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Bounds of the Structural Response Imprecisely-defined Systems under Earthquake Action

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Abstract: Structures with finite dynamic degree of freedom and fuzzy geometrical and physical parameters are analyzed under earthquake action, membership functions of quality characteristics have been established, deflections of the fuzzy results as ratios, of the deterministic are estimated with percents, safety criterion and its estimating is given. Experimental test is made by using orthogonal array Taguchi's method. For each α -cut level the equations of motion are solved using step by step integration in the time domain by the numerical Wilson-θ's method. Some operations with fuzzy parameters are provided by using extension principle (Zadeh, 1965). Numerical examples are given and the results are explained.

Key Words: Earthquake Action, Fuzzy Set, Taguchi Method, Finite Element Analysis

Introduction

As is known, the analysis under seismic action of special structures necessitates the use of actual records or simulated acceleration of ground. In this analysis, one of crucial points is, the estimation of sensitivity of reaction of a structure, with respect to the uncertainties in geometrical and physical parameters (height of a floor, mass, rigidity, damping etc....). In the applications, uncertainties are classified as being random and fuzzy. In the submitted work in a direction of each dynamic degree of freedom (d. d. f) mass, height of a floor, module of elasticity is accepted by fuzzy quantities, with triangular membership function and are replaced by the α - cuts. Thus, for every α -cut is constructed the equation of motion. Despite of the availability of investigations on fuzzy equations (Buckley, 1992), practical methods for the applications is rather limited. In some stages numerical realization of a considered (examined) problem, use of interval arithmetics for every α -cut (based on a principle of expansion Zadeh) is very difficult to be realized, because borders of reactions (target parameters) of the system are undefined. In this study the marked difficulties of transformation connected to the interval analysis have been overcome for every α -cut by application the Taguchi method .Thus factors, on whose variability the system is sensitive to, are replaced for each α -cut and for each stage of a experiment combination of constructed equation that is solved by a numerical Wilson-θ method. Membership functions for the system (structure) reactions, deviation with respected to the deterministic values, combination number equivalent to system critical state and equivalent time during the effective earthquake interval have been obtained.

Related to this study , random vibration of the system with fuzzy parameters in one d. d. f system application has been investigated in (Wang, 1990) ; eigen vector and eigen value problems for the multi degree of freedom systems with fuzzy parameters has been investigated in (Chen, 1997) and (Lullemend, 1999) respectively. Investigation of system reactions coupling fuzzy logic with statistics has been investigated in (Wadia - Foscetti,2000). Estimation of the relibilaty of the structure foundation - ground (with stochastic characteristics) systems under seismic action has been investigated in (Kasumov, 1999).

Formulation: More sensitive fuzzy parameters of reactions of structure, namely height of floor (L_i), elasticity module of columns (E_i), mass (m_i) in the direction of i- th d. d. f. are accepted to have two level triangular membership functions μ(x).

$$\mu(x)=\begin{cases}0, & if\ x\le x_L\ or\ if\ x_R\le x\\ f_1(x), & if\ x\ge x_L\quad x\le x_P\\ f_2(x), & if\ x\ge x_P\quad x\le x_R\end{cases}\qquad(1)$$

$$f_1(x)=\frac{x-x_L}{x_P-x_L}$$

$$f_2(x)=\frac{x_R-x}{x_R-x_P}$$

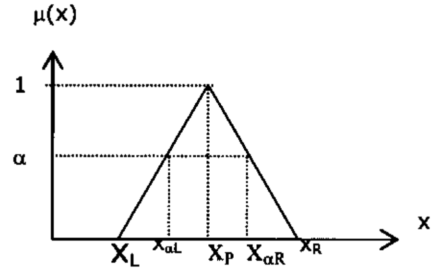

Fig.1: Fuzzy Number With Triangular Membership Function and its α-Cut

After that, parameters are fuzzyfied, using full factorial and orthogonal array testing plan for each α-cut level and are shown in Table 1, 2 (the rigidity of storey K_i (E_i, L_i) is periphery related with E_i and L_i factors). Testing plan interaction for factors between storeys using orthogonal array is shown in Table 3 (only for the imbricated storeys (d. d. f.), interaction of the rigidity factors are accepted).

