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Mathematical Aspect for Worm Grinding Using a Toroidal Tool

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Abstract: This paper is an attempt to improve the accuracy of the worm gearings with concave profile of the worm thread by using a new generating surface of grinding wheel to eliminate the lacks of the popular worm gearings and extensioning the field of investigation. The present study propose grind concave profile worms by means of toroidal tool – grinding wheel which section in the axial plane is an arc of parabola. The generating equation of the grinding wheel surface and the arrangement of this wheel with respect to the worm during grinding of its thread was determined.

Key words: Grinding accuracy, tool axial section, ellipse, parabola, radius-vector, arbitrary point

INTRODUCTION

The need for finishing operations on gears after the teeth have been cut in the blank arises from the newer-ending quest to improve the accuracy of the product, solve the question of quiet running, and enhance the strength of the gears so that the unit may be small and compact. The process relating to the finishing of the teeth after the main cutting operation must be chosen to produce a smooth and accurately finished tool profile that will provide the maximum line of contact with the minimum stress concentration at any one point.

The worm gearings with concave profile of the worm thread occupy singular place among the worm gearings. In contrast to worms of classical worm gearings (Archimedes, thread-convolute and involute worms), the worms of this type have a concave profile in the axial and normal sections. According to its loading capacity and efficiency, the worm gearings with concave profile of the worm threads approach to globoidal worms, and considerably exceed on these parameters the classical worm gearings with cylindrical worm.

In the previous study[1-5] the worms of concave profile were processed with grinding wheel of an arc of circle in the axial section of the generating surface. The arc radius ρ[1,2,5] was chosen equaled the worm pitch radius and the angle crossing the worm and tool axes equaled the helix angle on the ρ radius cylinder. The disadvantages of this worm gearing as pointed[3] are: the decreasing of the grinding wheel diameter by the resharpening process that decreases the shortest distance between the tool and worm axes and this deteriorates the worm ground surface; the hazard to seizure is not completely eliminated because of the form of the contact line in the worm gearing.

The contact line of the worm and the tool surfaces may be presented as a plane curve (not spatial) that coincides with the tool axial profile[4,6]. This attained through the special calculated parameters of the tool setting. In this case, the worm surface can be defined by simple equations. The form of the contact line of the worm and worm wheel in such worm gearings is more favorable than the above mentioned. The tooth under-cut must be assigned among the disadvantages of this worm gearing[3].

Illes[7] has suggested more improved variant of the worm gearing than that studied in Litvin[3] but has the same disadvantages.

For eliminating the lacks of the popular worm gearings with concave profile of the worm and extensioning the field of investigation, Abdullah[8] has suggested to perform a search for new generating surface of grinding wheel. In the capacity of axial section of this generating surface, it was suggested to use the arc of ellipse instead of arc of circle, which has applied in popular worm gearings with concave profile of the worm thread[8].

In this study, the continue search for newer generating surfaces of the grinding wheel for processing worms with concave profile of threads. The present study propose grind concave profile worms by means of toroidal tool–grinding wheel which section in the axial plane is an arc of parabola. Moreover, the generating equation of the grinding wheel surface and the arrangement of this wheel with respect to the worm during grinding of its thread was determined.

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Mathematical model: It is well known that the ellipse is related to the lines of the second order. In the group of lines of second order, the parabola and hyperbola can be also contained in. An important characteristic of this type of lines is the eccentricity (ε), which for ellipse ranged from 0 to 1 ($0<\varepsilon<1$), for parabola equals 1 ($\varepsilon=1$) and for hyperbola more than 1 ($\varepsilon>1$). Consequently, in the capacity of new axial section of the grinding wheel for processing worms with concave profile, the arc of parabola and hyperbola may be used.

In the present investigation the arc of the parabola as an axial section of the grinding wheel for processing worms with concave profile was examined and the equation for the generating surface of this grinding wheel was defined. The arrangement of this wheel with respect to the worm during grinding of its threads was also defined.

The ellipse equation was presented[8] as following:

$$\frac{X_p^2}{a_p^2} + \frac{Y_p^2}{b_p^2} = 1 \qquad (1)$$

where, a_p = the large semiaxis of the ellipse, located along X axis, b_p= the small semiaxis of the ellipse, located along Y axis.

