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Causality and Cointegration Tests in the Framework of a Single Zero-Non Zero Patterned Vector Time Series Modeling

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ABSTRACT

In cointegration theory vector error-correction models (VECMs) have become an important means of analysing cointegrating relations. Usual full-order VECMs assume all nonzero entries in their coefficient matrices. However applications of VECMs to economic and financial time series data have revealed that zero entries are indeed possible. If indirect causality or Granger non-causality exists among the variables, the use of a full-order VECM will incorrectly conclude only the existence of Granger causality among these variables. In addition, the statistical and numerical accuracy of the cointegrating vectors estimated in this mis-specified full-order VECM will be definitely doubtful. It is argued that the zero-non-zero (ZNZ) patterned VECM is a more straightforward and effective means of testing for both indirect causality and Granger non-causality. The note presents simulations and an application that demonstrate the usefulness of the ZNZ patterned VECM.

Key words: Zero-Non-Zero patterned modeling, cointegration theory, causality

INTRODUCTION

The use of vector autoregressive (VAR) models for investigating the cause and effect relationship, or simply the causality, which exists among economic and financial variables has become common in the literature. Granger (1969) introduced a definition of causality which is based entirely on the predictability of the objective variables, such that they make no explicit use of economic and financial laws to provide a priori restrictions on the structure. He then proposed to fit a VAR for empirical model building to detect Granger causality and Granger non-causality. Sims (1972, 1977) suggested that we should treat all variables as jointly dependent and then fit a vector autoregression to avoid imposing on the model spurious or false restrictions for causality testing. Subsequently Hsiao (1979, 1982) developed procedures to test Granger causality and indirect causality in a class of VAR models. Geweke (1982) recommended a means of measuring two-way linear causality and instantaneous linear causality in VAR modeling. Penm and Terrell (1984a,b) proposed algorithms to study the ZNZ patterned VAR models, which allow for possible zero entries in the coefficient matrices of VAR models. The optimal ZNZ patterned VAR model was then used as a basis for detecting Granger causality, Granger non-causality and indirect causality.

However recent cointegration work has suggested that, if a time series system under study includes cointegrated variables, then this system may be more appropriately specified as a vector error-correction model (VECM) rather than a VAR [see Engle and Granger (1987)]. The VECM is identical to the VAR model with unit roots and can be evidenced as follows:

In VAR modeling, we have the following VAR model:

$$Z(t)+\sum_{\tau=1}^{p}\Gamma_\tau Z(t-\tau)=\Gamma^p(L)Z(t)=\varepsilon(t), \qquad (1.1)$$

where $\varepsilon(t)$ is a sx1 I(0) vector process with $E\{\varepsilon(t)\} = 0$ and

$$E\{\varepsilon(t)\varepsilon'(t-\tau)\}=\Theta, \quad \tau=0,$$

$$0, \quad \tau > 0,$$

where Γ_τ, τ= 1,2,...., p are sxs parameter matrices and

$$\Gamma^p(L) = I + \sum_{\tau=1}^{p} \Gamma_\tau L^\tau$$

L denotes the lag operator and the roots of $|\Gamma^p(L)| =$ lie outside or on the unit circle.
Further, we have the following relation:

$$\Gamma^p(L) = \Gamma^p(1) + (I-L)(I + \sum_{\tau=1}^{p-1} \Gamma_\tau^* L^\tau).$$

In accordance with the concept of cointegrated variables introduced by Granger (1981), z(t) is said to be I(1) if it contains at least one element which must be differenced before it becomes I(0). Then z(t) is said to be cointegrated of order I with the cointegrating vector, β, if β'z(t) becomes I(0), where z(t) has to contain at least two I(1) variables. Under this assumption the identical VECM for (1.1) can be expressed as:

$$\Gamma^* z(t-1) + \Gamma^{p-1}(L)\,\Delta z(t) = \varepsilon(t), \qquad (1.2)$$

where z(t) contains both I(0) and I(1) variables, Δ =(I-L), $\Gamma^* = \Gamma^p(1)$, $\Gamma^* z(t-1)$ is stationary and

$$\Gamma^{p-1}(L) = I + \sum_{\tau=1}^{p-1} \Gamma_\tau^* L^\tau.$$

The first term in (1.2) is the error-correction term, which concerns the long-term cointegrating relationship. Γ^{p-1} (L) Δz (t) is referred to as the VAR part of the VECM, describing the short-term dynamics.