Table 1: Testing plan Table with Full Factorial Combination

Factors / Number of Testing Combination	L_i	E_i	(K_i) L_i xE_i	m_i
1	1	1	1	1
2	1	1	1	2
3	1	2	2	1
4	1	2	2	2
5	2	1	2	1
6	2	1	2	2
7	2	2	1	1
8	2	2	1	2

In the Table number 1 and number 2 are left and right positions for α - cut level of fuzzy numbers.

A fuzzy relation f (AxB) from fuzzy set

$X = \left[\frac{\mu_X(x)}{x}\right]$ to a fuzzy set $Y = \left[\frac{\mu_Y(y)}{y}\right]$ is a

fuzzy subset of the Cartesian product XxY, which is a mapping from X to Y. A fuzzy relation f (AxB) is expressed as (Zadeh's extension principle):

$$f(AxB) = \sum_{x \in X} \sum_{y \in Y} \frac{\min(\mu_A(x), \mu_B(y))}{f(x,y)} \quad (2)$$

Table 2: Testing plan Table in Orthogonal Array

Factors / Number of Testing Combination	L_i	E_i	m_i
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Fuzzy arithmetic is based on the Zadeh's extension principle (2). The computational features of the extension principle can be achieved by using the α-cut representation of fuzzy numbers.

For each α-cut level fuzzy parameters for the testing combination number according to the critical situation, in interaction between two storeys (d.d.f.) equation of motion are arranged as:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [K]\{u\} = \{F_*\} \quad ;$$
$$F_{i*} = -m_i \ddot{x}_g(t) \quad (3)$$

Where [m],[c],[K] are mass, rigidity and damping matrix of system, if the problem is aimed to be solved using the finite element method, they are expressed in the form :

$$[m] = \sum [m_i]$$
$$[K] = \sum [K_i] \quad (4)$$

Where [m_i], [K_i] are mass and rigidity of the systems in the i-th d.d.f. When given vector {ξ} with damping ratio elements for the each d.d.f., damping coefficient matrix is expressed as:

$$[c] = [m] \sum_{i=1,n} a_i \left([m]^{-1}[K]\right)^i \quad (5)$$
$$\{a\} = [\omega]^{-1} 2\{\xi\};$$
$$\{\xi\} = (\xi_1 \xi_2 \ldots\ldots \xi_n)^T$$
$$[\omega] = \begin{bmatrix} \omega_1 & \omega_1^3 & \omega_1^5 & . & . & \omega_1^{2n-1} \\ \omega_2 & \omega_2^3 & \omega_2^5 & . & . & \omega_2^{2n-1} \\ . & . & . & . & . & . \\ \omega_n & \omega_n^3 & \omega_n^5 & . & . & \omega_n^{2n-1} \end{bmatrix}$$

Where ω_i , frequency of structure for i-th vibration mod ; n , total d.d.f. of system.

{F_*}, $F_{*i} = -m_i \ddot{x}_g(t)$, $\ddot{x}_g(t)$,

dynamic force in the i-th d.d.f. of the system evaluated under ground acceleration.

Solution: For each α-cut of the fuzzy parameters testing, combination number accepted for the critical position of the storey interaction, the equation of motion (3) is solved using step by step integration in the time domain using the Wilson-θ's method. As a critical situation ,the time, for the maximal overturning moment in the base of structure (in n_e-th testing $t_{*e} = t_{max\ (Mov)}$) is accepted. In critical situation $e_{No,*}$ (t_{*e}), for each α-cut of fuzzy parameters, response of the system is expressed in the form:

$$u_e, \dot{u}_e, \ddot{u}_e, F_{ue}(:, n_e) = u, \dot{u}, \ddot{u}, F_u(:, e_{No,*}) \quad (6)$$

Where the designation u(:,j) means j-th column of a matrix u.For the n_e=1-n_{ex} number testing combination, displacement (u), velocity $(\dot{u})$, acceleration $(\ddot{u})$, seismic force (F_u), overturning moment (M_{ov}),..are expressed in the form:

Table 3: Testing plan Table for Interaction Between Factors of 2 Storeys in Orthogonal Array

