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A Classical Model for Study of Non-polar Dielectric to Achieve Microwave Frequency Oscillator Using Electron Spin Resonance

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Abstract: The main aim of this study was to apply a classical model for study of non-polar dielectric to achieve a microwave oscillator using electron spin resonance. This dielectric has first been studied in an oscillating electric field in order to find out its electric susceptibility and then its average absorbed power. The transition frequency and the static value of susceptibility have also been found. It has also been deduced that, an electromagnetic wave traversing this dielectric is obeying an exponential law. In order to achieve a microwave oscillator, the sample has been considered in an oscillating magnetic field. An external magnetic induction, perpendicular to oscillating magnetic field has been applied to see how the Zeeman level could split into two sub-levels. The distribution of populations over these two sub-levels has also been determined. Finally, the sample containing lithium atoms as impurities was placed in a cavity resonator, tuned on the resonance frequency of the sample. It was found out that the sample was emitting an average power instead of absorbing, when the magnetic induction was switched from $+B_0$ to $-B_0$.

Key words: Classical modeling, spin resonance, non-polar dielectric, microwave frequency oscillator

INTRODUCTION

A non-polar dielectric, with known frequency characteristic of v_{12}, has been chosen as sample and put under the influence of an oscillating electric field, **E**. Each electron of the sample was considered to be experiencing a small displacement; **r** and three kind of forces such as; electric, return, and friction[1]. An expression for the electric susceptibility; χ, as a function of the number of vibrating electrons, their mass and the resonance frequency has been established. The expression showed that the susceptibility was a complex number; therefore an expression for the imaginary part has also been established. The average power absorbed by this sample has also been calculated as a function of static susceptibility; χ_0 and band pass, δv, within the resonance frequency.

The sample has then been put in an oscillating magnetic field; **H**. The splitting of the energy levels, in the domain of microwave frequency, has been achieved by applying an external magnetic induction; B_0, perpendicular to H_1. In this condition, the sample containing lithium atoms as paramagnetic centers has shown two Zeeman sub-levels. The distribution of populations over these two sub-levels has been found. The average absorbed power as a function of transition frequency v_{12}, magnetic field H_1 and the width of band pass δv, of the transition frequency v_{12}, has also been calculated.

In two previous steps some physical characteristics of sample being obtained, we have considered a cavity resonator, with the frequency tuned to the resonance frequency v_{12}, of the paramagnetic substance.

It was supposed that there existed permanently a radiation density, which was tending the sample to saturation of transition. By passing rapidly and adiabatically from $+B_0$ to $-B_0$, it was noticed that the sample is capable to provide power instead of drawing, in condition that the population inversion could be maintained.

The quality factor Q, of the sample being determined by other researchers, is of the order of a few hundred thousands[2-4], we were able to determine the number of paramagnetic centers in the sample. Finally in pulse method we were also able to calculate the average power of the emitted signal.

In this study, the emphasis was on the application of classical mechanics to achieve physical characteristics of a non-polar dielectric and to establish an ultra high frequency oscillator.

Analytic formulae: A very simple formalism, based on classical mechanics, is presented below.

Absorption phenomenon of a non-polar dielectric: The electrons have been considered as equivalent and vibrating with a frequency of $v_{12}=\omega_0/2\pi$, within their

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equilibrium positions. In an oscillating electric field: $\mathbf{E}=\mathbf{E}_1 \text{Cos}\omega t$, each electron gets a small displacement of $\mathbf{r}$ and therefore is subjected to three forces such as; electric force $e\mathbf{E}$, return force $-m_0\ \omega_0\ \mathbf{r}$ and friction force $-m_0\ \gamma\ d\mathbf{r}/dt$. The friction factor γ is generally very small.

In order to find a solution to $\mathbf{r}(t)$, relative to electron movement, non-relativistic classical mechanics has been applied. In fact we can have[1]:

$$m_0\ (d^2\ \mathbf{r}/dt^2 + \gamma\ d\mathbf{r}/dt + \omega_0{}^2\ r)=-eE_1\ e^{J\omega t} \quad (1)$$

Taking the corresponding characteristic equation into account, we can have:

$$r = -\gamma/2 \pm J(\omega_0{}^2 - \gamma^2/4)^{1/2} \quad (2)$$

We will obtain for $\mathbf{r}(t)$, the following equation:

$$\mathbf{r}\ (t) = e^{-1/2}\ (A\cos(\omega_0{}^2 - \gamma^2/4)^{1/2}\ t + B\sin((\omega_0{}^2 - \gamma^2/4)^{1/2}\ t - eE_1\ e^{J\omega t} / m_0\ ((\omega_0{}^2 - \omega^2) + J\omega\gamma$$

n_0, has been considered as the number of electrons per unity of volume and the correction of Lorentz field has been ignored. In average, over a macroscopic volume, the $\bar{A}$ and $\bar{B}$ mean values are zero. In fact there exists a friction term which can intervene only in the study of transient term which tends to zero for a sufficient long time.