Because z(t) is cointegrated of order 1, the long-term impact matrix Γ^* must be singular. As a result $\Gamma^* = \alpha\beta'$, where α and β are sxr matrices and the rank of Γ^* is r, r< s. The columns of β are the cointegrating vectors and the rows of U. are the loading vectors.

Engle and Granger (1987) noted that, for cointegrated systems, the VARs in first difference will be mis-specified and the VARs in levels will ignore important constraints on the coefficient matrices. Although these constraints may be satisfied asymptotically, efficiency gains and improvements in forecasts are likely to result by imposing them. Comparisons of forecasting performance of the VECMs versus VARs for cointegrated systems were reported in studies such as Engle and Yoo (1987) and LeSage (1990). The results of their studies consistently indicated that, in the short-term, there may be gains in using the unrestricted VAR models, but the VECMs produce long-term forecasts with smaller errors when the variables used in the models satisfy the statistical tests for cointegration. Subsequently Alm and Reinsel (1990), Reinsel and Alin (1992) and Johansen (1988, 1991) proposed various algorithms for the estimation of cointegrating vectors in the full-order VECM models, which contain all non-zero entries in the coefficient matrices. Since the early 1990s, abundant literature has utilised the full-order VECM models in analysing the short-term dynamics and the longterm cointegrating relationships for cointegrated time series.

One problem encountered in empirical research using cointegration theory is to provide satisfactory financial and economic interpretation for estimated cointegrating vectors. As demonstrated by Wickens (1996) it is important to introduce a prior information, as so to produce ZNZ patterns. To explicitly address this issue Penm *et al.* (1997) have presented a search algorithm in conjunction with model selection criteria to identify the optimal specification of a ZNZ patterned VECM for an I(l) system. This VECM, with allowance for possible zero entries in the coefficient matrices, is referred to as a ZNZ patterned VECM. Given the optimal ZNZ patterned VECM, the number of cointegrating vectors can be confirmed. Once the ZNZ patterned impact matrix has been determined, along with the number of cointegrating vectors in the system, a tree-pruning procedure is then proposed for the search for all acceptable ZNZ patterns of the cointegrating and loading vectors. After this, the dynamic ordinary least squares method suggested by Stock and Watson (1993) is utilised to estimate the acceptable patterned cointegrating vectors and the regression method with linear restrictions as recommended in Penm *et al.* (1997) is conducted to estimate the acceptable patterned loading vectors. Model selection criteria are again employed to determine the optimal ZNZ patterned cointegrating and loading vectors. This algorithm leads to a neat and effective analysis of the cointegrating relations in vector time series and can be applied to higher order integrated systems.

In this note we indicate that the use of full-order VECM models in analysing time series systems can produce misleading and invalid causal inferences. Specifically, if indirect causality or Granger non-causality exists among the variables, the use of a full-order VECM to detect Granger causal relations and investigate cointegrating relations among these variables is inappropriate. This is because indirect causality detection, Granger non-causality detection and the long-term cointegration investigation are crucially dependent on the positions of zero entries in the coefficient matrices. However full-order **VECM** models assume all nonzero elements in their coefficient matrices comprising Γ^* and Γ_τ^* for τ from 1 to p-1. Thus, as full-order models cannot show either indirect causality or Granger non-causality among the variables, these models will only show the existence of Granger causality. In addition, the statistical and numerical accuracy of the cointegrating vectors estimated in this misspecified full-order **VECM** will be diminished.

