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Optimal Cycle Time and Optimal Payment Time under Supplier Credit

Yung-Fu Huang
Department of Business Administration, Chaoyang University of Technology,
Taichung, Taiwan, Republic of China

Abstract: This study investigate the retailer's optimal cycle time and optimal payment time under supplier's trade credit policy and cash discount policy. Mathematical models have been derived for obtaining the optimal cycle time and optimal payment policy for item under supplier's trade credit and cash discount so that the annual total relevant cost is minimized. Furthermore, numerical examples have given to illustrate the results developed in this study and a lot of managerial insights have obtained.

Key words: Inventory, trade credit, cash discount

INTRODUCTION

In real world, the supplier often makes use of trade credit policy to promote his/her commodities. The effect of supplier credit policy on optimal order quantity has received the attention of many researchers[1-15].

Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. From the viewpoint of the supplier, the supplier hopes that the payment is paid from retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the retailer. The retailer can obtain the cash discount when the payment is paid before cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. Many articles are related to the inventory policy under cash discount and trade credit[16,17].

This study tries to use the payment rule[6] and the cash discount[17] to develop the retailer's inventory model, then, to determine the optimal cycle time and optimal payment time for the retailer under cash discount and trade credit so that the annual total relevant cost is minimized.

Model formulation: For convenience, following notation and assumptions are used[6,17].

Notation

D = Demand rate per year

A = Cost of placing one order

c = Unit purchasing price per item

s = Unit selling price per item

h = Unit stock holding cost per item per year excluding interest charges

I_e = Nterest earned per \$ per year

I_p = Interest charges per \$ investment in inventory per year

r = Cash discount rate, $0 < r < 1$

M_1 = The period of cash discount in years

M_2 = The period of permissible delay in payments in years, $M_1 < M_2$

T = the cycle time in years

$TRC_{11}(T)$ = The total relevant cost per unit time when payment is paid at time M_1 and $T \geq M_1$

$TRC_{12}(T)$ = The total relevant cost per unit time when payment is paid at time M_1 and $T \leq M_1$

$TRC_1(T)$ = The total relevant cost per unit time when payment is paid at time M_1 and $T > 0$

$$= \begin{cases} TRC_{11}(T) & \text{if } T \geq M_1 \\ TRC_{12}(T) & \text{if } T \geq M_1 \end{cases}$$

$TRC_{21}(T)$ = The total relevant cost per unit time when payment is paid at time M_2 and $T \geq M_2$

$TRC_{22}(T)$ = The total relevant cost per unit time when payment is paid at time M_2 and $T \leq M_2$

Corresponding Author: Yung-Fu Huang, No.168, Jifong E. Road, Wufong Township, Taichung County 41349, Taiwan, Republic of China Tel: + 886 4 247 39 477 Fax: + 886 4 247 29 772 E-mail: huf@mail.cyut.edu.tw

$TRC_2(T)$ = The total relevant cost per unit time when payment is paid at time M_2 and $T > 0$

$$= \begin{cases} TRC_{21}(T) & \text{if } T \geq M_2 \\ TRC_{22}(T) & \text{if } T \geq M_2 \end{cases}$$

$TRC(T)$ = The total relevant cost per unit time when $T > 0$

$$= \begin{cases} TRC_1(T) & \text{if the payment is paid at time } M_1 \\ TRC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$$

T_1^* = The optimal cycle time of $TRC_1(T)$
T_2^* = The optimal cycle time of $TRC_2(T)$
T^* = The optimal cycle time of $TRC(T)$
Q^* = The optimal order quantity=DT^*.

