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Further Results on Order Statistics from the Generalized Log Logistic Distribution

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Abstract: Further to our earlier results, we derive exact explicit expressions for the triple and quadruple moments of order statistics from the generalized log-logistic distribution.

Key words: Order Statistics; Recurrence Relations; Generalized log-logistic distribution; triple and quadruple moments

INTRODUCTION

Recently Adeyemi[1], Adeyemi and Ojo[2] initiated the study into the recurrence relations for moments of order statistics from the generalized log logistic distribution. We have obtained recurrence relations for single and product moments of order statistics from a symmetric, Adeyemi[3] and the, generalized log logistic distribution Adeyemi and Ojo[2].

In this paper, we present further results on our earlier studies by presenting recurrence relations for triple and quadruple moments of order statistics from the generalized log logistic distribution.

The probability density function (pdf) of the GLL (m_1, m_2) distribution is given by

$$xF'(x) = \gamma[F(x)]^{m_1}[1-F(x)]^{m_2}; \quad \gamma = \frac{\alpha}{B(m_1, m_2)} \tag{1.1}$$

Letting $\alpha = \frac{1}{\sigma}$ and $\beta = \frac{-\mu}{\sigma}\ln\left(\frac{m_1}{m_2}\right)$ It can be easily shown that the pdf of GLL (m_1, m_2) becomes

$$f(x) = \frac{1}{\sigma B(m_1, m_2)} \frac{e^{-\frac{\mu m_1}{\sigma}}(m_1/m_2)^{m_1} x^{\frac{m_1}{\sigma}-1}}{[1+(m_1/m_2)(e^{-\mu}x)^{\frac{1}{\sigma}}]^{m_1+m_2}} \tag{1.2}$$

Note that if $m_1=m_2=1$, GLL(m_1, m_2) becomes the log-logistic distribution. It is symmetric around $\ln(t) = \frac{-\beta'_z}{\alpha}$ if $m_1 = m_2$, positive skewed if $m_1 > m_2$ and negative skewed if $m_2 > m_1$.

Let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ denote the order statistics obtained when the n X_i's are arranged in increasing order of magnitude. We denote

$$\mu^{(a,b,c)}_{r,s,t:n} = E[X^a_{r:n} X^b_{s:n} X^c_{t:n}], \quad 1 \le r < s < t \le n \tag{1.3}$$

and

$$\mu^{(a,b,c,d)}_{r,s,t,u:n} = E[X^a_{r:n} X^b_{s:n} X^c_{t:n}], \quad 1 \le r < s < t < u \le n \tag{1.4}$$

Also

$$f_{r,s,t:n}(w,x,y) = C_{r,s,t:n}[F(w)]^{r-1}[F(x)-F(w)]^{s-r-1} [F(y)-F(x)]^{t-s-1}x[1-F(y)]^{n-t}f(w)f(x)f(y) \tag{1.5}$$

where

$$C_{r,s,t:n} = \frac{n!}{(r-1)!(s-r-1)!(t-s-1)!(n-t)!}$$

and

$$f_{r,s,t,u:n}(w,x,y,z) = C_{r,s,t,u:n}[F(w)]^{r-1} [F(x)-F(w)]^{s-r-1}[F(y)-F(x)]^{t-s-1} x[F(z)-F(y)][1-F(y)]^{n-u}f(w)f(x)f(y)f(z) \tag{1.6}$$

where

$$C_{r,s,t,u:n} = \frac{n!}{(r-1)!(s-r-1)!(t-s-1)!(u-t-1)!(n-u)!}$$

Adeyemi[3] and Adeyemi and Ojo[2] have obtained recurrence relations for $\mu^{(i)}_{r:n}$ and expressions for $\mu_{r,s:n}$ in both symmetric and general cases respectively.

In this paper, we obtain recurrence relations for $\mu^{(a,b,c)}_{r,s,t:n}$ and $\mu^{(a,b,c,d)}_{r,s,t,u:n}$ for positive integers m_1 and m_2.