Moreover, the arc of the ellipse (eq. 1) serves as the axial section, which may be circumscribed by the following equation:

$$X_p = -\sqrt{a_p^2 - Y_p^2 \times \frac{a_p^2}{b_p^2}} \qquad (2)$$

where, $Y_p \in [0; b_p]$

In the coordinate system O_p (Fig. 1), the point (M) can be determined by the coordinates M ($a_p \cos \upsilon_p$, $b_p \sin \upsilon_p$), where, υ_p = the angle between the positive direction of the X_p axis and radius vector defining this point of ellipse, measurable counterclockwise. Preserving the generality of the work[8], let us introduce the coordinate system O_e, in which the desired parabola will be defined. In this system, the X_e axis coincide with X_p, the Y_e is parallel to Y_p and the origin of coordinates O_e is displaced with respect to the origin O_p for a magnitude a_p in the contrariwise side from the X_p axis direction. In the system of coordinates O_e the point M is located on the arc of parabola, which may be described with the following equation:

$$X_e = \frac{Y_e^2}{2P_e} \qquad (3)$$

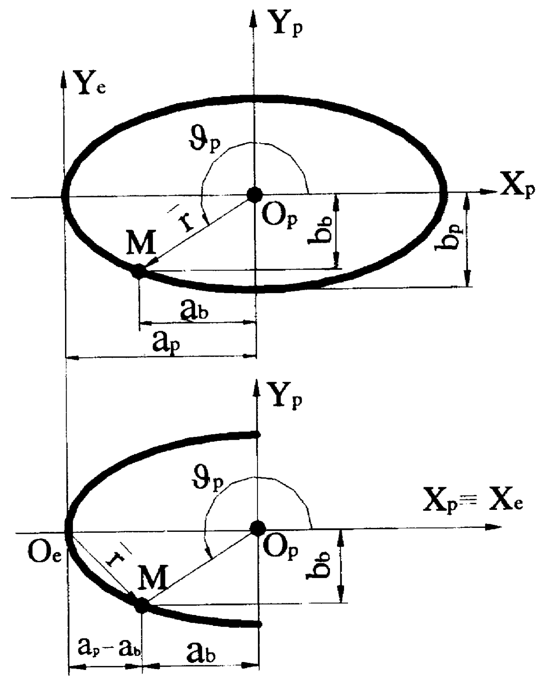

Fig. 1: The arrangement of coordinate system in which the eq. of parabola defined

where, $Y_e \in [0; b_p]$

Let us determine the parameter of the parabola for the point M, belonging to the arc of ellipse described in eq. (2) in the coordinate system O_p and the arc of parabola described in eq. (3) in the O_e system.

According to Fig. 1, the X coordinate of point M in the coordinate system O_e is defined as:

$$X_{em} = a_p + a_p \cos \vartheta_p \qquad (4)$$

where, υ_p = the angle between the positive direction of the axis X_p and radius vector defining this point of ellipse measurable counterclockwise[8].

According to Fig. 1, the Y coordinate of point M in the coordinate system O_e is defined as:

$$Y_{em} = b_p \sin \vartheta_p \qquad (5)$$

By substituting (4) and (5) in eq. (3) we obtain:

$$a_p + a_p \cos \vartheta_p = \frac{(b_p \sin \vartheta_p)^2}{2P_e} \qquad (6)$$

After the mathematical transformation of the eq. (6) we get:

$$p_e = \frac{b_p^2 \sin^2 \vartheta_p}{2a_p(1 + \cos \vartheta_p)} \qquad (7)$$

Under $\upsilon_p=225°$, the eq. (7) will have the following form:

$$p_e = \frac{b_p^2}{a_p \times 2(2-\sqrt{2})}$$

Figure 2 shows the relation between the parameter of the parabola P_e and the length of the large semiaxis of the corresponding ellipse a_p under r_0=46 mm. This relation is obtained on the basis of work[8] and eq. (7).

Therefore, the equation of the unknown parabola passing through point M and with regard to eq. (7) will be in the coordinate system O_e as the following:

$$X_e = \frac{Y_e^2}{\left[\frac{b_p^2 \sin^2 \vartheta_p}{2a_p(1+\cos \vartheta_p)}\right]} \quad (8)$$

The analysis of Fig. 2 shows that for every ellipse in the work[8] corresponds a unique parabola (eq. 8), where the increase of a_p calls to diminution of P_e.