Assumptions

1. Demand rate is known and constant
2. Shortages are not allowed
3. Time horizon is infinite
4. Replenishments are instantaneous with a known and constant lead time
5. $s \geq c$ and $I_p \geq I_e$
6. Supplier offers a cash discount if payment is paid within M_1, otherwise the full payment is paid within M_2
7. When the retailer must pay the amount of purchasing cost to the supplier, the retailer will borrow 100% purchasing cost from the bank to pay off the account. When $T \geq M_1$ or M_2, the retailer returns money to the bank at the end of the inventory cycle. However, when $T \leq M_1$ or M_2, the retailer returns money to the bank at $T = M_1$ or M_2
8. If the credit period is shorter than the cycle length, the retailer can sell the items, accumulate sales revenue and earn interests throughout the inventory cycle

The model: The annual total relevant cost consists of the following elements

1. Annual ordering cost = $\frac{A}{T}$.
2. Annual stock holding cost (excluding interest charges) = $\frac{DTh}{2}$.
3. Annual purchasing cost:

Case 1: Payment is paid at time M_1, the annual purchasing cost = $c(1-r)D$.
Case 2: Payment is paid at time M_2, the annual purchasing cost = cD.

Since the supplier offers a cash discount if payment is paid within M_1, there are two payment policies for the retailer. First, the payment is paid at time M_1 to get the cash discount, Case 1. Second, the payment is paid at time M_2 not to get the cash discount, Case 2. So the interest payable and interest earned, we will discuss these two cases as follows.

4. Cost of interest charges for the items kept in stock per year:

Case 1: Payment is paid at time M_1
Case 1.1: $T \geq M_1$

According to assumption (7), in this case, the retailer must borrow annual purchasing cost, $c(1-r)D$, from M_1 to the end of inventory cycle. So,

Cost of interest charges for the items kept in stock per year = $c(I-r)I_PD(T-M_1)$.

Case 1.2: $T \leq M_1$

In this case, cost of interest charges for the items kept in stock per year = 0.

Case 2: Payment is paid at time M_2
Case 2.1: $T \geq M_2$

According to assumption (7), in this case, the retailer must borrow annual purchasing cost, cD, from M_2 to the end of inventory cycle. So,

Cost of interest charges for the items kept in stock per year = $cI_PD(T-M_2)$.

Case 2.2: $T \leq M_2$

In this case, cost of interest charges for the items kept in stock per year = 0.

5. Interests earned per year:

Case 1: Payment is paid at time M_1
Case 1.1: $T \geq M_1$

According to assumption (8), in this case, the retailer can earn interests throughout the inventory cycle. So, Interests earned per year = $\frac{DTsI_e}{2}$.

Case 1.2: $T \le M_1$

According to assumption (8), in this case, the retailer can earn interests until the end of M_1. So, Interests earned per year = $DsI_e(M_1 - \frac{T}{2})$.

Case 2: Payment is paid at time M_2

Case 2.1: $T \ge M_2$

Same as above case 1.1, the retailer can earn interests throughout the inventory cycle. So, Interests earned per year = $\frac{DTsI_e}{2}$.

Case 2.2: $T \le M_2$

According to assumption (8), in this case, the retailer can earn interests until the end of M_2. So, Interests earned per year = $DsI_e(M_2 - \frac{T}{2})$.

From the above arguments, the annual total relevant cost for the retailer can be expressed as: Annual total relevant cost = ordering cost+stock-holding cost+purchasing cost+interest payable-interest earned.
We show that the annual total relevant cost is given by
Case 1: Payment is paid at time M_1

$$TRC_1(T) = \begin{cases} \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + c(1-r)I_pD(T-M_1) - \frac{DTsI_e}{2} & \text{if} \quad M_1 \le T \quad (1a) \\ \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DsI_e(M_1 - \frac{T}{2}) & \text{if} \quad 0 < T \le M_1 \quad (1b) \end{cases}$$

Let

$$TRC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + c(1-r)I_pD(T-M_1) - \frac{DTsI_e}{2} \tag{2}$$

and

$$TRC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DsI_e(M_1 - \frac{T}{2}). \tag{3}$$

At $T=M_1$, we find $TRC_{11}(M_1)=TRC_{12}(M_1)$. Hence $TRC_1(T)$ is continuous and well-defined. All $TRC_{11}(T)$, $TRC_{12}(T)$ and $TRC_1(T)$ are defined on $T > 0$.
Case 2: Payment is paid at time M_2

$$TRC_2(T) = \begin{cases} \frac{A}{T} + \frac{DTh}{2} + cD + cI_pD(T-M_2) - \frac{DTsI_e}{2} & \text{if} \quad M_2 \le T \quad (4a) \\ \frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_2 - \frac{T}{2}) & \text{if} \quad 0 < T \le M_2 \quad (4b) \end{cases}$$