Recurrence relations for triple moments: Theorem 2.1 for $1 \le r < s < t \le n - m_1 - i$ and a, b, c ≥ 1

$$\begin{aligned} A_1(i)\,\mu^{(a,b,c)}_{r+m_1+i,s+m_1+i,t+m_1+i:n} &= \frac{\alpha\gamma(r-1)!(n-t)!}{m_2}\mu^{(a,b,c)}_{r,s,t:n} \\ &= -\frac{s-r-1}{m_2}A_2(i)\,\mu^{(a,b,c)}_{r+m_1+i,s+m_1+i-1,t+m_1+i-1:n} \\ &= +\frac{r+m_1-1}{m_2}A_3(i)\,\mu^{(a,b,c)}_{r+m_1+i-1,s+m_1+i-1,t+m_1+i-1:n} \end{aligned} \tag{2.1}$$

where

$$A_1(i) = \sum_{i=0}^{m_2-1}\binom{m_2-1}{i}(-1)^i(r+m_1+i-1)!(n-t-m_1-i)!$$

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$$A_2(i) = \sum_{i=0}^{m_2} \binom{m_2}{i} (-1)^i (r+m_1+i-1)!(n-t-m_1-i+1)!$$

and

$$A_3(i) = \sum_{i=0}^{m_2} \binom{m_2}{i} (-1)^i (r+m_1+i-2)!(n-t-m_1-i+1)!$$

Proof

$$\gamma \mu_{r,\,s,\,t:n}^{(a,\,b,\,c)} = C_{r,\,s,\,t:n} \int_x \int_y \gamma x^b y^c [F(y)-F(x)]^{t-s-1} [1-F(y)]^{n-t} I_1(x) f(x) f(y)\, dx dy \tag{2.2}$$

having used (1.1), (1.3) and (1.5) where

$$I_1(x) = \int_w [F(w)]^{r+m_1-1} [1-F(w)]^{m_2} [F(x)-F(w)]^{s-r-1} w^{a-1} dw \tag{2.3}$$

Integrating by parts, we have

$$\begin{aligned} I_1(x) &= (s-r-1) \int_w w^a [F(w)]^{r+m_1-1} [1-F(w)]^{m_2} [F(x)-F(w)]^{s-r-2} F(w) dw \\ &+ m_2 \int_w w^a [F(w)]^{r+m_1-1} [1-F(w)]^{m_2-1} [F(x)-F(w)]^{s-r-1} F(w) dw \\ &- (r+m_1-1) \int_w w^a [F(w)]^{r+m_1-1} [1-F(w)]^{m_2} [F(x)-F(w)]^{s-r-1} F(w) dw \end{aligned} \tag{2.4}$$

by putting (2.4) in (2.3) and after simplification, we have the relation (2.1)
Theorem 2.2 For $1 \le r < s \le n-1$ and a, b, $c \le 1$

$$B_1(i)\ \mu_{r+m_1+i, r+1, s+1:n}^{(a,\,b,\,c)} = \frac{\alpha \gamma (r-1)!}{m_2}\ \mu_{r, r+1, s+1:n}^{(a,\,b,\,c)} - \frac{r+m_1-1}{m_2}\ B_2(i)\ \mu_{r+m_1+i-1, r+1, s+1:n}$$

where

$$B_1(i) = \sum_{i=0}^{m_2-1} \binom{m_2-1}{i} (r+m_1+i-1)!$$

and

$$B_2(i) = \sum_{i=0}^{m_2} \binom{m_2}{i} (r+m_1+i-2)! \tag{2.5}$$

$$\gamma \mu_{r+1, r+1, s+1:n}^{(a,\,b,\,c)} = C_{r, r+1, s+1:n} \int_x \int_y \gamma x^b y^c [1-F(y)]^{n-s-1}\ I_2(x) f(x) f(y)\, dx dy \tag{2.6}$$

