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Drilling Mechanics: Consequences and Relevance of Drill string Vibration on Wellbore Stability

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Abstract: Wellbore instability problems have been attributed mainly to rock-fluid interaction, especially when drilling shaley formations with water-base fluids. However, recent studies showed that other events during drilling may contribute much more decisively in causing the problems than the rock-fluid interaction and swelling. The Study presents a conceptual model to analyze wellbore stability based on energy. The Drill string vibration concern as an important problem and is fairly complex since it is generally a combination of longitudinal, torsional and transverse modes, which are all present and coupled. Because of the complexity and the importance of Drill string vibrations in the drilling operation, a full simulation covering all the relevant phenomena is not practical and a common approach has been used to study the vibration mechanisms individually. The consideration was subjected to the effect of the longitudinal and torsional vibration.

Key words: Torque, drag, vibration, longitudinal and torsional, differential equations

INTRODUCTION

Torque, Drag and Vibration analysis has been used for more than 10 years to drill wells “in study on papers”. Representations of the Drill string and bottomhole assembly (BHA) that do not include the individual component stiffness have been used with some success in analyzing and preventing problems in extended reach drilling. As curve rates have increased in directional wells, the effects of the previously neglected stiffness have led to errors in the side force calculations that underpin all torque, drag and vibration analysis. Full finite-element analysis can solve this problem. Additionally, tortuosity can be added to provide a rippled effect to a well plan that simulates the micro doglegs that occur in the drilled well, enabling more realistic bounds to be placed on the drag and torque losses and the forces experienced by the Drill string. Identification of the real cause of the problems is essential to reduce the exploration costs in challenged environments, such as the ones faced today by the oil industry. Field observations, in the form of downhole and surface vibrations measurement, have indicated that Drill string are subjected to severe vibrations. The primary causes of these vibrations are bit/formation and Drill string/borehole interactions. On the other hand, measurements of these vibrations may provide valuable information about the drilling assembly and formation characteristics. Therefore, vibrations must be fully understood and their effects should be minimized in any approach to drilling optimization.

Finnie and Bailey[1,2] made an early contribution to the subject with experimental and analytical studies covering longitudinal and torsional natural frequencies in the Drill string. Paslay and Bogey[3] considered the bit as source of vibrations due to intermittent contact of bit teeth. Some studies on Drill string dynamics were mostly concerned with longitudinal and torsional vibrations. Later, it was shown that the coupling mechanism between longitudinal and torsional vibrations could lead to large torsional vibrations[4]. Kyllingstad and Halsey[5] studied the stick-slip vibrations due to static friction at the bit/formation interface. Brett[6] showed that the dependence of bit torque feedback could help eliminate stick-slip vibrations. Parametric instabilities caused by fluctuation weight on the bit were studied by Dunayevsky *et al.*[7]. Another instability mechanism for coupled longitudinal and torsional vibrations was proposed by Elsayed *et al.*[8] which is due to vibrations in the phase angle between surface undulations of subsequent cutters. Lateral vibrations, which occur in the forms of whirling and parametric resonance, have also been studied extensively[7,9,10].

In this study, the consideration will be subjected to the effect of the longitudinal, torsional vibrations and to the establishment of the fundamental equations for the dynamic analysis of the Drill string. The mathematical

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models of the longitudinal and torsional vibrations as well as the model coupling these two vibrations will be performed. The proposed mathematical models will be used to study the entire force of the Drill string and the displacement to find out the appropriate position of the shock-sub and the other tools. Based on the results presented, recommendations to be done in future wells are described. Suggestions in terms of drilling fluids are also made, since this can contribute decisively to strongly reduce the incidence of the problems.

