Pakistan Journal of Applied Sciences 2(8): 843-844, 2002

Generalized Logistic Distribution: An Application to the Maximum Annual Rainfalls

[1]M.T.Rasool, [2]M. Arshad and [3]M.I. Ahmad
[1]Government College, Samundri, District Faisalabad, Pakistan
[2]College of Management Sciences, University of Azad Kashmir, Kotli
[3]Department of Mathematics and Statistics Sultan Qaboos University, Musqat, Oman

Abstract: An application for Generalized Logistic Distribution (GLD) estimation by Probability Weighted Moments (PWM) was made on 24-hours maximum rainfall events recorded for different cities of Pakistan to Faisalabad, Mianwali Khanpur Khushab, Murree, Bhawalpur, Jehlum Islamabad and Sialkot. The goodness of fit of GLD estimated by PWM method was examined by Anderson Darlings' g Statistic. The hypothesis of GLD as the assumed distribution was accepted for every city.

Key Words: Generalized Logistic Distribution, Probability Weighted Moments

Introduction

In the past, two parameter distributions such as Extreme Value, Pareto and Lognormal etc. were used to analyse hydrologic and meteorological data sets but in recent years, three parameters generalized forms of the distributions such as Generalized Extreme Value (Hosking *et al.*, 1985), Generalized Pareto (Moharram *et al.*, 1993), three parameter Lognormal (LaRiccia *et al.*, 1983) etc. were studied.

The use of three parameter distributions involves an additional parameter. Three parameter distributions are seen to be superior than two parameter distributions in the study of extreme events. For instance (Sangel and Kallio 1977), (McMohan and Srikanthan 1981) recommended Log-Pearons-3 distribution for flood frequency analysis in Australia. United States Water Resources Council (1977) also suggested Log-Pearson-3 distribution. British Natural Environment Research Council (NERC, 1975) recommended the Generalized Extreme Value Distribution for flood and rainfall analysis in Britian.

Currently favored methods of estimation of the parameters and quantiles of the distribution are method of moments, method of probability weighted moments, method of ordinary least squares and method of generalized least squares etc.

Direct application of probability weighted moments method can be used only for estimating parameters of several distributions whose cumulative distribution function can be expressed in inverse form (Greennwood *et al.*, 1979). However,(Song *et al.*, 1988) pointed out that probability weighted moments method can be applied to the Pearson's type III distribution, although. This distribution is not expressible in an explicit inverse form. (Ding *et al.*, 1988) suggest the formula for computing probability weight moments in terms of a non-simple sample (with historical information). Such studies have extended the domain of function of probability weight moments method.

In the present study, the Generalized Logistic Distribution is applied to 24 hours annual maximum rainfall data available for different cities of Pakistan. The parameters of the distribution are estimated by the method of probability weight moments. The goodness of fit is tested by Anderson Darlings' Test Statistic.

Materials and Methods

The Generalized Logistic Distribution belongs to Burr system of distributions (Burr, 1942). The probability density function of the Generalized Logistic Distribution is given as:

$$f(x)=\frac{\alpha^{-1}e^{-(1-k)y}}{(1+e^{-y})^2}$$

where $y = -k^{-1}\log\left\{1-\frac{k(x-\xi)}{\alpha}\right\} \quad k \neq 0$

$$= \frac{x-\xi}{\alpha} \qquad k = 0$$

$\xi + \alpha/k \le X \le \infty$ if $k < 0$
$-\infty < X < +\infty$ if $k = 0$
$-\infty < X \le \xi + \alpha/k$ if $k > 0$

where
ξ = location parameter;
α = scale parameter;
k = shape parameter;

The Probability Weight Moments Estimates of the Generalized Logistic Distribution: Rasool *et al.*, (1994) has estimated the probability weight moments of the Generalized Logistic Distribution as under:

$k = -\lambda_3/\lambda_2$,
$\alpha = \lambda_2/\Gamma(1-k)\,\Gamma(1+k)$
$\xi = \lambda_1 - \alpha\,[1 - \Gamma(1-k)\,\Gamma(1+k)] / k$
where
$\lambda_1 = \xi + \alpha[1 - \Gamma(1-k)\,\Gamma(1+k)] / k$
$\lambda_2 = \alpha\,\Gamma(1-k)\,\Gamma(1+k)$
$\lambda_3 = -k\,\alpha\,\Gamma(1-k)\,\Gamma(1+k)$

Table 1: Estimates of the Parameters (S.D)* by Probability Weighted Moments of Generalized Distribution for Annual Maximum Rainfall Data

