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An Inventory Model under Conditional Trade Credits Depending on Payment Time

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Abstract: The present study investigate the retailer's optimal cycle time and optimal payment time under supplier credits including conditionally permissible delay in payments and cash discount depending on retailer payment time. That is, the retailer can obtain fully permissible delay in payments and cash discount if the payment is paid before the period of full delay payments permitted by the supplier. Otherwise, the retailer will just obtain partially permissible delay in payments within the period of partial delay payments permitted by the supplier. The supplier uses this policy to attract retailer to pay the payment as soon as possible to shorten its collection period. Mathematical models have been derived for obtaining the optimal cycle time and optimal payment time for item so that the annual total relevant cost is minimized. One theorem is developed to efficiently determine the optimal replenishment and payment policy for the retailer.

Key words: Inventory, conditional trade credit, cash discount, payment time

INTRODUCTION

The traditional EOQ (Economic Order Quantity) model assumes the payment for the quantity ordered is made when the quantity is received. However, in practice it is found that supplier allows a certain fixed credit period to the retailer and do not charge any interest from the retailer on the amount owed during this credit period. Before the end of credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of credit period. In real world, the supplier often makes use of this policy to promote his/her commodities. Recently, the research area of trade credit has been considered. At first, Goyal^[1] derived an EOQ model under the condition of trade credit. Huang and Chung^[2] extended Goyal's model^[1] to cash discount policy for early payment. The retailer can obtain the cash discount when the payment is paid within the cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. Huang^[3] relaxed the assumption that the selling price was equal to the purchasing price by Huang and Chung^[2]. The inventory policy under trade credit and cash discount can be found by Huang^[4-6] and Lin and Huang^[7].

Earlier research assumed that the retailer could obtain the cash discount when the payment was paid before the cash discount period offered by the supplier. Otherwise, the retailer would pay full payment within the trade credit

period. What the above statement describes is just one of ways of attracting the retailer to pay the payment as soon as possible. This study tries to develop another more effective supplier's credit policy to shorten its collection period. We assume that the retailer can obtain fully permissible delay in payments and cash discount if the payment is paid before the period of full delay payments permitted by the supplier. Otherwise, the retailer will just obtain partially permissible delay in payments within the period of partial delay payments permitted by the supplier. The supplier uses the credits policy depending on payment time to attract the retailer to pay the payment as soon as possible to shorten its collection period. Under this condition, we model the retailer's inventory system as a cost minimization problem and provide one theorem to efficiently determine the retailer's optimal replenishment and optimal payment policy.

Model formulation:

Notation:

D= Demand rate per year

A= Cost of placing one order

c= Unit purchasing price

h= Unit stock holding cost per year excluding interest charges

r= Cash discount rate, $0 \leq r < 1$

α = The fraction of the total amount owed payable at the time of placing an order, $0 \leq \alpha \leq 1$

I_e = Interest earned per \$ per year

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I_k = Interest charges per \$ investment in inventory per year

M_1 = The period of full delay payments permitted in years

M_2 = The period of partial delay payments permitted in years, $M_1 < M_2$

T = The cycle time in years

$TRC_{11}(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T \geq M_1$

$TRC_{12}(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T \leq M_1$

$TRC_1(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T > 0$

$$= \begin{cases} TRC_{11}(T) & \text{if } T \geq M_1 \\ TRC_{12}(T) & \text{if } T \leq M_1 \end{cases}$$

$TRC_{21}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T \geq M_2/\alpha$.

$TRC_{22}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $M_2 \leq T \leq M_2/\alpha$.

$TRC_{23}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T \leq M_2$.

$TRC_2(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T > 0$.

$$= \begin{cases} TRC_{21}(T) & \text{if } T \geq \frac{M_2}{\alpha} \\ TRC_{22}(T) & \text{if } M_2 \leq T \leq \frac{M_2}{\alpha} \\ TRC_{23}(T) & \text{if } T \leq M_2 \end{cases}$$

$TRC(T)$ = The annual total relevant cost when $T > 0$

$$= \begin{cases} TRC_1(T) & \text{if the payment is paid at time } M_1 \\ TRC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$$

T_1^* = The optimal cycle time of $TRC_1(T)$

T_2^* = The optimal cycle time of $TRC_2(T)$

T^* = The optimal cycle time of $TRC(T)$

Q^* = The optimal order quantity = DT^* .