Longitudinal and torsional vibrations differential equations: It is well known that Drill string vibration may lead to fatigue failures and abrasive wear of tubular damage to the drill bit and the borehole wall. The measurements of these vibrations provide valuable information about the drilling assembly and formation characteristics and their effects should be minimized in any approach to drilling optimization. The longitudinal and torsional vibrations system includes: the derrick, the cable, the traveling block, the rotary hose and the Drill string, with neglecting the vibrations of the transmission and power systems, the torsional vibrations includes only the Drill string. The assumptions proposed according to the tough of the rocks drilled and the recommendations needed certainly to the operation are:

1. The axis of the Drill string is concordant with the well axis
2. The drilling fluid is Newtonian fluid
3. The dynamic effect of the drilling fluid is neglected
4. The lateral vibrations are neglected
5. The changes in temperature and the static characteristics are also ignored

Analyzing all the above factors the longitudinal and torsional vibrations differential Equations are established. Form the assumptions (1), (2) (3) and (5)

$$\left.\begin{aligned} &F = F_t k \\ &h = h_t k \\ &r = r k \\ &r_o = s k \\ &M = M_t k \\ &m = m_t k \\ &\omega = \omega k \\ &T = 0 \end{aligned}\right\}$$

Where, h_t is the external force per unit length of the Drill string, m_t is the external torque per unit length of the Drill string, ω is the rotating speed of the Drill string. Then the fundamental Equations modified as follows:

$$\left.\begin{aligned} &\frac{\partial F_t}{\partial s} + h_t = \frac{\partial^2 (A\rho s)}{\partial t^2} \\ &= \pi\left(R_o^2 - R_t^2\right) \end{aligned}\right\} \quad \left.\begin{aligned} &\frac{\partial M_t}{\partial s} = -m_t + I_o \frac{\partial \omega}{\partial t} \\ &I_o = A\rho\left(R_o^2 - R_t^2\right)/2 \end{aligned}\right\} \quad \begin{aligned} &F_t = EA\left(\frac{\partial s}{\partial l} - 1\right) \\ &M_t = GJ \frac{\partial \gamma}{\partial s} \end{aligned}$$

From the assumptions (3) and (4) the external force and the external moment are:

$$h_t = \frac{2\pi\mu}{\ln\left(\dfrac{D_w}{2R_o}\right)} \frac{\partial s}{\partial t} \qquad m_t = -\frac{4\pi\mu D_w^2 R_o^2}{D_w^2 - 4R_o^2}\omega$$

where:
μ is the dynamical viscosity of drilling fluid. D_w is the borehole diameter. The displacement assumed to be represented by the following formula: u=s−l. The angular displacement is assumed as: $\Psi = \omega_o t + \gamma$.

Continuity conditions: The continuity conditions are the torque and the angular displacement of the upper stages that are equal to those of the lower stage respectively at the connection point:

$$\left.\begin{aligned} &G_{j-1} J_{j-1} \frac{\partial \gamma_{j-1}}{\partial l}\bigg|_{l=L_{j-1}} = G_j J_j \frac{\partial \gamma_j}{\partial l}\bigg|_{t=0} \\ &\gamma_{j-1}(L_{j-1}, t) = \gamma_j(0, t) \end{aligned}\right\}$$

Initial conditions: In the drilling operation the vibrations is periodic, therefore the initial conditions are as follows:

$$\left.\begin{aligned} &\gamma_j(l,t) = \gamma_j(l, t + T_p) \\ &\frac{\partial \gamma_j(l,t)}{\partial t} = \frac{\partial \gamma_j(l, t + T_p)}{\partial t} \end{aligned}\right\}$$

Boundary conditions

The kelly: The Kelly rotates at a constant speed, the torsion angle is considered to be zero.

$$\gamma_4(0, t) = \phi(t) = 0$$

Where φ(t), is the torsional angle of the Drill string at rotary table.

The bit: the torque on bit causes the torsional vibrations.

$$\gamma_m(L_m, t) = \phi_b(t) \qquad G_m J_m \frac{\partial \gamma_m(l,t)}{\partial t}\bigg|_{l=L_m} = T_{ob}$$

where, T_{ob} is the fluctuating torque on bit. $\phi_b(t)$ is the torsional angle of a bit. Because of the periodicity of Drill string vibrations, therefore,

$$\begin{aligned} &T_{jn}(t) = e^{in\omega t} \\ &\lambda_{jn} = -\alpha_{jn} + i\beta_{jn} \end{aligned} \qquad \text{where:} \quad \omega = \frac{2\pi}{T_p},$$