Factors / Number of Testing Combination	L_i	E_i	m_i	Testing Results
1	1	1	1	
2	1	2	2	
3	2	1	2	
4	2	2	1	
5	2	2	2	

Factors No	L_{i+1}	E_{i+1}	m_{i+1}
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1
5	2	2	2

$$u_e, \dot{u}_e, \ddot{u}_e, F_{ue} \ldots = \underbrace{\left[\quad\right]}_{n_{ex} = \text{number of testing combination}} \Big\} k = \text{number of } d.d.f. \quad (7)$$

$$M_{OV} = \overbrace{\left[\quad\right]} \Big\} 1\,row \quad (8)$$

Right ($M_{ov,R}$) and left ($M_{ov,L}$) values of the overturning moment and numbers of testing for this values $e_{No,L}$, $e_{No,R}$ are found for i_i-th α-cut, based on extension principle:

$[M_{ov,L}, e_{No,L}]=\min(M_{ov,e})$
$[M_{ov,R}, e_{No,R}]=\max(M_{ov,e})$ (9)

Response of the structures (displacement, velocity, acceleration,...) which correspond to testing numbers $e_{No,L}$, $e_{No,R}$ for the left and right values for each i_i-th α-cut level are evaluated as (the designation u(:,[i j]) means i-th and j-th columnes of a matrix u) :

$$u_{i,e}(:[2(i_i-1)\ \ 2i_i])=[\underbrace{u_e(:e_{NoL})}_{left}\ \ \underbrace{u_e(:e_{NoR})}_{right}] \quad (10)$$

....

$$M_{ovl_i,e}(:[2(i_i-1)\ \ 2i_i])=[\overbrace{M_{ovL}}^{left}\ \ \overbrace{M_{ovR}}^{right}] \quad (11)$$

After i_i=1- m_{ns} time realization matrices (10), (11) is expressed in the form:

$$\left.\begin{matrix} u_{i,e} \\ \dot{u}_{i,e} \\ \ddot{u}_{i,e} \\ F_{u,l_i,e} \\ \ldots \end{matrix}\right\} = \underbrace{\begin{bmatrix} \overbrace{}^{1st\,\alpha-section} & \overbrace{}^{2nd\,\alpha-section} & \overbrace{}^{m_{ns}\,\alpha-section} \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \end{bmatrix}}_{m_{ns}\,x2\ \ number\,columns} \Big\} k\ \ number\,d.d.f. \quad (12)$$

$$M_{ovl_i,e} = \overbrace{\left[\quad\right]} \Big\} 1\,row \quad (13)$$

According to these results, the membership functions of the system response and its deflections regarding deterministic response, are calculated easily.

Numerical Analyses: The shear building shown in Fig.2 with its physical and geometrical parametres given below is investigated:

$$L_1 = (L_{1L} = 0.09L_{1P} \quad L_{1P} = 4.572 \quad L_{1R} = 1.01L_{1P});(metre)$$
$$L_2, L_3, L_4, L_5 = (L_{2L} = 0.09L_{2P}\ L_{2P} = 3.0481\ L_{2R} = 1.01L_{2P});$$
$$m_1 = (m_{1L} = 0.95m_{1P}\ m_{1P} = 23826.8\ m_{1R} = 1.05m_{1P});(Newton)$$
$$m_2, m_3, m_4, m_5 = (m_{2L} = 0.95m_{2P} \quad m_{2P} = 11563.0 \quad m_{2R} = 1.05m_{2P});$$

$$J_1, J_2, J_3, J_4, J_5 = (2J_0 2J_0\, 2J_0\ 2J_0\ 2J_0)^T; J_0 = 2.0695.10^{-4} m^4$$
$$\{\xi\} = (0.05 \quad 0.05 \quad 0.05 \quad 0.05 \quad 0.05)^T$$
$$E_i = [E_{iL} = 0.95E_{iP} \qquad E_{iP} = 1.0349\ 10^{11} \qquad E_{iR} = 1.05E_{iP}];(N/m^2);(i = 1 \sim 5)$$

First 10.19 seconds records for the East-West component of 1940 El Centro earthquake is accepted as ground acceleration. Integration time step:Δt=0.02, parameter of θ=1.4 (method becomes unconditionally stable) is accepted.