having used (1.1), (1.3) and (1.5) where

$$I_2(x) = \int_w [F(w)]^{r+m_1-1} [1-F(w)]^{m_2}\ w^{a-1} dw \tag{2.7}$$

Integrating by parts, we have

$$\begin{aligned} I_2(x) &= m_2 \int_w w^a [F(w)]^{r+m_1-1} [1-F(w)]^{m_2-1}\ f(w) dw \\ &- (r+m_1-1) \int_w w^a [F(w)]^{r+m_1-2} [1-F(w)]^{m_2}\ f(w) dw \end{aligned} \tag{2.8}$$

substracting (2.8) into (2.6) and simplyfying the resulting expression yields the relation (2.5).
Theorem 2.3 For $1 \le r < s < t \le n$ and a, b, $c \ge 1$

$$C_1(i,j)\ \mu_{r+m_1-i-1, s+2m_1-i, t+m_2+2m_1-i-1:n+2m_1+2m_2-i-j-1}^{(a,\,b,\,c)} = \frac{\gamma b}{C_{r,s,t:n}(t+m_2-s-1)}\ \mu_{r,s,t:n}^{(a,\,b,\,c)}$$

$$+\frac{(s+m_1-r-1)}{(t+m_2-s-1)}C_2(i,j)\ \mu^{(a,\,b,\,c)}_{r+m_1-i-1,s+2m_1-i,-1,t+m_2+2m_1-i-1:n+2m_1+2m_2-i-j-1}$$

where

$$C_1(i,j)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i-1)!(s+m_1-r-1)!}{(n+2m_1+2m_2-i-j-1)!}$$
$$x(t+m_2-s-2)!(n+m_2-t-j)!$$

and

$$C_2(i,j)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i-1)!(s+m_1-r-2)!}{(n+2m_1+2m_2-i-j-1)!}\tag{2.9}$$
$$x(t+m_2-s-1)!(n+m_2-t-j)!$$

Proof

$$\gamma\mu_{r,\,s,\,t:n}=C_{r,\,s,\,t:n}\int\int w^{a}x^{b}y^{c}[F(w)]^{r-1}[1-F(w)]^{n-t}K(w,y)f(w)f(y)dwdy\tag{2.10}$$

where

$$K(w,y)=\int_w^y[F(x)-F(w)]^{s-r-1}[F(y)-F(x)]^{t-s-1}[F(x)]^{m_1}[1-F(x)]^{m_2}x^{b-1}dx\tag{2.11}$$

having used (1.1), (1.3) and (1.5). Upon writing F(x) = F(x) − F(w) + F(w) and 1-F(x) = F(y) − F(x) + 1 − F(y) and using binomial expansion, we have

$$K(w,y)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\int_w^y[F(x)-F(w)]^{s+m_1-r-1}[F(y)-F(x)]^{t-m_2-s-1}\tag{2.12}$$
$$x[F(w)]^{m_1-i}[1-F(y)]^{m_2-j}x^{b-1}dx$$

Integrating (2.12) by parts, we have

$$K(w,y)=\frac{t+m_2-s-1}{b}\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\int_w^y[F(w)]^{m_1-i}[F(x)-F(w)]^{s+m_1-r-1}$$
$$x\,[F(y)-F(x)]^{t+m_2-s-2}[1-F(y)]^{m_2-j}f(x)dx$$

$$-\frac{s+m_1-r-1}{b}\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\int_w^y[F(w)]^{m_1-i}[F(x)-F(w)]^{s+m_1-r-2}$$
$$x\,[F(y)-F(x)]^{t+m_2-s-1}[1-F(y)]^{m_2-j}f(x)dx$$

By putting the above expression into (2.10) and after simplification, we have the relation (2.9).

