

A Note on Retailer's Ordering Policy and Payment Policy under Trade Credit and Cash Discount

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Abstract: In present study, the model of Huang, 2004 has been modified and discussed with retailer's ordering policy under conditions of cash discount and trade credit. Mathematical model has been modified for obtaining the optimal cycle time and optimal payment policy for item under cash discount and trade credit so that the annual total cost is minimized. Then, a theorem is provided to efficiently determine the optimal cycle time and optimal payment policy. Finally, a numerical example is solved to illustrate the results obtained.

Key words: Cash discount, trade credit, inventory

INTRODUCTION

In practice, the supplier frequently offers the retailer a fixed delay period, that is the trade credit period, in settling the accounts. In real world, the supplier often makes use of this policy to stimulate his/her customer's demand.

Goyal^[1] established a single-item inventory model under permissible delay in payments. Chung^[2] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi^[3] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang *et al.*^[4] extended this issue to varying rates of deterioration. Liao *et al.*^[5] and Sarker *et al.*^[6] investigated this topic in the presence of inflation. Jamal *et al.*^[7] and Chang and Dye^[8] extended this issue with allowable shortage. Chang *et al.*^[9] extended this issue with linear trend demand. Chen and Chuang^[10] investigated a light buyer's inventory policy under trade credit by the concept of discounted cash flows. Hwang and Shinn^[11] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal *et al.*^[12] and Sarker *et al.*^[13] addressed the optimal payment time under permissible delay in payment with deterioration. Teng^[14] assumed that the selling price was not equal to the purchasing price to modify Goyal's model^[1]. Chung *et al.*^[15] discussed this issue under the assumptions that the selling price is not equal to the purchasing price and

different payment rules. Khouja and Mehrez^[16] investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Shinn and Hwang^[17] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung and Huang^[18] examined this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang^[19] extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer.

Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. From the viewpoint of the supplier, the supplier hopes that the payment is paid from retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the retailer. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. Recently, Huang and Chung^[20] extended Goyal's model^[1] to cash discount policy for early payment. Huang^[21] relaxed the assumption that the selling price was equal to the purchasing price in Huang and Chung^[20].

Model reformulation: For convenience, we adopt the same notation and assumptions as in Huang^[21].

Notation

- D = annual demand
- A = cost of placing one order
- c = unit purchasing price per item
- s = unit selling price per item
- h = unit stock holding cost per item per year excluding interest charges
- I_e = interest which can be earned per \$ per year
- I_p = interest charges per \$ investment in inventory per year
- r = cash discount rate, 0 < r < 1
- M₁ = the period of cash discount
- M₂ = the period of trade credit, M₁ < M₂
- T = the cycle time
- TVC₁₁(T) = the total relevant cost per unit time when payment is paid at time M₁ and T ≥ M₁
- TVC₁₂(T) = the total relevant cost per unit time when payment is paid at time M₁ and T ≤ M₁
- TVC₁(T) = the total relevant cost per unit time when payment is paid at time M₁ and T > 0

$$= \begin{cases} TVC_{11}(T) & \text{if } T \geq M_1 \\ TVC_{12}(T) & \text{if } T \leq M_1 \end{cases}$$

- TVC₂₁(T) = the total relevant cost per unit time when payment is paid at time M₂ and T ≥ M₂
- TVC₂₂(T) = the total relevant cost per unit time when payment is paid at time M₂ and T ≤ M₂
- TVC₂(T) = the total relevant cost per unit time when payment is paid at time M₂ and T > 0

$$= \begin{cases} TVC_{21}(T) & \text{if } T \geq M_2 \\ TVC_{22}(T) & \text{if } T \leq M_2 \end{cases}$$

TVC(T) = the total relevant cost per unit time when T > 0

Then, the annual total relevant cost functions is formulated.