Table 4: Results of Deterministic Analyzes for Critical (max|M_{ov}|)($e_{No,*}$) Testing Situation

d.d.f . N_o	u(m)	F_u(N)	M_{ov},(Nm) t_{Mov},(sn)
1	-0.0533	-26167.1357	M_{ov}=
2	-0.0869	-38318.5721	=-3524639.571
3	-0.1156	-62516.4520	
4	-0.1362	-76877.5522	t_{Mov}=4.76
5	-0.1469	-83009.1439	

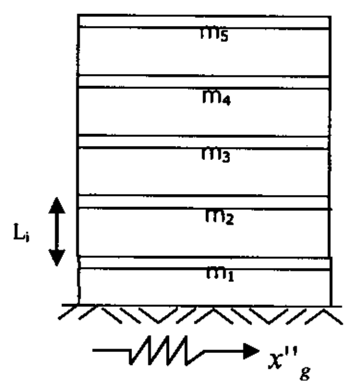

Fig.2: 5 Story Shear Building

Table 5: Maximum System Response with Fuzzy Parameters L(%1),E(%5),m(%5), (Case "A")

d.d.f No	Results of the fuzzy analyzes consequant for testing case: min, max(M_{ov}), ($e_{No,L}$,$e_{No,R}$) (max. interval from membership functions is given)					Deviation of fuzzy results regarding to deterministic results (for the left and right deviation values)in percent				
	u_L (m)	u_R (m)	$F_{u,L}$ (N)	$F_{u,R}$ (N)	$M_{ov,L,R}$ $t_{Mov,L,R}$	u_{Ls}	u_{Rs}	$F_{u,Ls}$	$F_{u,Rs}$	$M_{ovL,R,S}$
1	-0.0560	0.0623	-28626.2068	59104.4959	$M_{ov,L}$=-3772498.71	5.0459	16.8049	9.3976	125.8730	$M_{ovL,S}$ = 7.0322
2	-0.0911	0.0981	-38318.5721	46704.9441	$M_{ov,R}$= 3987892.38 $t_{Mov,L}$=4.76	4.7589	12.8642	0	21.8859	$M_{ovR,S}$ =13.1433 min,max M_{ov}
3	-0.1215	0.1280	-64568.5358	64467.0802	$t_{Mov,R}$=6.04	5.1257	10.7580	3.2825	3.1202	consequant
4	-0.1439	0.1498	-82840.4293	80876.1898		5.6912	9.9798	7.7563	5.2013	testing No: e_L=1 e_R=5
5	-0.1557	0.1613	-89903.6976	92454.5232		6.0050	9.8227	8.3058	11.3787	

Table 6: Maximum System Response with Fuzzy Parameter L(1%), (Case"B")

d.d.f. No	Results of the fuzzy analyzes consequant for testing case:min,max(M_{ov}), ($e_{No,L}$,$e_{No,R}$) (max. interval from membership functions is given)					Deviation of fuzzy results regarding to deterministic results (for the left and right deviation values)in percent				
	u_L (m)	u_R (m)	$F_{u,L}$ (N)	$F_{u,R}$ (N)	$M_{ov,L,R}$ $t_{Mov,L,R}$	u_{Ls}	u_{Rs}	$F_{u,Ls}$	$F_{u,Rs}$	$M_{ovL,R,S}$
1	-0.0549	0.0593	-27658.1708	57987.8509	$M_{ov,L}$=-3592855.91	2.8630	11.2088	5.6981	121.6056	$M_{ovL,S}$ = 1.9354
2	-0.0896	0.0932	-38610.4342	42555.6348	$M_{ov,R}$= 3774490.09	3.0121	7.1825	0.7617	11.0575	$M_{ovR,S}$ = 7.0887
3	-0.1196	0.1217	-62589.0867	59784.2993	$t_{Mov,L}$=4.76	3.4310	5.2999	0.1162	4.3703	min, max M_{ov}
4	-0.1414	0.1428	-78895.6469	78443.1992	$t_{Mov,R}$=6.04	3.8218	4.8381	2.6251	2.0365	consequant
5	-0.1528	0.1540	-85622.5692	90040.5798		4.0004	4.8680	3.1484	8.4707	testing No: e_L=1 e_R=3