From macroscopic point of view we can write[5]:

$$P = \chi\ \epsilon_0\ \bar{E} \quad (4)$$

From microscopic point of view we can write:

$$P=n_0\ \alpha\ E=n_0\ \alpha\ (\bar{E}+E_{Lorentz} + E_z) \quad (5)$$

In isotropic medium E_z is zero and on the other hand; $E_{lorentz}= P/3\epsilon_0$ can be ignored because it is very small.

In this condition $\bar{E}$ = Eand we can have:

$$P=n^0\ e^2\ E/(m_0\ ((\omega_0{}^2 - \omega_2) + J\omega\gamma)) = \chi\ \epsilon_0\ E \quad (6)$$

We draw finally:

$$\chi = \chi' - J\ \chi'' = n^0\ e^2\ ((\omega_0{}^2 - \omega^2) -J\gamma\omega) / (m_0\epsilon_0((\omega_0{}^2 - \omega^2) + \gamma^2\ \epsilon^2)) \quad (7)$$

The average power $\bar{P}$ absorbed by the non-polar dielectric per unity of volume in an oscillating field, can also be estimated. In fact the electrical energy localized in a volume dv is:

$$dU/dv = \int_0^D E\,dD \quad (8)$$

In a sufficient long time, we have:

$$d\bar{U}/dv = \int_0^D \bar{E}\,d\bar{D} \quad (9)$$

As; $$D=\epsilon_0\ E+P \quad (10)$$

Therefore we can have:

$$\Delta\bar{U}_0 = d\bar{U}/dv = \oint \bar{E}\,d\bar{P} = \bar{E}\,\Delta\bar{P} \quad (11)$$

This leads to the calculation of average power:

$$\bar{P} = \Delta\bar{U}/\Delta t = \text{Re}[E].\ \text{Re}[\Delta P/\Delta t] \quad (12)$$

Whereas:

$$\bar{P} = \epsilon_0 E_1^2 .\ \chi''.\ \omega/2 \quad (13)$$

The existence of χ'' enables us to assimilate the dielectric a conductor with conductivity such as:

$$\sigma = \epsilon_0\ \chi''\ \omega \quad (14)$$

In which the average dissipated power is:

$$\bar{P} = \sigma E^2/2 \quad (15)$$

As γ is generally very small, therefore $\bar{P}$ cannot be noticeable unless at the vicinity of ω_0. If, in this case we consider:

$$\omega + \omega_0 \approx 2\omega \quad (16)$$

Then:

$$\chi'' = (n_0\ e^2 / m_0\epsilon_0)\ (\gamma\omega/((\omega_0{}^2 - \omega^2) + \gamma^2\omega^2) \quad (17)$$

Taking (16) into account, we have; $(\omega + \omega_0 \approx 2\omega\ (\omega_0 - \omega)$ then, (17) becomes:

$$\chi'' = (n_0\ e^2\ \gamma) / (4m_0\epsilon_0\ ((\omega_0{}^2 - \omega^2) + \gamma^2/4)\ \omega) \quad (18)$$

The average dissipated power is then:

$$\bar{P} = (n_0 e^2\gamma .E_1^2)/(8m_0((\omega_0^2 - \omega^2) + \gamma^2/4) \quad (19)$$

