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Unitary Transformation of the Time-dependent Hamilton System for the Linear, the V-shape and the Triangular Well Potentials into the Quadratic Hamiltonian System

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Abstract: In the present study, the unitary relation between time-dependent Hamiltonian system for the linear, the V-shape and the triangular well potentials and quadratic Hamiltonian system is obtained. Exact wave functions for the three types of time-dependent system by making use of the unitary transformation approach was derived. The results are complete so that they can be used to investigate various quantum-mechanical properties of the time-dependent systems.

Key words: Unitary relation, linear potential, V-shape potential, triangular well potential, wave function

INTRODUCTION

The linear potential is one of the many cases in which a schematic potential can be applied to various part in the real circumstance sufficiently to provide plausible solutions at the expense of mathematical work as well as harmonic potential. The system of the linear potential can be easily extended to the V-type and the triangular well potential. The Hamiltonian systems of a particle whose mass explicitly depend on time and moving under the time-dependent linear potential are considered in this study. During the past several decades, the time-dependent Hamiltonian systems attracted much attention since they can be solved analytically and have plentiful applications such as damped harmonic oscillator[1,2], harmonic oscillator with time-variable mass and/or frequency[3,4], inverted oscillator[5,6] and time-dependent linear potential system[7,8]. Fortageh *et al.*[9] described the realization of a miniaturized magnetic guide for neutral atom that can be treated by approximately linear potential. Castro approached the problem of confining the fermions in 2-dimensions with a linear potential in the Dirac equation[10]. Cocke and Reichl investigated the spectrum of radiation emitted from a sinusoidally driven particle by an electromagnetic field in a triangular well potential[11].

Taking regard of this inclination, the quantum-mechanical states of the time-dependent Hamiltonian systems of a particle in the linear, the V-shape and the triangular well potentials are studied. Exact normalized solution of the Schrödinger equation for a particle in a triangular well potential has been presented[12]. There are several techniques to a treat time-dependent Hamiltonian systems such as invariant operator method[3,13], propagator method[14,15] and unitary transformation method[4,16]. Unitary transformation method in order to transform the Hamiltonian of three-type potentials into the system of quadratic potential and to obtain their solutions of the Schrödinger equation will use.

Linear potential: The Hamiltonian of a particle moving under linear potential, that explicitly depend on time, can be written as:

$$\hat{H}(\hat{x},\hat{p},t) = \frac{\hat{p}^2}{2m(t)} + \alpha(t)\hat{x}, \tag{1}$$

where, m(t) is time-dependent mass and α(t) is a positive time-dependent function. The linear potential is shown in Fig. 1. The slope of the graph in the figure varies with time depending on the magnitude of α(t). Recently, the quantum state of this shape of potential investigated by Guedes[7] and Feng[8]. Feng described the wave function of the system in terms of Airy function.

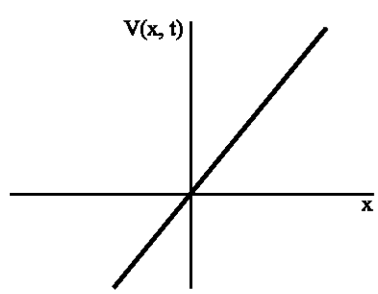

Fig. 1: Time-dependent linear potential

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The corresponding Schödinger equation can be expressed as:

$$\frac{ih}{2\pi}\frac{\partial}{\partial t}\langle x|\psi(t)\rangle = \hat{H}\left(\hat{x},\frac{h}{2\pi i}\frac{\partial}{\partial \hat{x}},t\right)\langle x|\psi(t)\rangle. \quad (2)$$

To simplify the problem, let us perform the unitary transformation of Eq. (1) as:

$$\hat{H}' = \hat{U}^{-1}\hat{H}\hat{U} - \frac{ih}{2\pi}\hat{U}^{-1}\frac{\partial\hat{U}}{\partial t}, \quad (3)$$

where, unitary operator $\hat{U}$ is given by:

$$\hat{U} = \hat{U}_1\hat{U}_2\hat{U}_3, \quad (4)$$

with

$$\hat{U}_1 = \exp\left(-\frac{2\pi i}{h}x_p(t)\hat{p}\right)\exp\left(\frac{2\pi i}{h}p_p(t)\hat{x}\right), \quad (5)$$

$$\hat{U}_2 = \exp\left(\frac{\pi i m\dot{\rho}(t)}{h\rho(t)}\hat{x}^2\right), \quad (6)$$

$$\hat{U}_3 = \exp\left[-\frac{i\pi}{2h}(\hat{x}\hat{p}+\hat{p}\hat{x})\ \ln[2\rho^2(t)]\right]. \quad (7)$$