Then Euler's formula can be written as follow: $e^{in\omega t}=\cos(n\omega t)+i\sin g(n\omega t)$. And the dynamic torque function is:

$$M_j(1,t) = G_j J_j \frac{\partial \gamma_j(1,t)}{\partial 1} = G_j J_j \left\{ \sum_{n=1}^{\infty} \left[\frac{\partial O_{jn}}{\partial 1} \cos(n\omega t) + \frac{\partial P_{jn}(1)}{\partial 1} \sin(n\omega t) \right] \right\}$$

where,

$$\left.\begin{aligned} \frac{\partial O_{jn}(1)}{\partial 1} &= \alpha_{jn}\{\cos(\alpha_{jn}1)[k_{jn}ch(\beta_{jn}1)+\gamma_{jn}sh(\beta_{jn}1)] \\ &-\sin(\alpha_{jn}1)[\mu_{jn}sh(\beta_{jn}1)+y_{jn}ch(\beta_{jn}1)]\} \\ &+\beta_{jn}\{\sin(\alpha_{jn}1)[k_{jn}sh(\beta_{jn}1)+\gamma_{jn}ch(\beta_{jn}1)] \\ &+\cos(\alpha_{jn}1)[\mu_{jn}ch(\beta_{jn}1)+y_{jn}sh(\beta_{jn}1)]\} \\ \frac{\partial p_{jn}(1)}{\partial 1} &= -\alpha_{jn}\{\sin(\alpha_{jn}1)[k_{jn}sh(\beta_{jn}1)+\gamma_{jn}ch(\beta_{jn}1)] \\ &+\cos(\alpha_{jn}1)[\mu_{jn}ch(\beta_{jn}1)+y_{jn}sh(\beta_{jn}1)]\} \\ &+\beta_{jn}\{\cos(\alpha_{jn}1)[k_{jn}ch(\beta_{jn}1)+\gamma_{jn}sh(\beta_{jn}1)] \\ &-\sin(\alpha_{jn}1)[\mu_{jn}ch(\beta_{jn}1)+y_{jn}sh(\beta_{jn}1)]\} \end{aligned}\right\}$$

The torsional angle and the torque at the upper end stage pipe can be expressed in Fourier series as follows:

$$\left.\begin{aligned} \gamma_j(0,t) &= \sum_{n=1}^{\infty} [v_{jn}\cos(n\omega t)+\delta_{jn}\sin(n\omega t)] \\ M_j(0,t) &= \sum_{n=1}^{\infty} [\sigma_{jn}\cos(n\omega t)+\tau_{jn}\sin(n\omega t)] \end{aligned}\right\}$$

Longitudinal vibrations mathematical model: The mathematical model can be constructed by solving the longitudinal vibrations periodic movement differential equation along the axis of the Drill string. In this model the Drill string consists of *m* stages pipes, therefore, the differential equation for the *j*th stage pipe in local coordinate system is as follows:

$$\frac{\partial^2 u_j}{\partial t^2} = a_j^2 \frac{\partial^2 u_j}{\partial 1^2} - c_j \frac{\partial u_j}{\partial t}$$

$(0\le 1\le L_j, j=1,2,....,m)$;

Where:

$$c_j = \frac{2\pi\mu}{\rho_j A_j \ln(\frac{D_w}{2R_{oj}})} \quad \text{and} \quad a_j^2 = \frac{E_j}{\rho_j}$$

It obtained:

$$\sigma_{ln} = \frac{1}{a_{mn}^2+b_{mn}^2}[(v_{mn}-c_{mn}v_{ln}-d_{mn}\delta_{ln})a_{mn} - (\delta_{mn}+d_{mn}v_{ln}-c_{mn}\delta_{ln})b_{mn}]$$

$$\tau_{ln} = \frac{1}{a_{mn}^2+b_{mn}^2}[(v_{mn}-c_{mn}v_{ln}-d_{mn}\delta_{ln})b_{mn} + (\delta_{mn}+d_{mn}v_{ln}-c_{mn}\delta_{ln})a_{mn}]$$

to the end, all the parameters of torsional vibration are conformed (Fig. 1, 2 and 3).