Further, we demonstrate that Granger non-causality, indirect causality and the **ZNZ** patterned cointegrating vectors can be detected in the context of a single **ZNZ** patterned **VECM** framework that can contain zero entries for time series of integrated order I(d), $d \geq 1$. The Granger causal relations are not only detected from the coefficient matrices on the lagged difference terms but also from the error-correction terms.

The remainder of this note is organised as follows: Section 2 reviews causality patterns in **VAR** modeling and describes zero entries in a **ZNZ** patterned **VAR** and its identical **VECM** for an I(d) system. Causality detection in **VECM** modeling is also discussed. Section 3 conducts the simulations to support the equivalence of testing for Granger causality in a **ZNZ** patterned **VAR** and its identical **VECM**. This section also demonstrates an application concerning the detection of both Granger non-causality and indirect causality and **ZNZ** patterned cointegrating vectors in the **VECM** applied to a balanced growth case. Some concluding remarks are provided in Section 4.

MATERIALS AND METHODS

Causality patterns in var modeling

In **VAR** modeling of (1.1), if we consider a bivariate system where $z(t) = [Z_1(t)\ Z_2(t)]'$ then the following natural way of defining a causal ordering may be developed.

Consider $\gamma_{ij}^{P}(L) = \sum_{\tau=1}^{P} \alpha_{ij}^{\tau} L$, where $\gamma_{ij}^{P}(L)$ is the (I, j)-th entry of Γ^P **(L)**.

Definition (a): z_1 (t) Granger non-causes z_2 (t) if and only if $\gamma_{21}^{P}(L) = 0$ and $\{\alpha_{12}^{P} = 0 \text{ or } \neq 0\}$, $\tau = 1,...,p$.

That means $\Gamma^P(L) = \begin{vmatrix} \gamma_{11}^P(L) & \gamma_{12}^P(L) \\ 0 & \gamma_{22}^P(L) \end{vmatrix}$ and the following p coefficients, α_{12}^{τ}, $\tau = 1,...,p$ can be either zero or nonzero.

Further, There exist 2^P different patterns of $\gamma_{12}^{P}(L)$ in this bivariate system, indicating that z_1 (t) Granger non-causes z_2 (t) .

Definition (b): z_2 (t) Granger non-causes z_1 (t) if and only if $\gamma_{12}^{P}(L) = 0$ and $\{\alpha_{21}^{\tau} = 0 \text{ or } \neq 0\}$, $\tau = 1,..., p$.

That means $\Gamma^P(L) = \begin{vmatrix} \gamma_{11}^P(L) & 0 \\ \gamma_{21}^P(L) & \gamma_{22}^P(L) \end{vmatrix}$ and the following p coefficients, $\alpha_{21}^{\tau}, \tau = 1,...,p$, p, can be either zero or nonzero.

Definition (c): z_2 (t) Granger causes z_1 (t) if and only if $\gamma_{12}^{P}(L) \neq 0$ and $\{\alpha_{21}^{\tau} = 0 \text{ or } \neq 0\}, \tau = 1,...,p;$ z_1(t) Granger causes Z_2 (t) if and only if $\gamma_{21}^{P}(L) \neq 0$ and $\{\alpha_{12}^{\tau} = 0 \text{ or } \neq 0\}, \tau = 1,...,p;$

More general causal patterns can be treated using definitions suggested by Hsiao (1982). Consider the following trivariate system:

$$\begin{vmatrix} \gamma_{11}^P(L) & \gamma_{12}^P(L) & \gamma_{13}^P(L) \\ \gamma_{21}^P(L) & \gamma_{22}^P(L) & \gamma_{23}^P(L) \\ 0 & \gamma_{32}^P(L) & \gamma_{33}^P(L) \end{vmatrix} \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) \end{bmatrix} = \varepsilon(t)$$

which describes z_1 (t)causing Z_3 (t) but only through Z_2 (t). We call this situation indirect causality from z_1 (t)to z_3 (t). In this trivariate system the above indirect causality implies

$\gamma^P_{31}(L)=0, \gamma^P_{21}(L)\neq 0$ and $\gamma^P_{32}(L)\neq 0$. Also $\{\alpha^\tau_{12}=0 \text{ or } \neq 0\}$, $\{\alpha^\tau_{13}=0 \text{ or } \neq 0\}$ and $\{\alpha^\tau_{23}=0 \text{ or } \neq 0\}$, $\tau=1,...,P$.