$$TVC_{11}(T) = \frac{A}{T} + \frac{hDT}{2} + c(1-r)D + \frac{c(1-r)I_p D (T - M_1)^2}{2T} - \frac{sI_e DM_1^2}{2T} \tag{1}$$

$$TVC_{12}(T) = \frac{A}{T} + \frac{hDT}{2} + c(1-r)D - sDI_e(M_1 - \frac{T}{2}), \tag{2}$$

$$TVC_{21}(T) = \frac{A}{T} + \frac{hDT}{2} + cD + \frac{cI_p D (T - M_2)^2}{2T} - \frac{sI_e DM_2^2}{2T} \tag{3}$$

and

$$TVC_{22}(T) = \frac{A}{T} + \frac{hDT}{2} + cD - sDI_e(M_2 - \frac{T}{2}). \tag{4}$$

$$= \begin{cases} TVC_1(T) & \text{if the payment is paid at time } M_1 \\ TVC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$$

- T₁* = the optimal cycle time of TVC₁(T)
- T₂* = the optimal cycle time of TVC₂(T)
- T* = the optimal cycle time of TVC(T).

Assumptions

1. Demand rate is known and constant.
2. Shortages are not allowed.
3. Time period is infinite.
4. The lead time is zero.
5. I_p ≥ I_e and s ≥ c.
6. During the time the account is not settled, a deposit is made of the unit selling price of generated sales revenue into an interest bearing account. At the end of this period, the retailer pays off all items sold, keeps profits and starts paying for the interest charges on the items in stocks. It is assumed that the interest earned from profits is too small to neglect it.
7. Supplier offers a cash discount if payment is paid within M₁, otherwise the full payment is paid within M₂. The account is settled when the payment is paid.

However, we want to modify the item of interest payable in TVC₁₁(T) in Huang^[21]. Since the supplier offers cash discount if payment is paid within M₁. Therefore, in this case, the retailer will pay the annual purchasing cost, c(1-r)D, to the supplier. Then, the annual cost of interest charges for the items kept in stock is based on the annual purchasing cost, c(1-r)D. So, we modify the item of interest payable in TVC₁₁(T) in Huang^[21]

$$\text{to } \frac{cI_p D (T - M_1)^2}{2T} \rightarrow \frac{c(1-r)I_p D (T - M_1)^2}{2T}$$

So,

$$TVC_1(T) = \begin{cases} \frac{A}{T} + \frac{hDT}{2} + c(1-r)D + \frac{c(1-r)I_p D(T-M_1)^2}{2T} - \frac{sI_e DM_1^2}{2T} & \text{if } M_1 \leq T \\ \frac{A}{T} + \frac{hDT}{2} + c(1-r)D - sDI_e(M_1 - \frac{T}{2}) & \text{if } 0 < T \leq M_1 \end{cases} \quad (5a)$$

$$(5b)$$

and

$$TVC_2(T) = \begin{cases} \frac{A}{T} + \frac{hDT}{2} + cD + \frac{cI_p D(T-M_2)^2}{2T} - \frac{sI_e DM_2^2}{2T} & \text{if } M_2 \leq T \\ \frac{A}{T} + \frac{hDT}{2} + cD - sDI_e(M_2 - \frac{T}{2}) & \text{if } 0 < T \leq M_2 \end{cases} \quad (6a)$$

$$(6b)$$

At $T=M_1$, we find $TVC_{11}(M_1) = TVC_{12}(M_1)$. Hence $TVC_1(T)$ is continuous and well-defined. All $TVC_{11}(T)$, $TVC_{12}(T)$ and $TVC_1(T)$ are defined on $T > 0$. And at $T=M_2$, we find $TVC_{21}(M_2) = TVC_{22}(M_2)$. Hence $TVC_2(T)$ is continuous and well-defined. All $TVC_{21}(T)$, $TVC_{22}(T)$ and $TVC_2(T)$ are defined on $T > 0$.