Table 7: Maximum System Response with Fuzzy Parameter E(5%), (Case "C")

d.d.f No	Results of the fuzzy analyzes consequant for testing case: min,max(M_{ov}), ($e_{No,L}$,$e_{No,R}$) (max. interval from membership functions is given)					Deviation of fuzzy results regarding to deterministic results (for the left and right deviation values)in percent				
	u_L (m)	u_R (m)	$F_{u,L}$ (N)	$F_{u,R}$ (N)	$M_{ov,L,R}$ $t_{Mov,L,R}$	u_{Ls}	u_{Rs}	$F_{u,Ls}$	$F_{u,Rs}$	$M_{ovL,R,S}$
1	-0.0560	0.0609	-28177.0207	59916.7063	$M_{ov,L}$=-3559461.43	5.0549	14.2654	7.6810	128.9769	$M_{ovL,S}$= 0.9880
2	-0.0910	0.0959	-38318.5721	45545.1874	$M_{ov,R}$= 3992408.32	4.7333	10.3045	0	18.8593	$M_{ovR,S}$= 13.2714
3	-0.1215	0.1252	-62516.4520	63451.4039	$t_{Mov,L}$=4.76	5.0735	8.3531	0	1.4955	min,max M_{ov}
4	-0.1438	0.1468	-79890.5679	81951.9576	$t_{Mov,R}$=6.04	5.6195	7.7989	3.9192	6.6006	consequa
5	-0.1556	0.1583	-87703.0938	93801.8323		5.9233	7.7860	5.6547	13.0018	testing No: e_L=1 e_R=2

Table 8:Maximum System Response with Fuzzy Parameter m(5%), (Case "D")

d.d.f. No	Results of the fuzzy analyzes consequant for testing case:min,max(M_{ov}), ($e_{No,L}$,$e_{No,R}$) (max. interval from membership functions is given)					Deviation of fuzzy results regarding to deterministic results (for the left and right deviation values) in percent				
	u_L (m)	u_R (m)	$F_{u,L}$ (N)	$F_{u,R}$ (N)	$M_{ov,L,R}$ $t_{Mov,L,R}$	u_{Ls}	u_{Rs}	$F_{u,Ls}$	$F_{u,Rs}$	$M_{ovL,R,S}$
1	-0.0549	0.0612	-28504.7352	57791.2565	$M_{ov,L}$=-3726854.21	2.8720	14.6812	8.9333	120.8543	$M_{ovL,S}$= 5.7372
2	-0.0899	0.0962	-38318.5721	43499.0479	$M_{ov,R}$= 3815161.08 $t_{Mov,L}$=4.76	3.4051	10.6843	0	13.5195	$M_{ovR,S}$= 8.2426 min,max M_{ov}
3	-0.1208	0.1257	-63454.8423	60583.1947	$t_{Mov,R}$=6.04	4.4716	8.7187	1.5010	3.0924	consequant
4	-0.1435	0.1473	-84000.3972	78305.2889		5.3548	8.1627	9.2652	1.8572	testing No: e_L=1 e_R=2
5	-0.1554	0.1589	-92371.2870	89643.9132		5.7862	8.1503	11.2784	7.9928	

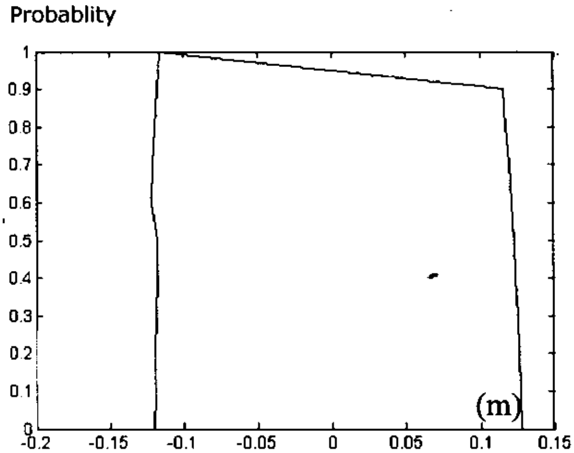

Fig 3. Membership function of the displacement(u)in the 5th d.d.f. (Case "A")