Thus, for these upper diagonal coefficients, we have 2^{3p} different patterns in this system which indicate indirect causality from z_1 (t) to Z_3 (t) .

The greater the number of components, z_i (t), I = 1,2,..., the more complicated are the causal patterns that may be detected. For more detail we again refer to Penm and Terrell (1984b).

Zero entries in a znz patterned var and its equivalent **VECM** for an I(d) system
In an I(d) system the equivalent **VECM** derived from (1.1) can be expressed as follows:

$\Gamma^p(1)\ z(t-1) + \Gamma^{p-1}(1)\ \Delta z(t-1) + ... + \Gamma^{p-d+1}(1)\ \Delta^{d-1} z(t-1) + \Gamma^{p-d}(L)\Delta^d z(t) = \varepsilon(t)$, Where $\Gamma^{p}(1)\ \Delta^{l} z(t-1)$ are stationary, l= 0,...,d-1. The first d terms are the error correction terms, while $\Gamma^{p-d}(L)\Delta^d z(t)$ is said to be the autoregressive part of the model.

Further, we have the following relations:

$$\Gamma^k (L) = \Gamma^k (I)L + \Gamma^{k-1} (L)(I - L),\ k = p, p-1, ..., p-d+ l. \qquad (2.2)$$

Since Granger causality detection is crucially dependent on the positions of off-diagonal zero entries in the coefficient matrices, we therefore focus on the positions where $I \neq j$.
If the (I, j)-th entries of Γ^k (L), Γ^k (1) and Γ^{k-1} (L) are γ_{ij} (L), γ_{ij} (1) and c_{ij} (L) respectively, we have

$$\gamma_{ij} (L) = \gamma_{ij} (I)L + c_{ij} (L)(I-L),\ I\neq j. \qquad (2.3)$$

Now we define c_{ij} (L) by

$$c_{ij} (L) = c_1 L + ... + C_{k-1} L^{k-1},$$

and thus

$$c_{ij} (L)(1-L) = c_1 L + (c_2 - c_1)L_2 + --- + (c_{k-1} - c_{k-2}) L_{k-1} - c_{k-1} L^k. \qquad (2.4)$$

If $\gamma_{ij}(L)= 0$, then γ_{ij} (1) will also be zero. From (2.3) we have $c_{ij}(L)(1-L)= 0$ and (2.4) produces $c_1 = O$, $C_2 - C_1 = 0$, --- $C_{k-1} - C_{k-2} = O$, $C_{k-1} = O$, which lead to $c_l = O$, I= 1,..., k-1 and therefore c_{ij} (L)= 0.

At this point, if the (I, j)-th entry of Γ^k (L) is zero, then the (I, j)-th elements of both Γ^k (1) and Γ^{k-1} (L) are zeros. Therefore we can conclude that if every (I, j)-th entry is zero for all coefficient matrices in a **VAR** then all (I, j)-th coefficient elements in the error correction terms and in the vector autoregressive part of the **VECM**, will also be zeros.

Analogously it is evident that if the (I, j)-th elements of all Γ^k (1), k= p, p-1,..., p-d+ 1 and Γ^{k-1} (L) in (2. 1) are zeros then the (I, j)-th entry of Γ^P (L) in the equivalent **VAR** will be zero. Therefore we can conclude that if all (I, j)-th coefficient elements in the error correction terms and all (I, j)-th coefficient elements in the vector autoregressive part of the **VECM** are zeros, then every (I, j)-th entry is zero for all coefficient matrices in a **VAR**.

