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Solution of Fredholm-Volterra Integral Equation of the Second Kind in Series Form

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Abstract: This study considers an integral equation of Fredholm-Volterra type, where the Fredholm integral term is measured with respect to the position, while Volterra integral term is measured with respect to the time. Also, we obtained the solution of Fredholm-Volterra integral equation in series form.

Key words: Integral Equation-Fredholm-Volterra-Solution in series form

INTRODUCTION

Certain problems of mathematical physics lead to a Fredholm integral equation and others to Volterra integral equation. Many authors[1-7] discussed the analytical solution for different types of integral equation. Other authors[8-11] obtained the numerical solution for integral equations with different types of kernels. Many applied papers[12,13] solve integrodifferential equation for an elastic infinite plate with a curvilinear hole having different types of poles by using Cauchy integral method in complex plane. For solving the Fredholm-Volterra integral equation we must use[14].

Formulation of the problem: In this study it is considered the Fredholm-Volterra integral equation as

$$\mu\phi(x,t)+\int_{\Omega}k(x,t)\phi(y,t)dy+\int_{0}^{t}F(\tau)\,d\tau=[\gamma(t)+x\beta(t)-f_{*}(x)]=f(x,t) \quad (1)$$

under the conditions:

$$N_1(t)=\int_{\Omega}\phi(x,t)dx;$$

$$N_2(t)=\int_{\Omega}x\phi(x,t)dx \quad (2)$$

Where, the kernel k(x,t)ϵ c((Ω))x(Ω)) is symmetric and satisfies the condition.

$$\left\{\int\int_{\Omega}k^2(x,y)dxdy\right\}^{\frac{1}{2}}=\text{constant}<\infty$$

The functions $\gamma(t)$, $\beta(t)$ and F(t) are positive and continuous functions belong to the class C(0,T) and $f_*(x)$ is a continuous function belongs to $L_2(\Omega)$, $\mu\in(0,\infty)$ and the unknown function $\phi(x,t)$ is continuous and belongs to the space $L_2(\Omega)\times C(0,T)$.

The integral equation (1) is investigated from the contact problem of frictionless impressed of a rigid surface (G, ν) having an elastic material occupying the domain Ω, where. $f_*(x)$, $x = x(x_1, x_2, x_3)$ describing the surface of stamp. This stamp is impressed into an elastic layer surface by a variable force $N_1(t)$, whose eccentricity of application e(t) and a variable known moment $N_2(t)$, $0\le t\le T\le\infty$, that case rigid displacements $\gamma(t)$, $x\beta(t)$ respectively, Here G is the displacement. magnitude, ν is the Poisson's coefficient, μ is composed of several physical properties and F(t) represent the characterized resistance function of the material.

For t=0, equation (1) reduced to

$$\mu\phi(X,0)+\int_{\Omega}k(x,y)\phi(y,0)\,dy=[\gamma(0)+x\beta(0)-f_{*}(x)]=f(x,0) \quad (3)$$

which represents Fredholm integral equation of the second kind solved in[5].

Solution of the problem: In the problem (1) the unknown function is $\phi(x,t)$, if we write

$$\phi(x,t)=\phi_0(x,t)+\phi_1(x,t) \quad (4)$$

where, $\phi_0(x,t)$ and $\phi_1(x,t)$ are the complementary and particularly solution of (1), respectively.

Using (4) in (1) and (3), Then subtract the result, we have.

$$\mu[\varphi_j(x,t)-\varphi_j(x,0)]+\int_{\Omega}k(x,y)[\varphi_j(y,t)-\varphi_j(y,0)]dy+\int_{\Omega}F_j(\tau\tau)_j(x,\tau)d\tau=\delta_j[\gamma(t)-\gamma(0)+(\beta(t)-\beta(0))x]$$

$$(j=0,1) \quad (5)$$

Where, $\delta_0=1$, $\delta_1=0$

We can assume the approximate solution of (2.1) in the following series expression form.

$$\phi_j(x,t)=\sum[A^j_{2k}(t)\phi_{2k}(x)+A^j_{2k-1}(t)\phi_{2k}(x)] \quad (6)$$

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Here, in (6) the solution is represented in the form of even and odd terms, respectively.

Theorem.1:

[14,15]The integral operator

$$K\phi = \int_{\Omega} k(x,y)\phi(y)dy \qquad (7)$$

for symmetric and positive kernel, is compact and self adjoint operator from $L_2(\Omega)$ to $L_2(\Omega)$ so, we can write (7) as

$$\lambda_n k\,\phi = \phi_n$$

Where, λ_n and ϕ_n are the eigenvalue and eigenfunction of the integral operator (7), respectively.

Substituting from (6) into (5) and then using theorem 1, we have the following

$$A_k^{(0)}(t) + \alpha_k \int_0^t A_k^{(1)}(\tau)F(\tau)d\tau = A_k^{(1)}(0) \qquad ;\ (j=1)$$

$$A_{2k}^{(0)}(t) + \alpha_{2k} \int_0^t A_{2k}^{(0)}(\tau)F(\tau)F(\tau)d\tau = \alpha_{2k}d_{2k}[\gamma(t) - \gamma(0)]$$

(j=0)

$$A_{2k-1}^{(0)}(t) + \alpha_{2k-1} \int_0^t A_{2k-1}^{(0)}(\tau)F(\tau)d\tau = \alpha_{2k-1}d_{2k-1}[\beta(t) - \beta(0)],\ (j=1) \qquad (8)$$

where:

$$A_{2k}^{(0)}(0) = 0, \qquad \sum_{k=1} d_{2k} \quad \phi_{2k} = 1,$$

Equations (8) represents Volterra integral equations of the second kind with continuos kernel $F(t) \in C(0,T)$ and the solution of (8) can be obtained directly[4,15],

One must note that the value of $A_k^{(1)}(0)$ can be obtained by using (6) in (3) to have $A^{(1)}{}_k(0) = \alpha_k\gamma(0)$ In view of the second and third equations of (8).

The general solution of (6) can be adapted in the form

$$\phi_N(x,t) = \sum_{k-1}^{N} \left[A_k^{(0)}(t) + A_k^{(0)}(t)\right]\phi_K(x)$$

where, $A_k^{(0)}(t)$ and $A_k^{(0)}(t)$ must satisfy the inequality[15].

Theorem . 2: If $f(x,t) \in L_2(\Omega) \times C(0,T)$, $K(x,y) \in ((\Omega) \times (\Omega))$,

then Lim $\quad \lim_{n \to \infty} \| \phi_N(x,t) - \phi(x,t) \| \to 0$

Where, $\phi(x,t)$ is a unique solution of (1).

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