City	ξ	α	K	A_n^2
Faisalabad	60.00(3.96)	13.73(2.35)	-0.198(0.145)	0.1592
Mianwali	53.78(8.49)	17.36(5.63)	-0.335(0.321)	0.2966
Khanpur	24.77(6.81)	12.13(4.49)	-0.327(0.366)	0.265
Khushab	54.69(7.06)	15.61(4.60)	-0.299(0.293)	0.2431
Murree	79.47(3.36)	11.22(2.24)	-0.039(0.148)	0.3508
Bahawalpur	35.61(3.44)	11.30(2.16)	-0.144(0.338)	0.3117
Jehlum	69.43(8.04)	15.58(5.05)	-0.254(0.308)	0.2332
Islamabad	95.22(9.39)	21.84(6.36)	-0.092(0.212)	0.2289
Sialkot	75.05(14.5)	25.34(9.48)	-0.319(0.374)	0.3047

*SD stands for standard deviation of the estimates and given in the brackets.

Anderson Darling Test Statistic: Anderson and Darling (1952) introduced the class of statistics defined by:

$$\int_{-\infty}^{+\infty} [F_n(x) - F(x)]^2 \phi(x) dF(x)$$

where $\phi(x)$ is a pre-assigned weighted function.
In the particular case

$$\phi(x) = [F(x)\{1 - F(x)\}]^{-1}$$

is usually known as Anderson-Darling test statistic and denoted by A_n^2

This statistic emphasizes discrepancies in both tails and should be powerful against alternatives in which $F_n(x)$ and $F(x)$ disagree near the tail of $F(x)$. In rainfall intensities analysis such discrepancies often of prime importance. Therefore, to test the estimated parameters, Anderson Darlings test statistic is used.
For computational purposes, we use an equivalent form

$$A_n^2 = -\Sigma[(2i-1)/n)]\{\log F(x_i) + \log(1-F(x_{n-1-i}))\}$$

where in our application F(.) will involve estimated parameters.

Rainfall Data: The study utilizes data recorded from nine cities of Pakistan for which almost continuous daily rainfall records were available from the Regional Meteorological Officer, Lahore. The 24-hours annual maximum rainfalls (in millimeters) was sorted out from this record and used to estimate the parameters of the Generalized Logistic Distribution by Probability Weighted Moments (PWM).

Results and Discussion

The Anderson Darlings statistics were calculated and compared with the table values given by (Ahmad 1988) and found to be non-significant for the Generalized Logistic Distribution. The estimates of the parameters by (PWM) and the standard deviations of the estimated of the Generalized distribution for annual maximum rainfall data for different cities of Pakistan are given in Table1.

References

Ahmad, M.I. 1988. Application of Statistics in Flood Frequency Analysis. Dept. of Math. Sci., Div. of Statistics, Univ. St. Andrews, U.K.

Anderson, T.W. and D.A. Darling, 1952. Asymptotic Theory of Certain Goodness of Fit Criteria Based on Stochastic Processes. Ann-Math. Stat., 23: 193-212.

Burr, I.W., 1942. Cumulative Frequency Function. Ann. Math. Stat., 13, 215.

Ding, J. and R. Yang, 1988. The Determination Of PWM With the Incorporation of Extraordinary Values into Sample Data and Their Application to Estimating Parameters for the Pearson Type III Distribution. J. Hydrol., 101: 63-81.

Greenwood, J.A., J.M. Landwehr, M.C. Matalas and J.R. Wallis, 1979. PWM; Definition and Relation to Parameters of Several Distributions Expressible in Inverse Form. Water Resources Res., 15: 1049-1054.

Hosking, J.R.M., 1985. Estimation of Generalized Estimate Value(GEV) Distribution by the Method of PWM. Technometrics 27: 251-261.

Larricia, N. and R.P. Kinderman, 1983. An Asymptotically Efficient Closed Form Estimation for the Three Parameter Lognormal Distribution. Comm. Statist. Theor. Meth., 12: 243-261.

McMohan, T.A. and R. Srikanthan, 1981. Log Pearson III Distribution is it Applicable to Flood Frequency Analysis of Astralian Streams? J. Hydral., 52: 139-147.

N.E.R.C. 1975. Flood Studies Report. Vol.I Natural Environ. Res. Council HMSO, London.

Rasool, M.T. 1994. Estimation of the Generalized Logistic Distribution(GLD) by PWM. Univ. Agric., Faisalabad, Pakistan.

Sangel, B.P. and R.W. Kallio. 1977. Magnitude and Frequency of Flood in Southern Ontario. Tech. Bull., 99. Fish and Environ., Ottawa.

Song, D. and J. Ding, 1988. The Application of PWM in Estimating the Parameters of Pearson Type III Distribution. J. Hydrol., 101: 47-61.

US Water Resources Council, 1977. Guidelines for Determining Flood Flow Frequency. Hydrol. Comm. US Water Resource Council, Washington D.C.