In Eqs. (5)-(7), ρ(t) is a solution of the classical equation:

$$\ddot{\rho}(t) + \frac{\dot{m}}{m}\dot{\rho}(t) + \frac{\Omega^2}{4m^2}\frac{1}{\rho^3(t)} = 0, \quad (8)$$

where, Ω^2 is a real positive constant and $x_p(t)$ and $p_p(t)$ are particular solutions of the classical equation of motion of the system in coordinate and momentum space, respectively. Note that $x_p(t)$ and $p_p(t)$ satisfy the following differential equations:

$$\ddot{x}_p(t) + \frac{\dot{m}(t)}{m(t)}\dot{x}_p(t) = -\frac{\alpha(t)}{m(t)}, \quad (9)$$

$$\dot{p}_p(t) = -\alpha(t). \quad (10)$$

After transformation, Eq. (3) becomes:

$$\hat{H}' = g(t)\hat{I}'(\hat{x}) - \frac{1}{2}m(t)\dot{x}_p^2(t) + \alpha(t)x_p(t), \quad (11)$$

where:

$$g(t) = \frac{1}{2m(t)\rho^2(t),} \quad (12)$$

$$\hat{I}'(\hat{x}) = -\frac{h^2}{8\pi^2}\frac{\partial^2}{\partial\hat{x}^2} - \frac{1}{2}\Omega^2\hat{x}^2. \quad (13)$$

Note that Eq. (13) is same as the Hamiltonian of the parabola potential. Thus, we can obtain the quantum solution of the time-dependent linear potential system by solving the parabola potential system.

The Schrödinger equation related to the transformed Hamiltonian is:

$$\frac{ih}{2\pi}\frac{\partial}{\partial t}\langle x|\psi'(t)\rangle = \hat{H}'\langle x|\psi'(t)\rangle. \quad (14)$$

To separate variables $\hat{x}$ and t, let us express the wave function in the form:

$$\langle x|\psi'(t)\rangle = f(t)\langle x|\phi'\rangle. \quad (15)$$

By inserting Eqs. (11) and (15) into Eq. (14), we obtain two separated equations as:

$$\hat{I}'(\hat{x})\langle x|\phi'\rangle = \lambda\langle x|\phi'\rangle, \quad (16)$$

$$\frac{\partial f(t)}{\partial t} = \frac{2\pi}{ih}\left[\lambda g(t) - \frac{1}{2}m(t)\dot{x}_p^2(t) + \alpha(t)x_p(t)\right]f(t), \quad (17)$$

where λ is a separation constant. The substitution of Eq. (13) into Eq. (16) with a scale transformation as:

$$\hat{y} = \sqrt{\frac{4\pi\Omega}{h}}\,\hat{x}, \quad (18)$$

leads to

$$\frac{\partial^2\langle x|\phi'\rangle}{\partial\hat{y}^2} + \left(\Lambda + \frac{1}{4}\hat{y}^2\right)\langle x|\phi'\rangle = 0, \quad (19)$$

where:

$$\Lambda = \frac{2\pi\lambda}{h\Omega}. \quad (20)$$

The standard solution of Eq. (19) can be expressed in the form[17]:

$$\langle x|\phi'\rangle\pm = \frac{[\cosh(\pi\Lambda)]^{1/4}}{2\sqrt{\pi}}\left\{\left|\Gamma\left(\frac{1}{4}-\frac{i\Lambda}{2}\right)\right|\prod_1(\Lambda,\hat{x})\pm\sqrt{2}\left|\Gamma\left(\frac{3}{4}-\frac{i\Lambda}{2}\right)\right|\prod_2(\Lambda,\hat{x})\right\} \quad (21)$$

where:

$$\prod_1(\Lambda,\hat{x}) = \exp\left(-\frac{1}{4}\hat{q}^2\right)\,{}_1F_1\left(\frac{i\Lambda}{2}+\frac{1}{4},\frac{1}{2},\frac{1}{2}\hat{q}^2\right), \quad (22)$$

$$\prod_2(\Lambda,\hat{x}) = \hat{q}\exp\left(-\frac{1}{4}\hat{q}^2\right)\,{}_1F_1\left(\frac{i\Lambda}{2}+\frac{3}{4},\frac{3}{2},\frac{1}{2}\hat{q}^2\right), \quad (23)$$

with

$$\hat{q} = \hat{y}e^{i\pi/4} = \sqrt{\frac{4\pi\Omega}{h}}\hat{x}e^{i\pi/4}. \quad (24)$$

Equation (22) and (23) can also be expressed in terms of the parabolic cylinder functions[17,18].

$$\prod_1(\Lambda,\hat{x}) = \frac{\sqrt{\pi}\,2^{i\Lambda/2-3/4}}{\Gamma(1/4-i\Lambda/2)\cos[\pi(1/4+i\Lambda/2)]}[D_{-i\Lambda-1/2}(\hat{q})+D_{-i\Lambda-1/2}(-\hat{q})], \quad (25)$$

$$\prod_2(\Lambda,\hat{x}) = \frac{\sqrt{\pi}\,2^{i\Lambda/2-5/4}}{\Gamma(3/4-i\Lambda/2)\sin[\pi(1/4+i\Lambda/2)]}[D_{-i\Lambda-1/2}(\hat{q})+D_{-i\Lambda-1/2}(-\hat{q})]. \quad (26)$$

From Eq. (17), f(t) can be easily read as:

$$f(t) = \exp\left[-\frac{2\pi i}{h}\int_0^t\left(\frac{\lambda}{2m(t')\rho^2(t')}-\frac{1}{2}m(t')\dot{x}_p^2(t')+\alpha(t')x_p(t')\right)dt'\right]. \quad (27)$$

Then, by substitution of Eqs. (21) and (27) into Eq. (15), the full wave function corresponding to the transformed Hamiltonian can be expressed as:

$$\langle x|\psi'(t)\rangle\pm = \frac{[\cosh(\pi\Lambda)]^{1/4}}{2\sqrt{\pi}}\left\{\left|\Gamma\left(\frac{1}{4}-\frac{i\Lambda}{2}\right)\right|\prod_1(\Lambda,\hat{x})\pm\sqrt{2}\left|\Gamma\left(\frac{3}{4}-\frac{i\Lambda}{2}\right)\right|\prod_2(\Lambda,\hat{x})\right\}$$
$$\times\exp\left[-\frac{2\pi i}{h}\int_0^t\left(\frac{\lambda}{2m(t')\rho^2(t')}-\frac{1}{2}m(t')\dot{x}_p^2(t')+\alpha(t')x_p(t')\right)dt'\right]. \quad (28)$$

The wave function $\langle x|\psi(t)\rangle\pm$ corresponding to the untransformed Hamiltonian, i.e., original Hamiltonian Eq. (1) can be derived from[16]:

$$\langle x|\psi(t)\rangle\pm = \hat{U}\langle x|\psi'(t)\rangle\pm. \quad (29)$$

Making use of Eq. (4), this can be easily evaluated as:

$$\langle x|\psi(t)\rangle\pm = \frac{[\cosh(\pi\Lambda)]^{1/4}}{2\sqrt{\pi}}\frac{1}{\sqrt{2\rho^2(t)}}\left\{\left|\Gamma\left(\frac{1}{4}-\frac{i\Lambda}{2}\right)\right|\tilde{\prod}_1(\Lambda,\hat{x})\pm\sqrt{2}\left|\Gamma\left(\frac{3}{4}-\frac{i\Lambda}{2}\right)\right|\tilde{\prod}_2(\Lambda,\hat{x})\right\}$$
$$\times\exp\left[\frac{2\pi i}{h}P_p(\hat{x}-x_p)+\frac{\pi i m\dot{\rho}(t)}{h\rho(t)}(\hat{x}-x_p)^2\right]$$
$$\times\exp\left[-\frac{2\pi i}{h}\int_0^t\left(\frac{\lambda}{2m(t')\rho^2(t')}-\frac{1}{2}m(t')\dot{x}_p^2(t')+\alpha(t')x_p(t')\right)dt'\right], \quad (30)$$