Assumptions:

- Demand rate is known and constant.
- Shortages are not allowed.
- Time horizon is infinite.
- Replenishments are instantaneous.
- $I_k \geq I_e$.
- Supplier offers a cash discount and fully delayed payment to the retailer if payment is paid within M_1 , otherwise just partially delayed payment if payment is paid within M_2 .

- If payment is paid within M_1 , when the account is settled the retailer starts paying for the interest charges on the items in stock. If payment is paid behind M_1 but within M_2 , as the order is received, the retailer must make a partial payment αcDT to the supplier. Then the retailer must pay off the remaining balance $(1-\alpha)cDT$ at the end of the partially permissible delay period M_2 .
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account.

The model: The annual total relevant cost consists of the following elements.

- Annual ordering cost = $\frac{A}{T}$
- Annual stock holding cost (excluding interest charges) = $\frac{DT h}{2}$
- Annual purchasing cost: Since the supplier offers a cash discount if payment is paid within M_1 , there are two payment policies for the retailer.

Case 1: Payment is paid at time M_1 , the annual purchasing cost = $c(1-r)D$.

Case 2: Payment is paid at time M_2 , the annual purchasing cost = cD .

- Cost of interest charges for the items kept in stock per year:

Case 1: Payment is paid at time M_1 , according to assumption (7); there are two cases in terms of interests charged per year.

Case 1.1: $T \geq M_1$.

Annual interests payable = $c(1-r)I_k D(T-M_1)^2/2T$

Case 1.2: $T \leq M_1$

In this case, annual interests payable = 0.

Case 2: Payment is paid at time M_2 , according to assumption (7); there are three cases in terms of interests charged per year.

Case 2.1: $\frac{M_2}{\alpha} \leq T$, shown in Fig. 1.

Annual interests payable = $cI_k \left[\frac{DT^2}{2} - (1-\alpha)DTM_2 \right] / T$

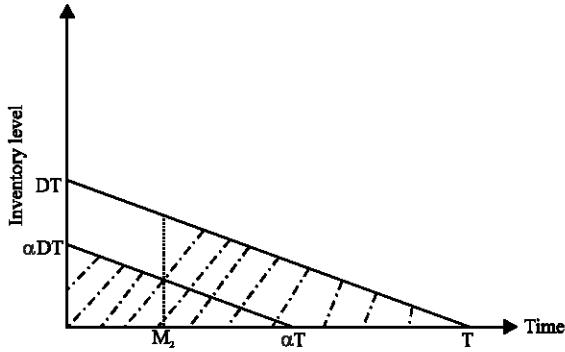


Fig. 1: The inventory level and the total amount of interest payable when $M_1/\alpha \leq T$

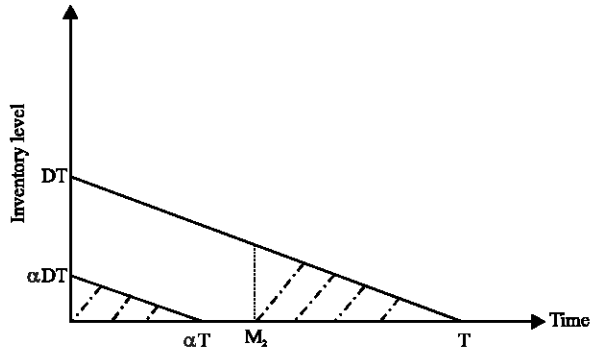


Fig. 2: The inventory level and the total amount of interest payable when $M_2 \leq T \leq M_2/\alpha$

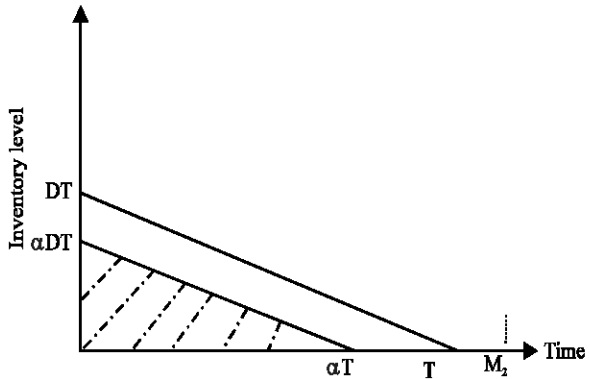


Fig. 3: The inventory level and the total amount of interest payable when $0 < T \leq M_2$