Corollary 2.1 Setting s=r+1, t= r+2 we have

$$C_3(i,j)\ \mu^{(a,\,b,\,c)}_{r+m_1-i-1,r+2m_1-i+1,r+m_2+2m_1-i+1:n+2m_1+2m_2-i-j-1}=\frac{\gamma b}{m_2}\ \mu^{(a,\,b,\,c)}_{r,r+1,r+2:n}$$
$$+\ \frac{m_1}{m_2}\ C_4(i,j)\mu^{(a,\,b,\,c)}_{r+m_1-i-1,r+2m_1-i,r+m_2+2m_1-i+1:n+2m_1+2m_2-i-j-1}$$

where

$$C_3(i,j)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i+1)!m_1!(m_2-1)!(n+m_2-r-j-2)!n!}{(r-1)!(n-r-2)!(n+2m_1+2m_2-i-j-1)!}$$

and

$$C_4(i,j)=\sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i+1)!(m_1-1)!m_2!(n+m_2-r-j-2)!n!}{(r-1)!(n-r-2)!(n+2m_1+2m_2-i-j-1)!}\tag{2.13}$$

Corollary 2.2 For $s-r \geq 2$ and t=s+1

$$C_5(i,j)\ \mu^{(a,\,b,\,c)}_{r+m_1-i-1,s+2m_1-i,s+m_2+2m_1-i:n+2m_1+2m_2-i-j-1}=\frac{\gamma b}{m_2}\ \mu^{(a,\,b,\,c)}_{r,s,s+1:n}$$
$$+\ \frac{s+m_1-r-1}{m_2}\ C_6(i,j)\mu^{(a,\,b,\,c)}_{r+m_1-i-1,s+2m_1-i-1,s+m_2+2m_1-i:n+2m_1+2m_2-i-j-1}$$

where

$$C_5(i,j) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i-1)!(s+m_1-r-1)!}{(r-1)!(s-r-1)!(n-s-1)!(n+2m_1+2m_2-i-j-1)!}$$
$$\text{x }(m_2-1)!(n+m_2-s-j-1)!n!$$

and

$$C_6(i,j) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j}\frac{(r+m_1-i-1)!(s+m_1-r-2)!}{(r-1)!(s-r-1)!(n-s-1)!(n+2m_1+2m_2-i-j-1)!}$$
$$\text{x }m_2!(n+m_2-s-j-1)!n! \qquad (214)$$

Remark 2.1 In theorems 2.1, 2.2 and 2.3 if $m_1 = m_2 = m$ we obtain relations for triple moments of order statistics from a symmetric generalized log-logistic distribution studied by Adeyemi[3].

Remark 2.2 In theorems 2.1, 2.2 and 2.3 if $m_1 = m_2 = 1$ we obtain relations fro triple moments of order statistics from the ordinary log-logistics distribution studied by Ali and Khan[4].

Recurrence relations for quadruple moments

Theorem 3.1. For $1 \le r < s < t < u \le n$ and $a, b, c, d \ge 1$

$$H_1(i,j,k)\mu^{(a,b,c,d-1)}_{r+m_1-i-j-k,s+2m_1-2(i+j)-k,t+3m_1-3i-2j-k,u+4m_1-3i-2j-k-1:n+4m_1+m_2-2(i+j)-k} =$$
$$\frac{n+m_2-u}{u-t-1}H_2(i,j,k)$$
$$\text{x }\mu^{(a,b,c,d-1)}_{r+m_1-i-j-k-1,s+2m_1-k-2(i+j+1),t+3m_1-k-2j-3(i+1),u+4m_1-k-2(i+j+2):n+4m_1+m_2-k-2(i+j)-3}$$
$$-\frac{m_1}{u-t-1}H_3(i,j,k)$$
$$\text{x }\mu^{(a,b,c,d-1)}_{r+m_1-i-j-k-1,s+2m_1-2(i+j)-k-2,t+3m_1-3i-2j-k-3,u+4m_1-3i-2j-k-4:n+4m_1+m_2-3i-2j-k-3}$$
$$-\frac{\gamma(d-1)}{u-t-1}\mu^{(a,b,c,d-1)}_{r,s,t,u:n}$$

where

$$H_1(i,j,k) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_1-i-1}\sum_{k=0}^{m_1-i-j}\binom{m_1}{i}\binom{m_1-i}{j}\binom{m_1-i-j}{k}$$
$$\text{x }\frac{(r+m_1-i-j-k-1)!(s+m_1-r-i-j-1)!}{(n+m_2+4m_1-k-3i-2j)!}$$
$$\text{x }(t-s+m_1-j-1)!(u-t+m_1-2)!(n+m_2-u)!$$