where:

$$\tilde{\prod}_1(\Lambda,\hat{x}) = \exp\left(-\frac{1}{4}\hat{Q}^2\right){}_1F_1\left(\frac{i\Lambda}{2}+\frac{1}{4},\frac{1}{2},\frac{1}{2}\hat{Q}^2\right), \tag{31}$$

$$\tilde{\prod}_2(\Lambda,\hat{x}) = \hat{Q}\exp\left(-\frac{1}{4}\hat{Q}^2\right){}_1F_1\left(\frac{i\Lambda}{2}+\frac{3}{4},\frac{3}{2},\frac{1}{2}\hat{Q}^2\right), \tag{32}$$

with

$$\hat{Q} = \sqrt{\frac{2\pi\Omega}{\rho^2(t)h}}(\hat{x}-x_p)e^{i\pi/4}. \tag{33}$$

Now we consider for a special case that m(t) and α(t) are given by:

$$m(t) = m_0 e^{\beta t}, \tag{34}$$

$$\alpha(t) = \alpha_0[2+\cos(\omega t+\delta)], \tag{35}$$

where, m_0 and α_0 are positive real constants. Then, the solution of Eqs. (8), (9) and (10) can be represented as:

$$\rho(t) = \sqrt{\frac{\Omega}{m_0\beta}}e^{-\beta t/2}, \tag{36}$$

$$x_p(t) = \frac{2\alpha_0}{m_0\beta}e^{-\beta t}\left(\frac{1}{\beta}+t\right)+\frac{\alpha_0 e^{-\beta t}}{m_0(\omega^2+\beta^2)}\left[\cos(\omega t+\delta)+\frac{\beta}{\omega}\sin(\omega t+\delta)\right], \tag{37}$$

$$p_p(t) = -\frac{2\alpha_0}{\omega}(\omega t+\delta)-\frac{\alpha_0}{\omega}\sin(\omega t+\delta). \tag{38}$$

In terms of these three equations Eq. (30) can be explicitly described.

V-shape potential and triangular well potential: A symmetric time-dependent V-shape potential is depicted in Fig. 2. This system is bound so that the quantum-mechanical energy eigenvalues are discrete. The corresponding Hamiltonian can be expressed as:

$$\hat{H}(\hat{x},\hat{p},t) = \frac{\hat{p}^2}{2m(t)}+|\alpha(t)|\hat{x}. \tag{39}$$

For a mathematical convenience, let's introduce $\alpha\pm(t)$ as:

$$\alpha\pm(t) = \pm\alpha(t), \tag{40}$$

where, $\alpha+(t)$ is only valid for positive $\hat{x}$ while $\alpha\_(t)$ is only valid for negative $\hat{x}$. Then, Eq. (39) can be written as:

$$\hat{H} = (\hat{x},\hat{p},t) = \frac{\hat{p}^2}{2m(t)}+\alpha\pm(t)\hat{x}. \tag{41}$$

We transform Eq. (14) with the unitary operator that given by:

$$\hat{U} = \hat{U}_1\hat{U}_2\hat{U}_3, \tag{42}$$

where:

$$\hat{U}_1 = \exp\left(-\frac{2\pi i}{h}X_p(t)\hat{p}\right)\exp\left(\frac{2\pi i}{h}P_p(t)\hat{x}\right), \tag{43}$$

$$\hat{U}_2 = \exp\left(\frac{\pi i m\dot{\eta}(t)}{h\eta(t)}\hat{x}^2\right), \tag{44}$$

$$\hat{U}_3 = \exp\left[-\frac{\pi i}{2h}(\hat{x}\hat{p} + \hat{p}\hat{x})\ln[2\eta^2(t)]\right]. \tag{45}$$