In order to estimate the transition frequency $\Delta\omega$ then δv, average dissipated power at $\omega=\omega_0$ and $\omega=0$ should be calculated. Therefore we can have successively:

$$\bar{P}_{max} = (n_0 e^2 . E_1^2)/(2 m_0 \gamma^2) \quad (20)$$

$$\bar{P}_{(0)} \approx (n_0 e^2 . E_1^2 \gamma)/(8 m_0 \omega_0^2) \quad (21)$$

Whereas:

$$\Delta\omega = \gamma \text{ and } \delta v = \gamma / 2\pi \quad (22)$$

At this stage, the static susceptibility; χ_0, which is defined at $\omega=0$ can be deduced. Therefore:

$$\chi(0) = \chi_0 = n_0 e^2 / (m_0 \epsilon_0 \omega_0^2) \quad (23)$$

The average dissipated power can be given as a function of χ_0 and also of δv at vicinity of resonance frequency; v_{12}. We can have:

$$\bar{P}(0) \approx (n_0 e^2 . E_1^2 \gamma)/(8 m_0 ((\delta\omega)^2 + \gamma^2/4) \ (\epsilon_0 \omega_0^2 \chi_0 E_1^2)/2\gamma \quad (24)$$

As at the vicinity of $\omega 0$; $\delta\omega << \Delta\omega$, therefore:

$$\bar{P} = (\epsilon_0 \omega_0^2 \chi_0 E_1^2)/2\Delta\omega \quad (25)$$

At this stage we have verified that the energy of electromagnetic wave of the length; λ_{12}, traversing the dielectric medium, was obeying an exponential rule. When λ is tending to λ_{12}, then the real part of χ tends toward zero. The energy of electromagnetic wave is then:

$$W = \epsilon_0 E_1^2 / 2 \quad (26)$$

On the other hand, as the average power is:

$$\bar{P} = d\omega/dt = -\omega \chi'' \epsilon_0 E_1^2 / 2 \quad (27)$$

Then the relative variation of energy can be written as:

$$dW / W = -(2\pi/\lambda_{12}).\ v.\ \chi''\ dt \quad (28)$$

Whereas:

$$W = \omega_0 \operatorname{Exp}\left(-\frac{2\pi\chi''}{\lambda_{12}} x \right) \quad (29)$$

Electron magnetic resonance: Regarding a dielectric medium, the results obtained so far can be applied to a paramagnetic medium, which is put under the effect of an oscillating magnetic field;

$$H = H_1 \operatorname{Cos} \omega t \quad (30)$$

for which the energy levels are separated by very high frequency and can be obtained by the effect of a uniform external magnetic induction field B_0, perpendicular to H_1. In this case χ_0 is presenting the static magnetic susceptibility and E_1 should be replaced by the magnetic field H_1. At this stage and admitting the previous hypothesis, we consider a paramagnetic substance containing n_0 paramagnetic centers per unity of volume.

These paramagnetic centers represent the lithium atoms of atomic mass and atomic number of; M=7 and Z=3, respectively. The electronic configuration of lithium ${}_3^7Li$ is: ($1S^2$, $2S^1$). According to Hund rules[6,7], we have:

L=0; S=1/2; J=L+S; j=1/2; and $M_J=\pm 1/2$
(two Zeeman levels).

H is given as:

$$H = \mu_B.\ B_0\ (L+2S) \quad (31)$$

And:

$$E = \pm \mu_B.\ B_0 \quad (32)$$

We take only the Zeeman energy into account; and spin-orbit coupling energy. The Columbian force does not intervene. The distribution of population over two Zeeman sub-levels can be obtained as:

$$n_1/n_2 = \operatorname{Exp}-2\ \mu_B\ B_0/KT \approx 1-2\ \mu_B\ B_0/KT \quad (33)$$

Whereas:

$$\chi_0 = ((n_1 - n_2) / H_0)\ \mu_B \quad (34)$$

From (33) and (34) we can get:

$$(n_1 - n_2) / (n_1 + n_2) = (\operatorname{Exp}2\ \mu_B B_0/KT-1)/(\operatorname{Exp}2\ \mu_B B_0/KT + 1) \quad (35)$$

Where:

$$(n1 - n2) \approx N.\ \mu_B.\ B_0/KT \quad (36)$$

And:

$$\chi_0 = N.\ \mu_0.\ \mu_B^2/KT \quad (37)$$

The average absorbed power, at the vicinity of resonance frequency, ω_0, if; $H_{Local} = H_{macroscopic}$, can be found as:

$$\bar{P} = (\mu_0 \omega_0^2 \chi_0 H_1^2)/2\Delta\omega = N((\mu_0 . \mu_B)^2 \pi V_{12}^2 H_1^2/(KT . \delta v) \quad (38)$$

Microwave oscillator based on electron spin resonance: The sample was silicon containing N lithium atoms per cubic centimeter, and placed in a resonating cavity in a uniform magnetic induction B_0. The cavity was tuned on the resonance frequency v_{12}, of this paramagnetic substance. The sample volume being the same as the cavity volume, it was supposing that there would be a permanent density of radiation tending the sample toward a saturation of transition[8].