$$H_2(i,j,k) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_1-i}\sum_{k=0}^{m_1-i-1}\binom{m_1}{i}\binom{m_1-i}{j}\binom{m_1-i-j}{k}$$
$$\text{x }\frac{(r+m_1-i-j-k-2)!(s+m_1-r-i-j-2)!}{(n+m_2+4m_1-k-2i-2j+1)!}$$
$$\text{x }(t-s+m_1-j-2)!(u-t+m_1-2)!(n+m_2-u)!$$

$$H_3(i,j,k) = \sum_{i=0}^{m_1-1}\sum_{j=0}^{m_1-i-1}\sum_{k=0}^{m_1-i-j-1}\binom{m_1-1}{i}\binom{m_1-i-1}{j}\binom{m_1-i-j-1}{k}$$
$$\text{x }\frac{(r+m_1-i-j-k-2)!}{(n+m_2+4m_1-3i-2j-k-3)!}$$
$$\text{x }(s+m_1-r-i-j-k-2)!(t-s+m_1-j-2)!$$
$$\text{x }(u-t+m_1-2)!(n+m_2-u)! \qquad (3.1)$$

Proof

$$\gamma\mu^{(a,b,c,d-1)}_{r,\,s,\,t:n} = C_{r,\,s,\,t,\,u:n}\int_w\int_x\int_y w^a x^b y^c [F(w)]^{r-1}[F(x)-F(w)]^{s-r-1}[F(y)-F(x)]^{t-s-1} \times J_1(y) f(w) f(x) f(y)\,dw\,dx\,dy \tag{3.2}$$

where

$$J_1(y) = \int_z z^{d-2}\,[F(z)-F(y)]^{u-t-1}\,[1-F(z)]^{n-m_2-u}\,[F(z)]^{m_2}\,dz \tag{3.3}$$

having used (1.1), (1.4) and (1.6). Upon integrating (3.3) by parts writing F(z) = F(z) − F(y) + F(y), F(y) = F(y) −F(x) − F(x) and F(x) = F(x) − F(w) + F(w) and using binomial expansion, we have

$$\begin{aligned} J_1(y) &= \frac{n+m_2-u}{d-1}\sum_{i=0}^{m_1}\sum_{j=0}^{m_1-1}\sum_{k=0}^{m_1-i-1}\binom{m_1}{i}\binom{m_1-1}{j}\binom{m_1-i-1}{k} \\ &\times \int_z z^{d-1}[F(w)]^{m_1-i-j-k-1}[F(x)-F(w)]^{m_1-i-j-1}[F(y)-F(x)]^{m_1-i-1} \\ &\times [F(z)-F(y)]^{u+m_1-t-2}[1-F(z)]^{n+m_2-u-1}f(z)\,dz \\ &- \frac{m_1}{d-1}\sum_{i=0}^{m_1-1}\sum_{j=0}^{m_1-i-1}\sum_{k=0}^{m_1-i-j-1}\binom{m_1-1}{i}\binom{m_1-i-1}{j}\binom{m_1-i-j-1}{k} \\ &\times \int_z z^{d-1}[F(w)]^{m_1-i-j-k-1}[F(x)-F(w)]^{m_1-i-j-1}[F(y)-F(x)]^{m_1-i-1} \\ &\times [F(z)-F(y)]^{u+m_1-t-2}[1-F(z)]^{n+m_2-u}f(z)\,dz \\ &- \frac{u-t-1}{d-1}\sum_{i=0}^{m_1}\sum_{j=0}^{m_1-i}\sum_{k=0}^{m_1-i-j}\binom{m_1}{i}\binom{m_1-i}{j}\binom{m_1-i-j}{k} \\ &\times \int_z z^{d-1}[F(w)]^{m_1-i-j-k-1}[F(x)-F(w)]^{m_1-i-j}[F(y)-F(x)]^{m_1-i} \\ &\times [F(z)-F(y)]^{u+m_1-t-2}[1-F(z)]^{n+m_2-u}f(z)\,dz \end{aligned} \tag{3.4}$$

Upon substituting (3.4) into (3.2) and simplifying, we have the relation (3.1).