In the above equation $\eta(t)$ is a classical solution of the following equation:

$$\ddot{\eta}(t) + \frac{\dot{m}}{m}\dot{\eta}(t) - \frac{\Omega_0^2}{4m^2}\frac{1}{\eta^3(t)} = 0, \tag{46}$$

where, Ω_0^2 is a real positive constant and $X_P(t)$ and $P_p(t)$ are particular solutions of the following equations:

$$\ddot{X}_p(t) + \frac{\dot{m}(t)}{m(t)}\dot{X}_p(t) = -\frac{\alpha\pm(t)}{m(t)}, \tag{47}$$

$$\dot{P}_p(t) = -\alpha\pm(t). \tag{48}$$

If converting Ω_0 into $i\Omega$, Eq. (46) becomes same as Eq. (8). Performing the same procedure as that of the previous case, we can transform $\hat{H}(\hat{x},\hat{p},t)$ with $\hat{U}$ to be

$$\hat{H}' = G(t)\hat{I}'(\hat{x}) - \frac{1}{2}m(t)\dot{X}_p^2(t) + \alpha\pm(t)X_p(t), \tag{49}$$

where:

$$G(t) = \frac{1}{2m(t)\eta^2(t)}, \tag{50}$$

$$\hat{I}'(x) = -\frac{h^2}{8\pi^2}\frac{\partial^2}{\partial\hat{x}^2} + \frac{1}{2}\Omega_0^2\hat{x}^2. \tag{51}$$

Note that Eq. (51) is same as the Hamiltonian of the simple harmonic oscillator. Therefore, we can obtain the quantum-mechanical solution of the bound symmetric time-dependent V-shape potential system by solving the system of the harmonic oscillator. The Schrödinger equation related to the transformed Hamiltonian is expressed in the form:

$$\frac{ih}{2\pi}\frac{\partial}{\partial t}\langle x|\psi'_n(t)\rangle = \hat{H}'\langle x|\psi'_n(t)\rangle. \tag{52}$$

Write

$$\langle x|\psi'_n(t)\rangle = h_1(t)\langle x|\phi'_n\rangle. \tag{53}$$

By using the same method as that of previous section, we arrive:

$$\hat{I}'(\hat{x})\langle x|\phi'_n\rangle = \lambda_n\langle x|\phi'_n\rangle, \tag{54}$$

$$\frac{\partial h_1(t)}{\partial t} = \frac{2\pi}{ih}\left[\lambda_n G(t) - \frac{1}{2}m(t)\dot{X}_p^2(t) + \alpha\pm(t)X_p(t)\right]h_1(t). \tag{55}$$

Since Eq. (51) is just same as that of the simple harmonic oscillator, the egenstate and eigenvalue of $\hat{I}'$ can be easily identified as:

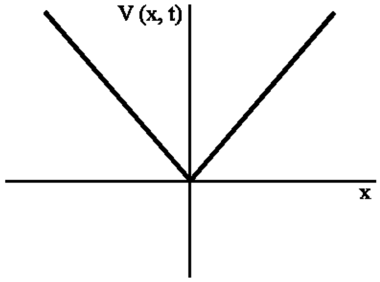

Fig. 2: Time-dependent V-shape potential. The value of V(x, t) is symmetric about x = 0

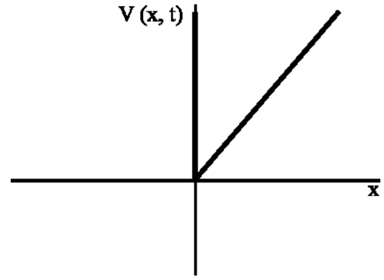

Fig. 3: Time-dependent infinite triangular well potential

$$\langle x|\phi'_n\rangle = \sqrt[4]{\frac{2\Omega_0}{h}}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{2\pi\Omega_0}{h}}\hat{x}\right)\exp\left(-\frac{\pi\Omega_0}{h}\hat{x}^2\right) \tag{56}$$

$$\lambda_n = \frac{h}{2\pi}\Omega_0\left(n+\frac{1}{2}\right), \tag{57}$$

where, H_n is nth order Hermite polynomial. The solution of Eq. (55) is:

$$h_1(t) = \exp\left[-i\frac{2\pi\lambda_n}{h}\int_0^t G(t')dt' + \frac{2\pi i}{h}\int_0^t\left(\frac{1}{2}m(t')\dot{X}_p^2(t') - \alpha\pm(t')X_p(t')\right)dt'\right]. \tag{58}$$