Let

$$TRC_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_pD(T-M_2) - \frac{DTsI_e}{2} \tag{5}$$

and

$$TRC_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_2 - \frac{T}{2}). \tag{6}$$

At $T=M_2$, we find $TRC_{21}(M_2)=TRC_{22}(M_2)$. Hence $TRC_2(T)$ is continuous and well-defined. All $TRC_{21}(T)$, $TRC_{22}(T)$ and $TRC_2(T)$ are defined on $T > 0$.

When the cash discount is neglected, our model is reduced to that of Chung *et al.*[6].

Optimality conditions: From equations (2), (3), (5) and (6) yield

$$TRC_{11}'(T) = \frac{-A}{T^2} + \frac{D[h + 2c(1-r)I_p - sI_e]}{2}, \tag{7}$$

$$TRC_{21}'(T) = \frac{-A}{T^2} + \frac{D(h + 2cI_p - sI_e)}{2} \tag{8}$$

$$TRC_{11}''(T) = TRC_{21}''(T) = \frac{2A}{T^3} > 0, \tag{9}$$

$$TRC_{12}'(T) = TRC_{22}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2} \tag{10}$$

and

$$TRC_{12}''(T) = TRC_{22}''(T) = \frac{2A}{T^3} > 0. \tag{11}$$

Equations (9) and (11) imply that all $TRC_{11}(T)$, $TRC_{12}(T)$, $TRC_{21}(T)$ and $TRC_{22}(T)$ are convex on $T > 0$. However, $TRC_{11}'(M_1) \neq TRC_{12}'(M_1)$ and $TRC_{21}'(M_2) \neq TRC_{22}'(M_2)$. So, both $TRC_1(T)$ and $TRC_2(T)$ are piecewise convex but not convex in general.

Decision rule of the optimal cycle time and optimal payment time: Let $TRC_i'(T_{ij}^*) = 0$ for all i = 1, 2 and j = 1, 2. We can obtain

$$T_{11}^* = \sqrt{\frac{2A}{D[h + 2c(1-r)I_p - sI_e]}} \quad \text{if} \quad h + 2c(1-r)I_p - sI_e > 0 \tag{11}$$

$$T_{21}^* = \sqrt{\frac{2A}{D(h + 2cI_p - sI_e)}} \quad \text{if} \quad h + 2cI_p - sI_e > 0 \tag{12}$$

and

$$T_{12}^* = T_{22}^* = \sqrt{\frac{2A}{D(h + sI_e)}} \tag{13}$$

Equation (11) implies that the optimal value of T for the case of $T \geq M_1$, that is $T_{11}^* \geq M_1$. We substitute Equation (11) into $T_{11}^* \geq M_1$, then we can obtain the optimal value of T

$$\text{if and only if } -2A + DM_1^2[h + 2c(1-r)I_p - sI_e] \leq 0 \tag{14}$$

Likewise, Equation (13) implies that the optimal value of T for the case of $T \leq M_1$, that is $T_{12}^* \leq M_1$. We substitute Equation (13) into $T_{12}^* \leq M_1$, then we can obtain the optimal value of T

$$\text{if and only if } -2A + DM_1^2(h + sI_e) \geq 0 \tag{15}$$

Similar disscussion, we can obain following results:

$$T_{21}^* \geq M_2 \text{ if and only if } -2A + DM_2^2(h + 2cI_p - sI_e) \leq 0 \tag{16}$$

$$T_{22}^* \leq M_2 \text{ if and only if } -2A + DM_2^2(h + sI_e) \geq 0 \tag{17}$$