The implications of the above proof are obvious. If z_j does not Granger-cause z_i then every (I, j)-th entry is zero for all coefficient matrices in the **VAR**. Also all (I, j)-th coefficient elements in the equivalent **VECM** are zeros.
Further, we can express (2.2) as follows:

$$\Gamma^p (L) \ = \Gamma^P (1) + \Gamma^{p-1} (L) - \Gamma^{p-1} (L)\ L \qquad (2.5.1)$$
$$\Gamma^{p-1} (L) \ = \Gamma^{P-1} (1) + \Gamma^{p-2} (L) - \Gamma^{p-2} (L)\ L \qquad (2.5.2)$$
.
.
.
$$\Gamma^{p-d+1} (L) = \Gamma^{P-d+1} (1) + \Gamma^{p-d} (L) - \Gamma^{p-d} (L)\ L . \qquad (2.5.3)$$

From (2.5.3) it is obvious that if the (I, j)-th element of Γ^{p-d+1} (1) is nonzero, then the (I, j)-th element of Γ^{p-d+1} (L) is nonzero. Also if the (I, j)-th element of Γ^{p-d} (L) is nonzero, then a zero (I, j)-th element of Γ^{p-d+1} (1) leads to a nonzero (I, j) element of Γ^{p-d+1} (L). Thus, we have proved that if there exists a nonzero (I, j)-th element in either Γ^k (1) or Γ^{k-1} (L), k= p, p-1,...,p-d+ 1 in (2.2), then the (I, j)-th element of Γ^k (L) is nonzero. This outcome shows that if any single (I, j)-th element is nonzero in any one of the d matrices, Γ^k (1) , k= p, p-1,..., p-d+ 1, or Γ^{p-d} (L) in the **VECM** in (2.1) is nonzero, then the (I, j)-th element of Γ^P (L) in the equivalent **VAR** is nonzero.

Analogously from (2.5.1) if the (I, j)-th element of Γ^P (L) is nonzero, then at least the (I, j)-th element is nonzero in one of the following d coefficient matrices, or Γ^P (L):

Γ^P (I), Γ^{P-1} (I),...,Γ^{p-d+1} (1).

Therefore we have just demonstrated that if z_j does Granger-cause z_i, then the (I, j)-th element of Γ^P (L) in the **VAR** is nonzero. In addition at least a single (I, j)-th coefficient element is nonzero in Γ^P (1), Γ^{P-1} (1) ,..., Γ^{p-d+1} (1), or Γ^{p-d} (L) in the equivalent **VECM**.

An indirect causality from z_j to z_i through z_m indicates z_j causing z_i but only through z_m. Hence, z_j Granger-causes z_m, z_m Granger-causes z_i and z_j does not Granger-cause z_i directly. We can easily demonstrate that the **VAR** in (1.1) has nonzero (m, j)-th and (I, m)th elements and a zero (I, j)-th element of Γ^P (L) . The identical indirect causality can also be shown in the equivalent **VECM**.

RESULTS AND DISCUSSION

Monte carlo simulationresults

Penm and Terrell (1984a, b) proposed a search method in conjunction with model selection criteria to select the optimal **ZNZ** patterned **VAR**. This method can be extended to select the optimal **ZNZ** patterned **VECM** for an I(I) system. The use of model selection criteria in determining the order of an I(I) system has been validated by Potscher (1989) and Paulsen (1984). They showed that, for a similar class of model selection criteria, results on consistency which are valid in the stationary case can be generalised to processes with roots which are on or within the unit circle.