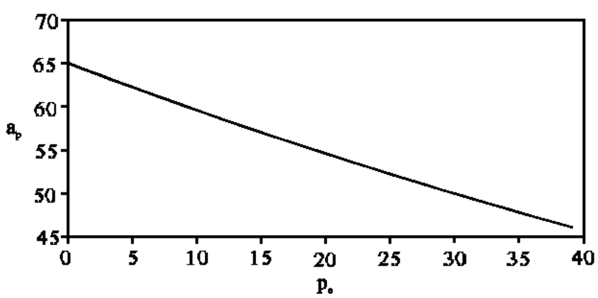

Fig. 2: The relationship between the parameter of the parameter of the parabola (P_e) and the length of the large semiaxis of the corresponding ellipse (a_p) under r_0=46 mm

For the subsequent calculations the eq. (8) represented in the matrix form.

The column of the radius vector defining arbitrary point of the lower parabola in coordinate system O_e is set in the following shape:

$$\bar{r} = \begin{vmatrix} X_e \\ -X_e\left[\frac{b_p^2 \sin^2 \vartheta_p}{2a_p(1+\cos \vartheta_p)}\right] \\ 0 \\ 1 \end{vmatrix} \quad (9)$$

where, $X_e \in [0; a_b]$

We find the eq. (9) in the coordinate system O_p:

$$\bar{r}_p = M_{pe}\,\bar{r}_e \quad (10)$$

where, M_{pe}= the conversion matrix from the system of coordinates O_e to O_p.

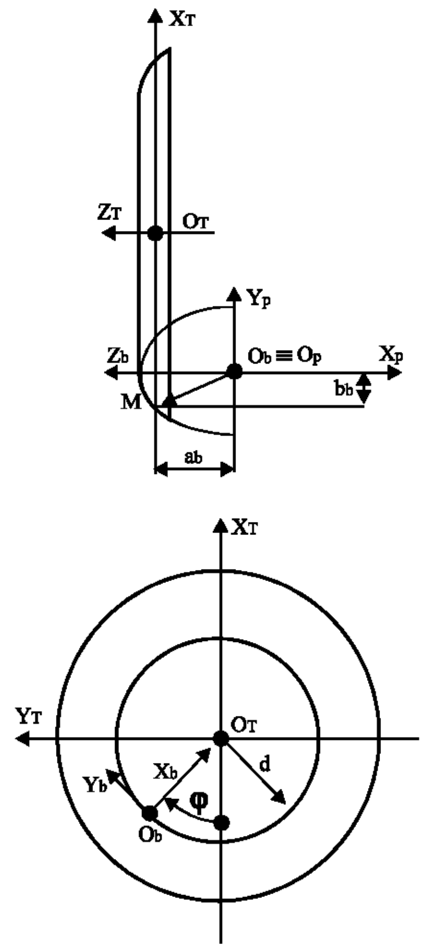

Fig. 3: The surface of the grinding wheel

According to Fig. 1, the matrix M_{pe} is defined as:

$$M_{pe} = \begin{vmatrix} 1 & 0 & 0 & -a_p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (11)$$

By substituting (9) and (11) in eq. (10) we obtain:

$$\bar{r}_p = \begin{vmatrix} X_e - a_p \\ -\sqrt{X_e\left[\frac{b_p^2 \sin^2 \vartheta_p}{2a_p(1+\cos \vartheta_p)}\right]} \\ 0 \\ 1 \end{vmatrix} \quad (12)$$

Figure 3 shows the axial section of the grinding wheel surface in the form of an arc of parabola, whereas, Fig. 4 shows the axial section of the worm and the tool cut by vertical plane passing through the mean point of the worm profile.

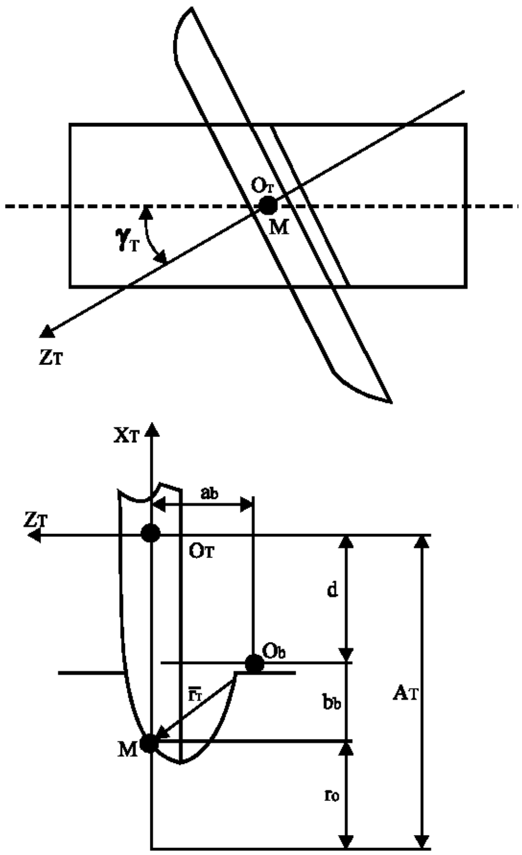

Fig. 4: The arrangement of the grinding wheel during processing of worm surface

Assuming that the line of the shortest distance between the axes of the worm and the tool as A_T and as in the work[8] this line passes through the mean point M of the worm profile. In this point, the length of the radius vector defining its position on the arc of parabola equals the pitch radius of the worm r_0.