Torsional vibrations mathematical model: In the case of the Drill string consists of *m* stages pipes, then the differential equation for the *j*th stage pipe in local coordinate system is as follows:

$$\frac{\partial^2 \gamma_j}{\partial t^2} = \frac{G_j}{\rho_j}\frac{\partial^2 \rho_j}{\partial 1^2} - d_j \frac{\partial \gamma_j}{\partial t}$$

$(0\le 1\le L_j, j=4,5,....,m)$ Where: $d_j = -\frac{4\pi\mu D_w^2 R_{oj}^2}{\rho_j J_j (D_w^2 - 4R_o^2)}$

It obtained:

$$\sigma_{ln} = \frac{1}{A_{mn}^2+B_{mn}^2}[(\frac{\sigma_{mn}}{E_m A_m} - C_{mn}v_{ln} - D_{mn}\delta_{ln})A_{mn} - (\frac{\tau_{mn}}{E_m A_m} + D_{mn}v_{ln} - C_{mn}\delta_{ln})B_{mn}]$$

$$\tau_{ln} = \frac{1}{A_{mn}^2+B_{mn}^2}[(\frac{\sigma_{mn}}{E_m A_m} - C_{mn}v_{ln} - D_{mn}\delta_{ln})B_{mn} + (\frac{\tau_{mn}}{E_m A_m} + D_{mn}v_{ln} - C_{mn}\delta_{ln})A_{mn}]$$

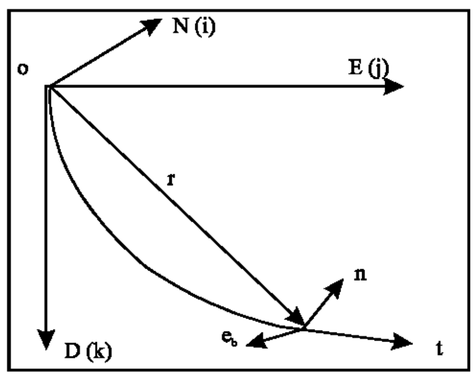

Fig. 1: Coordinate systems

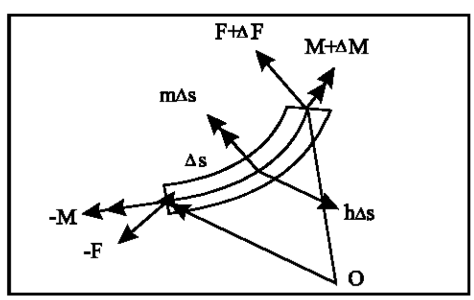

Fig. 2: Equilibrium of Drill string element

to the end, all the parameters of torsional vibration are conformed; (Fig. 1, 2 and 3).

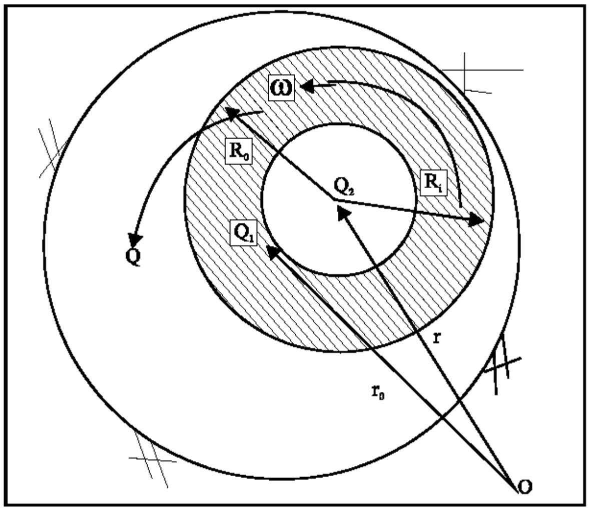

Fig. 3: Rotation and whirling motion

The theoretical work in this study forms the basics for the dynamic, kinetics behavior of Drill string, rock bit behavior to prevent early failure of Drill string and control the well orbit. This Method is applicable for theoretical analysis; optimizing Drill string design and determining shock-sub exact position.

In addition, this model helps in avoiding the Drill string failure. The increase in the Drill string stiffness leads to strong violent vibrations in the Drill string. So to satisfy the Drill string precondition requirements, it is strongly recommended to reduce the size of the drill collar and the drill pipe.

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