After the optimal **ZNZ** patterned **VECM** is selected, the rank of $\Gamma(1)$ is then computed using the singular value decomposition (**SDV**) method and the number of cointegrating vectors in the system will be known. The usefulness of this procedure in determining the rank of $\Gamma(1)$ is supported by a Monte Carlo experiment. In this experiment three simulations were carried out. Data were generated from **ZNZ** patterned **VAR**s and $\Gamma(1)$ of less than full rank. In all three simulations, $\Gamma(1)$ contains zero coefficients. The Monte Carlo results indicate that the Hannan-Quinn Criterion (**HQC**) employed in our paper is effective in identifying the order structures of the equivalent **VECM**s. After the correct order structure of a **VECM** is determined, the rank of $\Gamma(1)$ can be accurately determined by the **SDV** method. All acceptable zero-non-zero patterned Ps and their associated acceptable zero-non-zero patterned as can be obtained using the tree pruning algorithm presented in Penm *et al.* (1997). These as and βs are consistent with the structure of $\Gamma(1)$ determined earlier. The Monte Carlo results are reported as follows:

In the first Monte Carlo simulation, the following **VAR** model was used.

$$z_t + \begin{bmatrix} -0.6 & -0.8 \\ -0.1 & -0.8 \end{bmatrix} z_{t-1} + \begin{bmatrix} -0.5 & 0.0 \\ 0.0 & -0.5 \end{bmatrix} z_{t-4} + \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix} z_{t-5} = \varepsilon_t \qquad (3.1)$$

The equivalent **VECM** can be expressed as follows:

$$(I + \Gamma^* L^4)\, \Delta z_t + \Gamma z_{t-1} = \varepsilon_t , \qquad (3.2)$$

where **L** denotes the lag operator and d denotes first difference,

$$\Gamma^* = \begin{bmatrix} -0.5 & 0.0 \\ 0.0 & -0.5 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 0.4 & -0.8 \\ -0.1 & 0.2 \end{bmatrix} = \alpha\beta' = \begin{bmatrix} -0.4 \\ 1 \end{bmatrix} \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}$$

The rank of Γ is 1 and $\Theta = 0.0675 I_2$, where I_2 denotes a 2x2 identity matrix[1] . In the second simulation, we increased the number of variables to three. The **VAR** model used was as follows:

$$z_t + \begin{bmatrix} -0.6 & -0.8 & 0.0 \\ -0.6 & 0.2 & 0.0 \\ 0.6 & 0.4 & 0.3 \end{bmatrix} z_{t-1} = \varepsilon_t \qquad (3.3)$$

The equivalent VECM can be expressed as follows.

$$(I+ \Gamma^*)\Delta z_t + \Gamma z_{t-1} = \varepsilon_t, \qquad (3.4)$$

where

$$\Gamma^* = 0,\ \Gamma = \begin{bmatrix} 0.4 & -0.8 & 0.0 \\ -0.6 & 1.2 & 0.0 \\ 0.6 & 0.4 & 1.3 \end{bmatrix} \alpha\beta' = \begin{bmatrix} -0.8 & 0.0 \\ 1.2 & 0.0 \\ -1.2 & 1.3 \end{bmatrix} \begin{bmatrix} -0.5 & 1.0 & 0.0 \\ 0.0 & 1.23 & 1.0 \end{bmatrix}$$

The rank of Γ is 2 and $\Theta = 0.0675 I_3$, where I_3 denotes a 3x3 identity matrix.

In these simulations, the normal random number generator based on the algorithm of Marsaglia was used to generate univariate random deviates. These deviates were then converted to multivariate normal deviates with the given covariance matrix. Since the random number generator was effectively restarted for each replication, all experiments were essentially independent of each other. For each replication, 200 observations were generated.

In this Monte Carlo experiment, we first applied the procedure for selecting the optimal ZNZ patterned VECM and then used the singular value decomposition method to compute the rank of Γ.

In these cases we defined the ratio of singular values as DR(l) = S(l+ 1)/S(l), where l is an integer value from 1 to (the number of variables in the model -1), S(l) denotes the l-th singular value. If a sharp increase occurred at DR(r), then r would be selected as the rank of B [see Woodside (1971)].

For the model (3.1), the minimum, mean and maximum values of DR(1) for the 100 replications were plotted in Fig. 1. The unmistakable sharp rises at r= 1 clearly indicated that the rank of B was 1. Fig. 2 shows the plot for the minimum, mean and maximum values of DR(1) and DR(2) for the model (3.2). The sharp rises were unmistakable at r= 2. Therefore the rank of B was indicated as 2.

