gamma \sim 1$ meV. For GaAs-Al_xGa_{1-x}As systems ($m=0.067m_0$, $\tau_0=60$ ps), $B=20$ T gives $\hbar\omega_c=33$ meV and $\Gamma=0.73$ meV. The exchange energy, ϵ_{ex} , in Si inversion layers was calculated by Stern (1972). The exchange energy at the Fermi level was found to be at the order of ~ 10 meV. Therefore for high enough magnetic fields it is possible to make ϵ_{ex} and Γ smaller than the Landau level separation $\hbar\omega_c$ and hence the Landau energies

of the unperturbed Hamiltonian. In this study we did not concentrate on the effective magnetic field (or the local magnetic field which is the true magnetic field at the position of the electron). We expect a larger effective magnetic field which makes $\hbar\omega_c$ larger than the applied magnetic field values. A more complete report of this work will be published in the future.

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