Furthermore, we let

$$\Delta_{11} = -2A + DM_1^2[h + 2c(1-r)I_p - sI_e] \tag{18}$$

$$\Delta_{12} = -2A + DM_1^2[h + sI_e] \tag{19}$$

$$\Delta_{21} = -2A + DM_2^2\,[h + 2cI_p - sI_e] \tag{20}$$

and

$$\Delta_{22} = -2A + DM_2^2\,(h + sI_e] \tag{21}$$

Since $M_1 < M_2$, we can get $\Delta_{11} < \Delta_{21}$ and $\Delta_{12} < \Delta_{22}$ from Equation (18) to Equation (21). Summarized above Equation (14) to Equation (17), the optimal cycle time T* and optimal payment time (M_1 or M_2) can be obtained as follows.

Theorem 1

A. If $\Delta_{11} \geq 0$ and $\Delta_{12} > 0$, then TRC(T*) = min{$TRC_1(T_{12}^*)$, $TRC_2(T_{22}^*)$ }. Hence $T^* = T_{12}^* = T_{22}^*$ and optimal payment time is M_1 or M_2 associated with the least cost.

B. If $\Delta_{11} \geq 0$, $\Delta_{12} < 0$ and $\Delta_{22} \geq 0$, then TRC(T*)= min{$TRC_1(M_1)$, $TRC_2(T_{22}^*)$ }. Hence T* is M_1 or T_{22}^*, optimal payment time is M_1 or M_2 associated with the least cost.

C. If $\Delta_{11} \geq 0$ and $\Delta_{22} \leq 0$, then TRC(T*)= min{ $TRC_1(M_1)$, $TRC_2(M_2)$ }. Hence T* is M_1 or M_2, optimal payment time is M_1 or M_2 associated with the least cost.

D. If $\Delta_{11} < 0$, $\Delta_{12} \geq 0$ and $\Delta_{21} > 0$, then TRC(T*)= min{$TRC_1(T_{11}^*)$, $TRC_1(T_{12}^*)$, $TRC_2(T_{22}^*)$ }. Hence T* is T_{11}^* or $T_{12}^*=T_{22}^*$, optimal payment time is M_1 or M_2 associated with the least cost.

E. If $\Delta_{12} \geq 0$ and $\Delta_{21} \leq 0$, then TRC(T*)= min{$TRC_1(T_{11}^*)$, $TRC_1(T_{12}^*)$, $TRC_2(T_{21}^*)$, $TRC_2(T_{22}^*)$}. Hence T* is T_{11}^*, T_{21}^* or $T_{12}^*=T_{22}^*$, optimal payment time is M_1 or M_2 associated with the least cost.

F. If $\Delta_{11} < 0$, $\Delta_{12} < 0$, $\Delta_{21} > 0$ and $\Delta_{22} > 0$, then TRC(T*)= min{ $TRC_1(T_{11}^*)$, $TRC_2(T_{22}^*)$ }. Hence T* is T_{11}^* or T_{22}^*, optimal payment time is M_1 or M_2 associated with the least cost.

G. If $\Delta_{11} < 0$, $\Delta_{21} > 0$ and $\Delta_{22} \leq 0$, then TRC(T*)= min{$TRC_1(T_{11}^*)$, $TRC_2(M_2)$ }. Hence T* is T_{11}^* or M_2, optimal payment time is M_1 or M_2 associated with the least cost.

H. If $\Delta_{12} < 0$, $\Delta_{21} \leq 0$ and $\Delta_{22} > 0$, then TRC(T*)= min{$TRC_1(T_{11}^*)$, $TRC_2(T_{21}^*)$, $TRC_2(T_{22}^*)$ }. Hence T* is T_{11}^*, T_{21}^* or T_{22}^*, optimal payment time is M_1 or M_2 associated with the least cost.

I. If $\Delta_{21} \leq 0$ and $\Delta_{22} \leq 0$, then TRC(T*)= min{$TRC_1(T_{11}^*)$, $TRC_2(T_{21}^*)$ }. Hence T* is T_{11}^* or T_{21}^*, optimal payment time is M_1 or M_2 associated with the least cost.