Case 2.2: $M_2 \leq T \leq \frac{M_2}{\alpha}$, shown in Fig. 2.

$$\text{Annual interests payable} = cI_k \left[\frac{\alpha^2 DT^2}{2} + \frac{D(T-M_2)^2}{2} \right] / T$$

Case 2.3: $T \leq M_2$, shown in Fig. 3.

$$\text{Annual interests payable} = cI_k \left(\frac{\alpha^2 DT^2}{2} \right) / T$$

• Interests earned per year:

Case 1: Payment is paid at time M_1 , according to assumption (8); there are two cases in terms of interests earned per year.

Case 1.1 $T \geq M_1$.

$$\text{Interests earned per year} = cI_e \left(\frac{DM_1^2}{2} \right) / T$$

Case 1.2 $T \leq M_1$.

$$\begin{aligned} \text{Interests earned per year} &= cI_e \left[\frac{DT^2}{2} + DT(M_1 - T) \right] / T \\ &= cI_e DT \left(M_1 - \frac{T}{2} \right) / T \end{aligned}$$

Case 2: Payment is paid at time M_2 , according to assumption (8); there are two cases in terms of interests earned per year.

Case 2.1 $T \geq M_2$.

$$\text{Interests earned per year} = cI_e \left(\frac{DM_2^2}{2} \right) / T$$

Case 2.2 $T \leq M_2$.

$$\begin{aligned} \text{Interests earned per year} &= cI_e \left[\frac{DT^2}{2} + DT(M_2 - T) \right] / T \\ &= cI_e DT \left(M_2 - \frac{T}{2} \right) / T \end{aligned}$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

Annual total relevant cost = ordering cost + stock-holding cost + purchasing cost + interest payable - interest earned. We show that the annual total relevant cost is given by:

Case 1: Payment is paid at time M_1

$$\text{TRC}_1(T) = \begin{cases} \text{TRC}_{11}(T) & \text{if } T \geq M_1 \\ \text{TRC}_{12}(T) & \text{if } 0 < T \leq M_1 \end{cases} \quad (1a)$$

where:

$$\begin{aligned} \text{TRC}_{11}(T) &= \frac{A}{T} + \frac{DTh}{2} + c(1-r)D \\ &\quad + \frac{c(1-r)I_k D(T-M_1)^2}{2T} - \frac{cI_e DM_1^2}{2T} \end{aligned} \quad (2)$$

and

$$\text{TRC}_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DcI_e \left(M_1 - \frac{T}{2} \right) \quad (3)$$

At $T=M_1$, we find $\text{TRC}_{11}(M_1) = \text{TRC}_{12}(M_1)$. Hence $\text{TRC}_1(T)$ is continuous and well-defined. All $\text{TRC}_{11}(T)$, $\text{TRC}_{12}(T)$ and $\text{TRC}_1(T)$ are defined on $T > 0$.

Case 2: Payment is paid at time M_2

$$TRC_2(T) = \begin{cases} TRC_{21}(T) & \text{if } T \geq \frac{M_2}{\alpha} \\ TRC_{22}(T) & \text{if } M_2 \leq T \leq \frac{M_2}{\alpha} \\ TRC_{23}(T) & \text{if } 0 < T \leq M_2 \end{cases} \quad (4a)$$

where:

$$TRC_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_k DT \left[\frac{T}{2} - (1-\alpha)M_2 \right] / T - cI_e DM_2^2 / 2T \quad (5)$$

$$TRC_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_k D[\alpha^2 T^2 + (T - M_2)^2] / 2T - cI_e DM_2^2 / 2T \quad (6)$$

and

$$TRC_{23}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + \alpha^2 cI_k DT^2 / 2T - cI_e DT(M_2 - \frac{T}{2}) / T \quad (7)$$

Since

$$TRC_{21}(\frac{M_2}{\alpha}) = TRC_{22}(\frac{M_2}{\alpha})$$

and

$$TRC_{22}(M_2) = TVC_{23}(M_2), TRC(T)$$

is continuous and well-defined. All $TRC_{21}(T)$, $TRC_{22}(T)$, $TRC_{23}(T)$ and $TRC_2(T)$ are defined on $T > 0$.