Theorem 3.2. For $1 \le r < s < t < u \le n$ and a, b, c, $d \ge 1$

$$\begin{aligned} &H_4(i,j)\,\mu^{(a,b,c,d)}_{r+m_1-i,\,s+2m_1-i-1,\,t+2m_1+m_2-i-j,\,u+2m_1+2m_2-i-j:n+2m_1+3m_2-i-2j} \\ &= \frac{t+m_2-s-j-1}{s-m_1-r-1}\,H_5(i,j) \\ &\times \mu^{(a,b,c,d)}_{r+m_1-i,\,s+2m_1-i,\,t+2m_1+m_2-i-j-1,\,u+2m_1+2m_2-i-j-1:n+2m_1+3m_2-i-2j-1} \\ &- \frac{\gamma b}{C_{r,s,t,u:n}(s+m_1-r-1)}\,\mu^{(a,b,c,d)}_{r,s,t,u:n} \end{aligned}$$

where

$$H_4(i,j) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j} \times \frac{(r+m_1-i-1)!\,(s+m_1-r-2)!\,(t+m_2-s-j-1)!}{(n+2m_1+3m_2-i-2j)!} \times (u+m_2-t-1)!\,(n+m_2-u-j)!$$

$$H_5(i,j) = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2}\binom{m_1}{i}\binom{m_2}{j} \times \frac{(r+m_1-i-1)!\,(s+m_1-r-1)!\,(t+m_2-s-j-2)!}{(n+2m_1+3m_2-i-2j-1)!} \times (u+m_2-t-1)!\,(n+m_2-u-j)! \tag{3.5}$$

Proof

$$\gamma \mu^{(a,b,c,d)}_{r,\, s,\, t,\, u:n} = C_{r,\, s,\, t,\, u:n} \int_w \int_y \int_z [F(w)]^{r-1} [F(z) - F(y)]^{u-t-1} \\ x\ [1 - F(z)]^{n-u} K(w,y) f(w) f(y) f(z)\, dw\, dy\, dz \tag{3.6}$$

where

$$K(w,y) = \int_w x^{b-1}\ [F(x)]^{m_1}\ [1 - F(x)]^{m_2}\ [F(x) - F(w)]^{s-r-1}\ [F(y) - F(x)]^{t-s-1}\ dx$$

having used (1.1), (1.4) and (1.6). Expressing $1 - F(x)$ as $1 - F(y) + F(y) - F(x)$ and $1 - F(y)$ as $F(z) - F(y) + 1 - F(z)$, we have

$$K(w,y) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{1} \binom{m_2}{j} \\ x \int_x x^{b-1} [F(w)]^{m_1 - 1} [F(x) - F(w)]^{s+m_1-r-1} [F(y) - F(x)]^{t+m_2-s-j-1} [1 - F(y)]^{m_2} dx \tag{3.7}$$

By integrating (3.7) by parts, we obtain

$$K(w,y) = \frac{t + m_2 - s - j - 1}{b} \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{1} \binom{m_2}{j} \\ x \int_x x^{b} [F(w)]^{m_1 - i} [F(x) - F(w)]^{s+m_1-r-1} [F(y) - F(x)]^{t+m_2-s-j-2} \\ x^{b}\ [F(z) - F(y)]^{m_2} [1 - F(z)]^{m_2 - j} f(x) dx \\ - \frac{s + m_1 - r - 1}{b} \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{1} \binom{m_2}{j} \\ x^{b} \int_x x^{b} [F(w)]^{m_1 - i} [F(x) - F(w)]^{s+m_1-r-2} [F(y) - F(x)]^{t+m_2-s-j-2} \\ x\ [F(z) - F(y)]^{m_2} [1 - F(z)]^{m_2 - j} f(x) dx \tag{3.8}$$