The coordinate system O_b is an auxiliary, whereas, the arc of parabola prescribed eq. (12) given in coordinate system O_p. The origins of these coordinate systems are coinciding.

Correlating the grinding wheel with the coordinate system O_T as shown in Fig. 3.

The location of the center of coordinate system O_p in the coordinate system O_T is defined by the value a_b and b_b concerning to the mean point M located on the worm profile. These values are taken according to Abdullah[8].

The radius of the grinding wheel equals the sum of values b_b and d- the distance from the center of the coordinate system O_T to O_b measured along X_T axis as shown in Fig. 4.

The distance A_T between the axes is defined by the following relationship:

$$A_T = r_0 + d + b_b \tag{13}$$

The angle γ_T crossing the axes of the worm and the tool (Fig. 4) is chosen equaled the helix angle on the cylinder of the radius.

The generating surface of the grinding wheel will be formed by rotating the profile (8) around the axis of the grinding wheel according to Fig. 3. Here

$$\bar{r}_T = M_{Tp}\,\bar{r}_p \tag{14}$$

where, $\bar{r}$ = the column of the radius vector of the profile point in coordinate system O_T. M_{Tp} = the transformation matrix from coordinate system O_p to O_T.

The transformation matrix may be defined by the following:

$$M_{Tp} = M_{Tb}\,M_{bp} \tag{15}$$

where, M_{bp}= the transformation matrix from coordinate system O_p to O_b, and M_{Tp} = the transformation matrix from coordinate system O_p to O_T.

The transformation matrix from coordinate system O_p to O_b in accord to Fig. 3 is defined as:

$$M_{Tb} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{16}$$

The transformation matrix from coordinate system O_b to O_T corresponding to Fig. 4 is defined as:

$$M_{Tb} = \begin{vmatrix} \cos\varphi_T & \sin\varphi_T & 0 & -d\cos\varphi_T \\ -\sin\varphi_T & \cos\varphi_T & 0 & d\cos\varphi_T \\ 0 & 0 & 1 & -a_p\cos\varphi_p \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{17}$$

where φ_p = the rotating angle of the coordinate system O_b relatively to O_T (Fig.3).

Thus, the transformation matrix from the coordinate system O_p to O_T with regard to eq. (15), (16) and (17) is defined as:

$$M_{Tp} = \begin{vmatrix} 0 & \cos\varphi_T & -\sin\varphi_T & -d\cos\varphi_T \\ 0 & -\sin\varphi_T & \cos\varphi_T & d\sin\varphi_T \\ -1 & 0 & 0 & -a_p\cos\varphi_p \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{18}$$

Consequently, the surface of the grinding wheel on the basis of eq. (12), (14) and (18) can defined by the following equation:

$$\bar{r}_T = \begin{vmatrix} \cos\varphi_T \left[-\sqrt{X_e \left[\dfrac{b_p^2 \sin^2 \vartheta_p}{2a_p(1+\cos\vartheta_p)} \right]} \right] - d \\ \sin\varphi_T \left[d + \sqrt{X_e \left[\dfrac{b_p^2 \sin^2 \vartheta_p}{2a_p(1+\cos\vartheta_p)} \right]} \right] \\ -(X_e - a_p) - a_p \cos\vartheta_p \\ 1 \end{vmatrix} \qquad (19)$$

Using of eq. (19) the eq. For the worm thread surface machined by the mentioned tool can be defined and experimented.

As a result of the conducted investigation the following results were established:

1. The axial section of the grinding wheel used to machine worms with concave profile may be represented as an arc of parabola situated according to eq. (3)-(13).
2. There are a number of continuous series of parabolas, each of which passes through the mean point M of the worm profile and corresponds to ellipse examined in work of Abdullah[8].
3. The generating surface of the grinding wheel is defined by eq. (13).
4. The arrangement of the grinding wheel with respect to worm during grinding and its feed are produced corresponding to Fig. 4 and eq. (3)-(19).

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