Thus, by inserting Eqs. (56) and (58) with Eqs. (50) and (57) into Eq. (53), the transformed wave function can be read as:

$$\begin{aligned}\langle x|\psi'_n(t)\rangle = {} & \sqrt[4]{\frac{2\Omega_0}{h}}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{2\pi\Omega_0}{h}}\hat{x}\right)\exp\left(-\frac{\pi\Omega_0}{h}\hat{x}^2\right)\\ & \times\exp\left[-i\Omega_0\left(n+\frac{1}{2}\right)\int_0^t\frac{1}{2m(t')\eta^2(t')}dt'\right.\\ & \left.+\frac{2\pi i}{h}\int_0^t\left(\frac{1}{2}m(t')\dot{X}_p^2(t') - \alpha\pm(t')X_p(t')\right)dt'\right].\end{aligned} \tag{59}$$

The wave function $\langle x|\psi_n(t)\rangle$ related to untransformed Hamiltonian can be readily evaluated:

$$\begin{aligned}\langle x|\psi_n(t)\rangle = {} & \hat{U}\langle x|\psi_n'(t)\rangle\\ = {} & \sqrt[4]{\frac{\Omega_0}{\eta^2(t)h}}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{\pi\Omega_0}{\eta^2(t)h}}(\hat{x}-X_p)\right)\\ & \times\exp\left\{\frac{2\pi i}{h}P_p(\hat{x}-X_p) - \frac{\pi}{\eta(t)h}\left[\frac{\Omega_0}{2\eta(t)} - im\dot{\eta}(t)\right](\hat{x}-X_p)^2\right\}\\ & \times\exp\left[-i\Omega_0\left(n+\frac{1}{2}\right)\int_0^t\frac{1}{2m(t')\eta^2(t')}dt'\right.\\ & \left.+\frac{2\pi i}{h}\int_0^t\left(\frac{1}{2}m(t')\dot{X}_p^2(t') - \alpha\pm(t')X_p(t')\right)dt'\right].\end{aligned} \tag{60}$$

The system of asymmetric linear potential (in other words, infinite triangular well potential) is depicted in Fig. 3. Because the wave function must vanish at the potential wall ($\hat{x} = 0$), the allowed eigenstates are only odd wave functions[19]:

$$n \to 2k+1, \qquad k=0,1,2,\ldots. \tag{61}$$

Thus, Eqs. (57) and (60) should replaced by:

$$\lambda_k = \frac{h}{2\pi}\Omega_0\left(2k+\frac{3}{2}\right), \tag{62}$$

$$\begin{aligned}\langle x|\psi_k(t)\rangle &= \sqrt[4]{\frac{\pi\Omega_0}{\eta^2(t)h}}\frac{1}{\sqrt{2^{4k+1}k!(k+1/2)!}}H_{2k+1}\left(\sqrt{\frac{\pi\Omega_0}{\eta^2(t)h}}(\hat{x}-X_p)\right)\\ &\times\exp\left\{\frac{2\pi i}{h}P_p(\hat{x}-X_p)-\frac{\pi}{\eta(t)h}\left[\frac{\Omega_0}{2\eta(t)}-im\dot{\eta}(t)\right](\hat{x}-X_p)^2\right\}\\ &\times\exp\left[-i\Omega_0\left(2k+\frac{3}{2}\right)\int_0^t\frac{1}{2m(t')\eta^2(t')}dt'\right.\\ &\left.+\frac{2\pi i}{h}\int_0^t\left(\frac{1}{2}m(t')\dot{X}_p^2(t')-\alpha+(t')X_p(t')\right)dt'\right].\end{aligned} \tag{63}$$

The normalization factor in the above equation is somewhat altered compared to Eq. (60), since $\hat{x}$ is restricted only in the positive region. The importance of the triangular well potential as a basic application to the electrical and optical properties of the electron gas in heterojunction high electron mobility transistors (DH-HEMTs) probably became evident with the appearance of the semiconductor physics[20,21]. The triangular well potential can also be applied to the analysis of the quantized electronic structures of a MOSFET[22,23].

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