Given the zero-non-zero pattern of B and the rank of B, we then proceeded with obtaining all acceptable zero-non-zero patterned αs and βs using the proposed tree pruning algorithm. After this, we used the methods introduced in Penm *et al.* (1997) in conjunction of the HQC criterion to select the optimal α and β.

The outcomes of this Monte Carlo experiment are summarised in Table 1. Four statistics P 1, P2, P3 and P4- are reported:

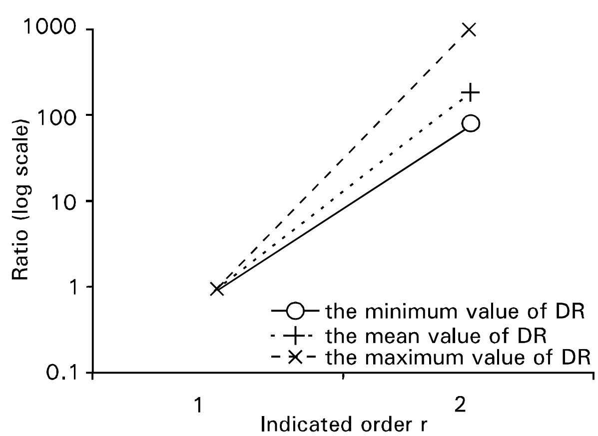

Fig. 1: Ratios of singular values DR for the first simulation (Note the rise at r--1)

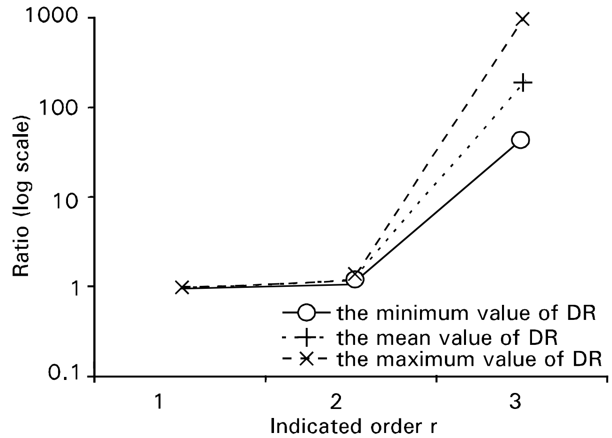

Fig. 2: Ratios of singular values DR for the second simulation (Note the rise at r--2)

PI is the proportion of replications in which the corrected zero-non-zero patterns of Γ^*, Γ, α and β were obtained by the proposed procedure;
P2 is the proportion of replications in which the corrected zero-non-zero patterns of Γ^*, Γ and α were found, but β was over-specified;
P3 is the proportion of replications in which Γ^* or Γ was over-specified; and P4 is the proportion of replications in which the results were not in P I, P2 and P3.

In the third simulation, the number of variables in the system was increased to 4. The VAR model used was as follows.

$$: + \begin{bmatrix} -0.6 & 0.0 & 0.0 & -0.8 \\ -0.6 & -1.8 & -0.65 & 1.2 \\ 0.6 & 1.6 & 0.3 & -1.2 \\ -0.6 & 0.0 & 0.0 & 0.2 \end{bmatrix} z_{t-1} = \varepsilon_t \qquad (3.5)$$

The equivalent VECM can be expressed as follows.

$$(I + \Gamma^*)\Delta z_t + \Gamma z_{t-1} = \varepsilon_t ,$$

where

$$\Gamma^* = 0,\ \Gamma = \begin{bmatrix} 0.4 & 0.0 & 0.0 & -0.8 \\ -0.6 & -0.8 & -0.65 & 1.2 \\ 0.6 & 1.6 & 1.3 & -1.2 \\ -0.6 & 0.0 & 0.0 & 0.2 \end{bmatrix} \alpha\beta' = \begin{bmatrix} 0.4 & 0.0 \\ -0.6 & -0.8 \\ 0.6 & 1.6 \\ -0.6 & 0.0 \end{bmatrix} \begin{bmatrix} 1.0 & 0.0 & 0.0 & -2.0 \\ 0.0 & 1.0 & 0.76 & 0.0 \end{bmatrix}$$

The rank of Γ is 2 and $\Theta = 0.0675 I_4$ where I_4 denotes a 4x4 identity matrix.