By substituting (3.8) into (3.6) and simplyfying the resulting expression, we obtain the relation (3.5)

Corollary 3.1. Setting s= r+1, t= r+2 and u= r+3, we have

$$H_6(i,j)\mu^{(a,b,c,d)}_{r+m_1-i,r+2m_1-i,r+2m_1+m_2-i-j+2,r+2m_1+2m_2-i-j+3:n+2m_1+3m_2-i-2j} = \\ \frac{m_2 - j}{m_1} H_7(i,j)\mu^{(a,b,c,d)}_{r+m_1-i,r+2m_1-i+1,r+2m_1+m_2-i-j+1,r+2m_1+2m_2-i-j+2:n+2m_1+3m_2-i-2j-1} \\ - \frac{\gamma b n!}{(r-1)!(n-r-3)!m_1} \mu^{(a,b,c,d)}_{r,r+1,r+2,r+3:n}$$

Where

$$H_6(i,j) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{i} \binom{m_2}{j} \\ x \frac{(r + m_1 - i - 1)!(m_1 - 1)!m_2!(m_2 - j)!(n + m_2 - r - j - 3)!}{(n + 2m_1 + 3m_2 - i - 2j)!}$$

$$H_7(i,j) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{i} \binom{m_2}{j} \\ x \frac{(r + m_1 - i - 1)!m_1!(m_2 - j - 2)!m_2!(n + m_2 - r - j - 3)!}{(n + 2m_1 + 3m_2 - i - 2j - 1)!} \tag{3.9}$$

Corollary 3.2. For s ≥r+2, t= s+1 and u= s+2, we have

$$H_8(i,j)\mu^{(a,b,c,d)}_{r+m_1-i,s+2m_1-i-1,s+2m_1+m_2-i-j+1,s+2m_1+2m_2-i-j+2:n+2m_1+3m_2-i-2j} = \\ \frac{m_2 - j}{s + m_1 - r - 1} H_9(i,j)\mu^{(a,b,c,d)}_{r+m_1-i,s+2m_1-i,s+2m_1+m_2-i-j,r+2m_1+2m_2-i-j+1:n+2m_1+3m_2-i-2j-1} \\ - \frac{\gamma b n!}{(r-1)!(s-r-1)!(n-s-2)!(s + m_1 - r - 1)} \mu^{(a,b,c,d)}_{r,s,s+1,s+2:n}$$

where

$$H_8(i,j) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{i} \binom{m_2}{j}$$

$$x \frac{(r+m_1-i-1)!(s+m_1-r-1)!(m_2-j-1)!m_2!(n+m_2-s-j-2)!}{(n+2m_1+3m_2-i-2j-1)!}$$

$$H_9(i,j) = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \binom{m_1}{i} \binom{m_2}{j}$$

$$x \frac{(r+m_1-i-1)!(s+m_1-r-1)!(m_2-j-1)!m_2!(n+m_2-s-j-2)!}{(n+2m_1+3m_2-i-2j-1)!} \quad (3.10)$$

Remark 3.1 In theorems 3.1 and 3.2 if we set $m_1=m_2=m$ we obtain relations for quadruple moments of order statistics from a symmetric generalized log-logistic distribution studied by Adeyemi[3].

Remark 3.1 In theorems 3.1 and 3.2 if we set $m_1=m_2=1$ we obtain relations for quadruple moments of order statistics from the ordinary log-logistic distribution studied by Ali and Khan[4].

REFERENCES

1. Adeyemi, S., 2002. Some recurrence relations for moments of order statistics from a symmetric Generalized Log-logistics distribution. InterStat, No. 1.
2. Adeyemi, S. and M.O. Ojo, 2002. Some recurrence relations for moments of order statistics from the generalized log-logistic distribution (submitted).
3. Adeyemi, S., 2002. Some recurrence relations for single and product moments of order statistics from the generalized pareto distribution. J. Stastistical Res., 2: 168-179.
4. Ali, M.M. and A.H. Khan, 1987. On order statistics from the log-logistic distribution. J. Statist. Plann. Infer., 17: 103-108.