Theorem 1 immediately determines the optimal cycle time T* and optimal payment time (M_1 or M_2) after computing the numbers Δ_{11}, Δ_{12}, Δ_{21} and Δ_{22}. Theorem 1 is an efficient solution procedure.

Table 1: Optimal solutions

Let A=\$50/order, D=3000units/year, c=\$10/unit, h=\$2/unit/year, Ip=\$0.15/\$/year, Ie=\$0.1/\$/year and M_2=0.1year

r	M_1	s	T*	Q*	TRC(T*)	Optimal payment time
0.01	0.01	12	M_2=0.1	300	30620.00	M_2
		16	T_{22}^*=0.09623	289	30559.23	M_2
		20	T_{22}^*=0.09129	274	30495.45	M_2
	0.015	12	M_2=0.1	300	30620.00	M_2
		16	T_{22}^*=0.09623	289	30559.23	M_2
		20	T_{22}^*=0.09129	274	30495.45	M_2
	0.02	12	M_2=0.1	300	30620.00	M_2
		16	T_{22}^*=0.09623	289	30559.23	M_2
		20	T_{22}^*=0.09129	274	30495.45	M_2
0.015	0.01	12	T_{11}^*=0.09422	283	30567.04	M_1
		16	T_{11}^*=0.09968	299	30508.92	M_1
		20	T_{11}^*=0.10621	319	30447.22	M_1
	0.015	12	T_{11}^*=0.09422	283	30544.88	M_1
		16	T_{11}^*=0.09968	299	30486.76	M_1
		20	T_{11}^*=0.10621	319	30425.05	M_1
	0.02	12	T_{11}^*=0.09422	283	30522.72	M_1
		16	T_{11}^*=0.09968	299	30464.59	M_1
		20	T_{11}^*=0.10621	319	30402.89	M_1
0.02	0.01	12	T_{11}^*=0.09441	283	30415.15	M_1
		16	T_{11}^*=0.0999	300	30356.90	M_1
		20	T_{11}^*=0.10648	319	30295.05	M_1
	0.015	12	T_{11}^*=0.09441	283	30393.10	M_1
		16	T_{11}^*=0.0999	300	30334.85	M_1
		20	T_{11}^*=0.10648	319	30273.00	M_1
	0.02	12	T_{11}^*=0.09441	283	30371.05	M_1
		16	T_{11}^*=0.0999	300	30312.80	M_1
		20	T_{11}^*=0.10648	319	30250.95	M_1

Numerical examples: To illustrate the results, let us apply the proposed method to solve the following numerical examples. For convenience, the numbers of the parameters are selected randomly. The optimal solutions for different parameters of r, M_1 and s are shown in Table 1.

The following inferences can be made based on Table 1. For fixed r and M_1, the larger the value of s is, the smaller the optimal cycle time and optimal order quantity will be as the optimal payment time is M_1. This result implies that the retailer will not order more quantity to take the benefits of the trade credit more frequently. For fixed r and s, the larger the value of M_1 is, the smaller the optimal total relevant cost per unit time will be as the optimal payment time is M_1; however, if the optimal payment time is M_2, the optimal total relevant cost per unit time is independent of the value of M_1. For fixed s and M_1, the value of r increasing, the retailer will be more possible to pay his/her full payment of the amount of purchasing cost quickly to get the cash discount. Since this policy can reduce the optimal total relevant cost per unit time for the retailer.

The supplier offers the trade credit to stimulate the demand of the retailer. Particularly, we investigate the effect of the cash discount. The supplier can also use the cash discount policy to attract retailer to pay the full payment of the amount of purchasing cost to shorten the collection period. This study discusses the inventory model under cash discount and trade credit and provides a very efficient solution procedure. Theorem 1 determines the optimal cycle time T* and optimal payment time after computing the numbers Δ_{11}, Δ_{12}, Δ_{21} and Δ_{22}. Then our model is reduced to the model of Chung *et al.*[6] when cash discount is neglected. Finally, numerical examples are given to illustrate the results developed in this study and a lot of managerial insights are obtained. In future research, we would like to extend to allow for shortages or finite replenishment rate.

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