In an adiabatic condition, the magnetic induction was switched rapidly from $+B_0$ to $- B_0$. In the range of ultra high frequency, the spontaneous emission being negligible, therefore passing from $+B_0$ to $- B_0$ in adiabatic condition, the populations' inversion would occur, that is:

$$(n_1 / n_2)_+ = (n_1 / n_2) \quad (39)$$

If this inversion is maintained, therefore all the energy previously supplied would be re-emitted, and the emission is stimulated: $W_{12} = W_{21}$. At this stage the quality factor of the cavity could be estimated as[2]:

$$Q = 2\pi \upsilon_{12} \frac{\frac{1}{2}\mu_0 H_1^2}{\bar{P}_{dis}} \quad (40)$$

In order to get a spontaneous oscillation, the dissipated power; $\bar{P}_{dis}$ should be smaller than the power provided by the sample, this means:

$$N((\mu_0 . \mu_B)^2 \pi v_{12}^2 H_1^2/(KT. \delta v) \geq 4\pi v_{12}\mu_0 H_1^2 / Q \quad (41)$$

(41) can be solved for N, then we have:

$$N \geq \frac{KT\delta v}{\mu_0 \mu_B^2 v_{12} Q} \quad (42)$$

On the other hand, as:

$$W_{12} = hv_{12} = 2\mu_B B_0 \quad (43)$$

Therefore:

$$N \geq \frac{KT.V.\delta v.h}{2Q.\mu_0 B_0 . \mu_B^3} = N_0 \quad (44)$$

In case of pulse method, if we consider $N = \alpha N_0$, with $\alpha > 1$ and that the energy is emitted in Δt second, then the average power of the emitted signal could also be estimated. In fact, in population inversion, the emitted number; n, of photons is:

$$n = N. V. \mu_B B_0 / KT = N.V.hv_{12} / 2KT \quad (45)$$

The emitted energy is then:

$$W = \frac{N.V.h^2 v_{12}^2}{2KT} \quad (46)$$

The average power emitted during the pulse duration; Δt, is

$$\bar{P}_e = \frac{W}{\Delta t} = \frac{N.V.(hv_{12})^2}{2KT.\Delta t} \quad (47)$$

Finally we can have for the average emitted power:

$$\bar{P}_e = \frac{\alpha.V.\delta v.h.B_0}{\mu_0 Q.\mu_B.\Delta t} \quad (48)$$

Numerical application

In case the cavity volume; $V=4.4$ cm^3, $B_0 = 3x10$Tesla, $Q=10^4$, $\delta v = 4$MHz, $\Delta t=10^{-2}$ sec., $N=10N_0$, we will obtain for the impurity paramagnetic centers, (44); 10^{22}m^{-3} and for the average power of the emitted signal; 3μW.

Results and Discussion

The idea of applying classical mechanics, for the study of non-polar dielectric, offered a very simple formalism. The dielectric was first studied in an oscillating electric field from which its electric susceptibility as a function of vibrating electrons, their mass and the resonance frequency has been found. The relationship showed that the electric susceptibility is a complex number. We were then guided to establish a relationship for the imaginary part in order to establish a relationship for absorbed average power.

This average power was then calculated for two limits; $\omega=\omega_0$ and $\omega=0$, in order to estimate the band pass; δv, of the dielectric. Taking the imaginary part of the susceptibility and the average power into account, we were able to show that the energy of electromagnetic wave of the length; λ_{12}, traversing the dielectric medium was obeying an exponential rule. The corresponding relationship has also been established.

The sample was then put under the influence of an oscillating magnetic field. By applying a magnetic induction perpendicular to magnetic field, the splitting of ultra high frequency energy levels into two Zeeman sublevels has been occurred. The distribution of populations over these two sub-levels was found. We were then to calculate the absorbed average power as a function of the transition frequency, magnetic field, and the width of band pass δv.

Finally the sample has been located into a cavity resonator, which was tuned to the resonance frequency of the sample. We supposed that there existed a permanent radiation density, which was tending the substance to saturation transition. It was noticed that the sample was capable to emit a signal with the same average power, when the magnetic induction was rapidly switched from $+B_0$ to $-B_0$.

Knowing the value of quality factor; Q, of the sample, we were able to determine the number of paramagnetic centers in the sample.

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