Determination of the optimal cycle time T^* and optimal payment policy: From equations (1), (2), (3) and (4) yield

$$TVC_{11}'(T) = \frac{-[2A + DM_1^2(c(1-r)I_p - sI_e)]}{2T^2} + \frac{D[h + c(1-r)I_p]}{2} \quad (7)$$

$$TVC_{11}''(T) = \frac{2A + DM_1^2[c(1-r)I_p - sI_e]}{T^3} \quad (8)$$

$$TVC_{12}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2} \quad (9)$$

$$TVC_{12}''(T) = \frac{2A}{T^3} > 0. \quad (10)$$

$$TVC_{21}''(T) = \frac{-[2A + DM_2^2(cI_p - sI_e)]}{2T^2} + \frac{D(h + cI_p)}{2} \quad (11)$$

$$TVC_{21}''(T) = \frac{2A + DM_2^2(cI_p - sI_e)}{T^3} \quad (12)$$

$$TVC_{22}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2} \quad (13)$$

and

$$TVC_{22}''(T) = \frac{2A}{T^3} > 0. \quad (14)$$

Equations (10) and (14) imply that all $TVC_{12}(T)$ and $TVC_{22}(T)$ are convex on $T > 0$. However, equation (8) implies that $TVC_{11}(T)$ is convex on $T > 0$ if $2A + DM_1^2[c(1-r)I_p - sI_e] > 0$. Equation (12) implies that $TVC_{21}(T)$ is convex on $T > 0$ if $2A + DM_2^2(cI_p - sI_e) > 0$.

Let $TVC_{11}'(T_{11}^*) = TVC_{12}'(T_{12}^*) = TVC_{21}'(T_{21}^*) = TVC_{22}'(T_{22}^*) = 0$, we can obtain

$$T_{11}^* = \sqrt{\frac{2A + DM_1^2[c(1-r)I_p - sI_e]}{D[h + c(1-r)I_p]}} \quad (15)$$

if $2A + DM_1^2[c(1-r)I_p - sI_e] > 0$,

$$T_{12}^* = \sqrt{\frac{2A}{D(h + sI_e)}} \quad (16)$$

$$T_{21}^* = \sqrt{\frac{2A + DM_2^2(cI_p - sI_e)}{D(h + cI_p)}} \quad (17)$$

and if $2A + DM_2^2(cI_p - sI_e) > 0$

$$T_{22}^* = \sqrt{\frac{2A}{D(h + sI_e)}} \quad (18)$$

Then, we find $T_{12}^* = T_{22}^*$.

Equation (15) implies that the optimal value of T for the case of $T \geq M_1$, that is $T_{11}^* \geq M_1$. We substitute equation (15) into $T_{11}^* \geq M_1$, then we can obtain the optimal value of T if and only if $-2A + DM_1^2(h + sI_e) \leq 0$.

Similarly, equation (16) implies that the optimal value of T for the case of $T \leq M_1$, that is $T_{12}^* \leq M_1$. We substitute equation (16) into $T_{12}^* \leq M_1$, then we can obtain the optimal value of T if and only if $-2A + DM_1^2(h + sI_e) \geq 0$.

Likewise, equation (17) implies that the optimal value of T for the case of $T \geq M_2$, that is $T_{21}^* \geq M_2$. We substitute equation (17) into $T_{21}^* \geq M_2$, then we can obtain the optimal value of T if and only if $-2A + DM_2^2(h + sI_e) \leq 0$.

Finally, equation (18) implies that the optimal value of T for the case of $T \leq M_2$, that is $T_{22}^* \leq M_2$. We substitute

Table 1: Comparisons with our solution and Huang's solution (changing the parameter r)

Let A=\$300/order, D=3000 units/year, c=\$30/unit, s=\$60/unit, h=\$1/unit/year, I_p=\$0.2/\$/year, I_c=\$0.1/\$/year, M₁=0.05 year and M₂=0.1 year