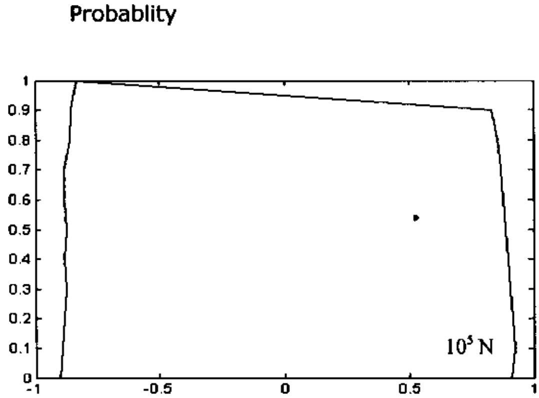

Fig 4. Membership function of the lateral force (F_u) in the 5th d.d.f.(Case"A")

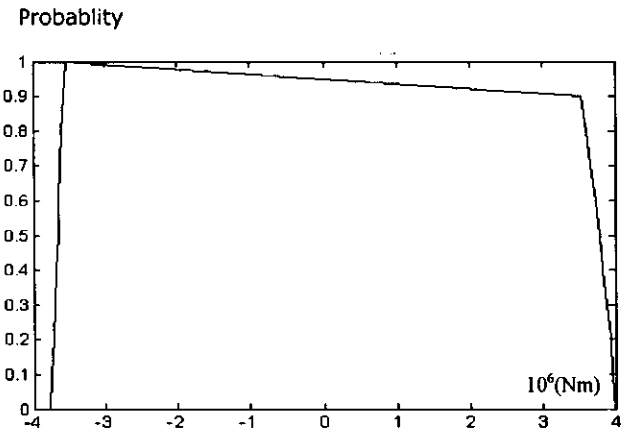

Fig 5. Membership function of the overturning moment(M_{ov}), (Case "A")

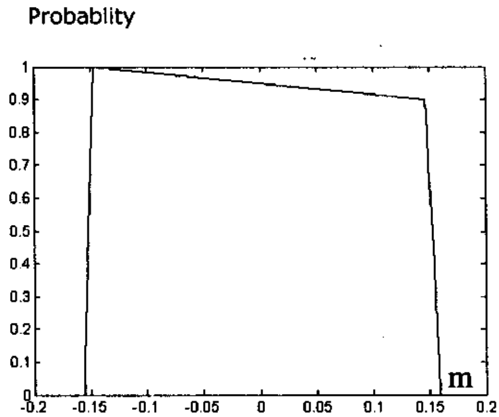

Fig 6. Membership fonction of the displacement (u) in the 5-th d.d.f. (Case D")

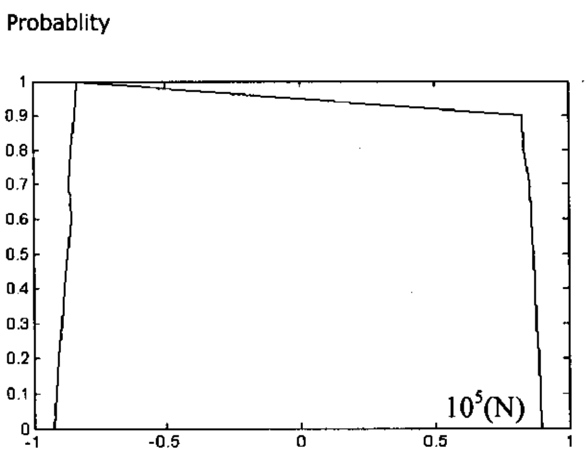

Fig 7. Membership fonction of the lateral force (F_u)in the 5-th d.d.f. (Case"D")