Optimality conditions:

From Eq. 2, 3 and 5-7 yield

$$TRC_{11}'(T) = \frac{-\{2A + cDM_1^2[(1-r)I_k - I_e]\} + D[h + c(1-r)I_k]}{2T^2} \quad (8)$$

$$TRC_{11}''(T) = \frac{2A + cDM_1^2[(1-r)I_k - I_e]}{T^3} \quad (9)$$

$$TRC_{12}'(T) = \frac{-A}{T^2} + \frac{D(h + cI_e)}{2} \quad (10)$$

$$TRC_{12}''(T) = \frac{2A}{T^3} > 0 \quad (11)$$

$$TRC_{21}'(T) = \frac{-(2A - cDM_2^2 I_e)}{2T^2} + \frac{D(h + cI_k)}{2} \quad (12)$$

$$TRC_{21}''(T) = \frac{2A - cDM_2^2 I_e}{T^3} \quad (13)$$

$$TRC_{22}'(T) = \frac{-[2A + cDM_2^2(I_k - I_e)] + D[h + cI_k(1 + \alpha)^2]}{2T^2} \quad (14)$$

$$TRC_{22}''(T) = \frac{2A + cDM_2^2(I_k - I_e)}{T^3} > 0 \quad (15)$$

$$TRC_{23}'(T) = \frac{-A}{T^2} + \frac{D[h + c(\alpha^2 I_k + I_e)]}{2} \quad (16)$$

$$TRC_{23}''(T) = \frac{2A}{T^3} > 0 \quad (17)$$

Equation 11, 15 and 17 imply that all $TRC_{12}(T)$, $TRC_{22}(T)$ and $TRC_{23}(T)$ are convex on $T > 0$. Equation 9 implies $TRC_{11}(T)$ is convex on $T > 0$ if

$$2A + cDM_1^2[(1-r)I_k - I_e] > 0.$$

Equation 13 implies $TRC_{21}(T)$ is convex on $T > 0$ if

$$2A - cDM_2^2 I_e > 0.$$

Furthermore, we have

$$TRC_{11}'(M_1) = TRC_{12}'(M_1),$$

$$TRC_{21}'(\frac{M_2}{\alpha}) = TRC_{22}'(\frac{M_2}{\alpha}) \text{ and}$$

$$TRC_{22}'(M_2) = TRC_{23}'(M_2).$$

Therefore, Eq. 1a and b imply that $TRC_1(T)$ is convex on $T > 0$ if $2A + cDM_1^2[(1-r)I_k - I_e] > 0$ and Eq. (4a-c) imply that $TRC_2(T)$ is convex on $T > 0$ if $2A - cDM_2^2 I_e > 0$.

Decision rule of the optimal cycle time T^* and optimal payment time

$$\text{Let } TRC_{11}'(T_{11}^*) = TRC_{12}'(T_{12}^*) = TRC_{21}'(T_{21}^*) \\ = TRC_{22}'(T_{22}^*) = TRC_{23}'(T_{23}^*) = 0.$$

We can obtain:

$$T_{11}^* = \sqrt{\frac{2A + cDM_1^2[(1-r)I_k - I_e]}{D[h + c(1-r)I_k]}} \quad (18)$$

if $2A + cDM_1^2[(1-r)I_k - I_e] > 0$

$$T_{12}^* = \sqrt{\frac{2A}{D(h + cI_e)}} \quad (19)$$

$$T_{21}^* = \sqrt{\frac{2A - cDM_2^2 I_e}{D(h + cI_k)}} \text{ if } 2A - cDM_2^2 I_e > 0 \quad (20)$$

$$T_{22}^* = \sqrt{\frac{2A+cDM_2^2(I_k-I_e)}{D[h+cI_k(1+\alpha^2)]}} \tag{21}$$

and

$$T_{23}^* = \sqrt{\frac{2A}{D[h+c(\alpha^2 I_k + I_e)]}} \tag{22}$$

Equation 18 implies that the optimal value of T for the case of $T \geq M_1$, that is $T_{11}^* \geq M_1$. We substitute Eq. 18 into $T_{11}^* \geq M_1$, then we can obtain the optimal value of T if and only if $-2A+DM_1^2(h+cI_e) \leq 0$.