r	Δ ₁	Δ ₂	Huang's solution		Our solution		PCP*
			T*	TVC _H (T*)	T*	TVC _O (T*)	
0.1	<0	<0	T ₁₁ * = 0.169031	83574.21	T ₁₁ * = 0.1761125	83571.36	0.003408
0.2	<0	<0	T ₁₁ * = 0.169031	74498.77	T ₁₁ * = 0.1842974	74486.77	0.016105
0.3	<0	<0	T ₁₁ * = 0.169031	65423.33	T ₁₁ * = 0.1938973	65394.80	0.043633
0.4	<0	<0	T ₁₁ * = 0.169031	56347.89	T ₁₁ * = 0.2053629	56294.01	0.095719
0.5	<0	<0	T ₁₁ * = 0.169031	47272.45	T ₁₁ * = 0.2193741	47182.49	0.190672
0.6	<0	<0	T ₁₁ * = 0.169031	38197.01	T ₁₁ * = 0.2370158	38057.56	0.366428
0.7	<0	<0	T ₁₁ * = 0.169031	29121.58	T ₁₁ * = 0.2601511	28915.27	0.713486
0.8	<0	<0	T ₁₁ * = 0.169031	20046.14	T ₁₁ * = 0.2923261	19749.35	1.502754

PCP(percentage cost penalty)=[{ TVC_H(T)- TVC_O(T*)}/ TVC_O(T*)}*100%

equation (18) into $T_{22}^* \leq M_2$, then the optimal value of T can be obtained if and only if $-2A+DM_2^2(h+sI_c) \geq 0$.

Furthermore, we let

$$\Delta_1 = -2A+DM_1^2(h+sI_c) \tag{19}$$

and

$$\Delta_2 = -2A+DM_2^2(h+sI_c). \tag{20}$$

Since $M_1 < M_2$, equations (19) and (20) we can get $\Delta_1 < \Delta_2$. From above arguments, the optimal cycle time T* and optimal payment policy (M₁ or M₂) can be obtained as follows:

Theorem 1

- A: If $\Delta_1 > 0$, then $TVC(T^*) = \min\{TVC_1(T_{12}^*), TVC_2(T_{22}^*)\}$. Hence $T^* = T_{12}^* = T_{22}^*$ and optimal payment time is M₁ or M₂ associated with the least cost.
- B: If $\Delta_2 < 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(T_{21}^*)\}$. Hence T* is T₁₁* or T₂₁*, optimal payment time is M₁ or M₂ associated with the least cost.
- C: If $\Delta_1 < 0$ and $\Delta_2 > 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(T_{22}^*)\}$. Hence T* is T₁₁* or T₂₂*, optimal payment time is M₁ or M₂ associated with the least cost.
- D: If $\Delta_1 = 0$ and $\Delta_2 > 0$, then $TVC(T^*) = \min\{TVC_1(M_1), TVC_2(T_{22}^*)\}$. Hence T* is M₁ or T₂₂*, optimal payment time is M₁ or M₂ associated with the least cost.
- E: If $\Delta_1 < 0$ and $\Delta_2 = 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(M_2)\}$. Hence T* is T₁₁* or M₂, optimal payment time is M₁ or M₂ associated with the least cost.

Theorem 1 immediately determines the optimal cycle time T* and optimal payment policy (M₁ or M₂) after computing the numbers Δ₁ and Δ₂. Theorem 1 is really very simple.

A numerical example: To illustrate the results, apply the proposed method to solve the following numerical example to investigate the percentage cost penalty (PCP) between our solution and Huang's solution^[21] changing the parameter of the cash discount rate r. For convenience, the numbers of the parameters are selected randomly.

From Table 1, the optimal cycle time in Huang's solution^[21] will not change when cash discount rate is increasing in this example. Since the optimal cycle time in Huang's solution^[21] in this case is independent of the cash discount rate r. In addition, it is also find the percentage cost penalty is increasing when the cash discount rate is increasing. So, the decision-maker may cause significant errors and penalties using wrong model when the cash discount rate is larger.

The supplier offers the trade credit to stimulate the demand of the retailer. However, the supplier can also use the cash discount policy to attract retailer to pay the full payment of the amount of purchasing cost to shorten the collection period. The credit term that contains cash discount is very realistic in real-life business situations. This note reformulates the retailer's inventory replenishment model in this situation and modifies Huang's model^[21]. Finally, it is concluded that the decision-maker may cause significant errors and penalties using wrong model when the cash discount rate is larger.

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