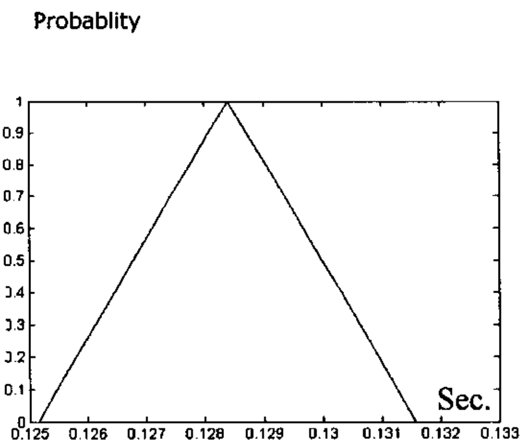

Fig 8. Membership fonction of the period (T) (5-th d.d.f.similer 5th mode) (Case "D")

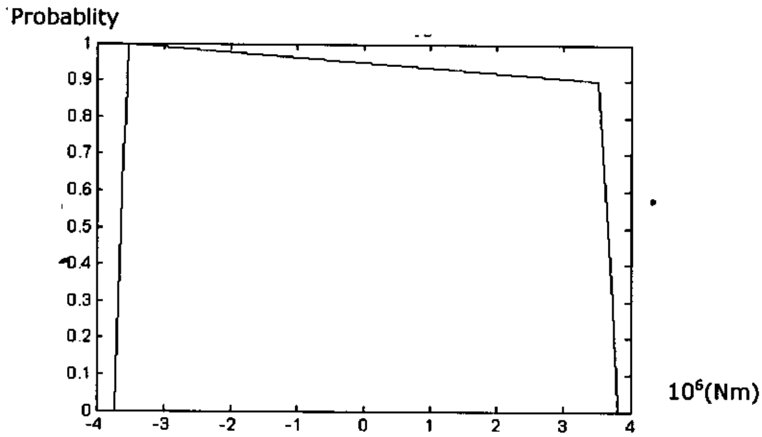

Fig. 9: Membership Fonction of the Overturning Moment, (Case"D")

Table 9: Deviation of Fuzzy Overturning Moment for the Analyzed Cases A, B, C, D

Cases	Deviation of Fuzzy Overturning Moment Regarding to Deterministic Results (for the Left and Right Deviation Values in Percent)	Testing Combination No for The $M_{ov,L}$ And $M_{ov,R}$ State Respectively
A(L(1%),E(5%),m(5%))	%7.03 $\leq M_{ov} \leq$% 13.14	$e_L = 1$ $e_R = 5$
B(L(1%))	%1.9314 $\leq M_{ov} \leq$%7.08	$e_L = 1$ $e_R = 3$
C(E(5%))	% 0.99 $\leq M_{ov} \leq$%13.27	$e_L = 1$ $e_R = 2$
D(m(5%))	%5.73 $\leq M_{ov} \leq$%8.24	$e_L = 1$ $e_R = 2$

Maximum changing interval characteristic reactions of the system with fuzzy parameters L(standard deviation from its average=1%), E(standard deviation from its average=5%), m(standard deviation from its average=5%) are given in Table 5 (case "A").
Maximum changing interval characteristic reactions of the system with fuzzy parameter L(standard deviation from its average=1% ; case "B"), the only elasticity module (standard deviation from its average=5%; case "C"), the only story masses (standard deviation from its average=5%; case "D") is given in Tables 6, 7, 8 respectivly.
The sensitivity of the structure to changes in the fuzzy parameters L, E, m for the overturning moment (comparing A, B, C, D cases) are shown below (Table 9):
That for this type structure, is a remarcable feature is more sensitive to change in the story height (case "B"). Distribution of the membership functions of the structure reactions approximately have the same identical character for the cases A, B, C, D. Membership functions for some structure reactions are given below (Fig.3-Fig.9) for the cases A, D.
As seen from the distribution of membership functions, with a probabilty of 0.7-0.9 structure response will be in the interval shown in the tables. Hence, as it is observed, in the structural analyses, it is necessitated to take into account the bounds (which exceed many times the 5% bounds) which include the highly probable (0.7-0.9) structural response that correspound to the permissible deviation (1-5%) of the structural parameters in the civil engineering field.

Conclusion

The method of analyses presented, provide a facility for determining the measures in evaluating the sensitivity of the structures, in other words, in determining the response bounds and consequent project work.

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