The Monte Carlo results are also presented in Table 1.

Table 1: Frequency distribution for models chosen by the HQC criterion

Model	P1	P2	P3	P4
(3.1)	0.92	0.0	0.08	0.0
(3.3)	0.85	0.15	0.0	0.0
(3.5)	0.78	0.10	0.08	0.04

An Application Concerning Granger Non-causality and Indirect ca Usality

In this section the balanced growth relationships among real private output zt, consumption ct and investment I, in the United States, presented by King *et al.* (1991) and Penm *et al.* (1997), are shown to demonstrate the usefulness of the proposed **VECM** modeling. We focus on the application of the **VECM** model for analysing Granger causal relationships, in particular Granger non-causality and indirect causality. As described in Permi *et al.* (1997), the optimal **VECM** presented in Fig. 3 has been selected by the Hannan-Quinn criterion, using the sample over 1978.1 to 1993. 1:

Since all (2,3)-th entries in the autoregressive part and in the error-correction terni of the **VECM** are zeros, this **VECM** indicates Granger non-causality from investment to consumption. Further, both the (2,l)-th and (1, 3)-th entries in the autoregressive part are non-zeros and these findings show one-way causation from real private output to consumption and from investment to real private output. We do, therefore, find that indirect causation holds from investment to consumption through real private output.

As described in Section 1 the full-order **VECM** models cannot detect Granger noncausality and indirect causality. This is because no zero entries are allowed in the fullorder **VECM** models.

$$\begin{bmatrix} \Delta \log y_t \\ \Delta \log c_t \\ \Delta \log i_t \end{bmatrix} = \begin{bmatrix} 0 & \substack{0.0527 \\ (1.95)} & \substack{0.1358 \\ (2.38)} \\ \substack{-0.3392 \\ (-3.03)} & 0 & 0 \\ \substack{0.0359 \\ (1.89)} & \substack{-0.0637 \\ (-1.25)} & 0 \end{bmatrix} \begin{bmatrix} \Delta \log y_{t-1} \\ \Delta \log c_{t-1} \\ \Delta \log i_{t-1} \end{bmatrix} + \begin{bmatrix} \substack{0.1263 \\ (3.15)} & \substack{-0.0979 \\ (-3.16)} & 0 \\ 0 & 0 & 0 \\ \substack{-0.0278 \\ (-2.53)} & 0 & \substack{0.0153 \\ (2.55)} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \end{bmatrix},$$

$$\text{the estimated } \alpha = \begin{bmatrix} 0 & \substack{0.01259 \\ (3.23)} \\ 0 & 0 \\ \substack{-0.0278 \\ (-2.55)} & 0 \end{bmatrix}, \text{ the estimated } \beta' = \begin{bmatrix} 1.0 & 0 & \substack{-0.6253 \\ (-3.15)} \\ 1.0 & \substack{-0.7737 \\ (-2.52)} & 0 \end{bmatrix},$$

Fig. 3: The Optimal **VECM** selected by the Hannan-Quinn criterion

where t-values are in parentheses and the estimates are GLS estimates.

In this note we have demonstrated that ZNZ patterned **VECM** models can be used as a basis for detecting Granger causality, Granger non-causality and indirect causality for time series of integrated order I(d). Conventional **VECM** models - the full-order **VECM** models - assume that they contain all nonzero entries in their coefficient matrices. Thus these models cannot detect either Granger non-causality or indirect causality, thereby leading to incorrect and invalid causal inferences.

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