Likewise, Eq. 19 implies that the optimal value of T for the case of $T \leq M_1$, that is $T_{12}^* \leq M_1$. We substitute Eq. 19 into $T_{12}^* \leq M_1$, then we can obtain the optimal value of T if and only if $-2A+DM_1^2(h+cI_e) \geq 0$.

Similar discussion, we can obtain following results:
 $M_2/\alpha \leq T_{21}^*$ if and only if

$$-(2A-cDM_2^2 I_e)+D(\frac{M_2}{\alpha})^2(h+cI_k) \geq 0.$$

$M_2 \leq T_{22}^* \leq M_2/\alpha$ if and only if

$$-(2A-cDM_2^2 I_e)+D(\frac{M_2}{\alpha})^2(h+cI_k) \geq 0.$$

and if and only if $-2A+DM_2^2[h+c(\alpha^2 I_k + I_e)] \leq 0$,

$T_{23}^* \leq M_2$ if and only if $-2A+DM_2^2[h+c(\alpha^2 I_k + I_e)] \geq 0$,

Furthermore, we let

$$\Delta_1 = -2A+DM_1^2(h+cI_e) \tag{23}$$

$$\Delta_2 = -(2A-cDM_2^2 I_e)+D(\frac{M_2}{\alpha})^2(h+cI_k) \tag{24}$$

and

$$\Delta_3 = -2A+DM_2^2[h+c(\alpha^2 I_k + I_e)]. \tag{25}$$

Since $M_1 < M_2$, we can get $\Delta_1 \geq \Delta_2 \geq \Delta_3$ from Eq. 23-25. Summarized above arguments, the optimal cycle time T* and optimal payment time (M_1 or M_2) can be obtained as follows:

Theorem 1:

(A) If $\Delta_1 \geq 0$, then $TRC(T^*) = \min\{TRC_1(T_{12}^*), TRC_2(T_{23}^*)\}$. Hence T* is T_{12}^* or T_{23}^* , optimal payment time is M_1 or M_2 associated with the least cost.

(B) If $\Delta_1 < 0$ and $\Delta_3 > 0$, then $TRC(T^*) = \min\{TRC_1(T_{11}^*), TRC_2(T_{23}^*)\}$. Hence T* is T_{11}^* or T_{23}^* , optimal payment time is M_1 or M_2 associated with the least cost.

(C) If $\Delta_2 > 0$ and $\Delta_3 \leq 0$, then $TRC(T^*) = \min\{TRC_1(T_{11}^*),$

$TRC_2(T_{22}^*)\}$. Hence T* is T_{11}^* or T_{22}^* , optimal payment time is M_1 or M_2 associated with the least cost.

(D) If $\Delta_2 \leq 0$, then $TRC(T^*) = \min\{TRC_1(T_{11}^*), TRC_2(T_{21}^*)\}$. Hence T* is T_{11}^* or T_{21}^* , optimal payment time is M_1 or M_2 associated with the least cost.

Theorem 1 immediately determines the optimal cycle time T* and optimal payment time (M_1 or M_2) after computing the numbers Δ_1 , Δ_2 and Δ_3 . Theorem 1 is an efficient solution procedure.

CONCLUSIONS

The supplier offers the permissible delay in payments to stimulate the demands of the retailer. But the supplier also hopes that the retailer can pay the payment as soon as possible. This study investigates the retailer's replenishment and payment policy under supplier offered cash discount and permissible delay in payments depending on retailer payment time. The supplier can handle the fraction of the delay payment permitted and cash discount rate to attract the retailer to pay the payment as soon as possible to shorten its collection period. This policy is a valuable and realistic alternative to the supplier.

Then, we develop the retailer's inventory model and provide a very efficient solution procedure. Theorem 1 effectively determines the optimal cycle time T* and optimal payment time after computing the numbers Δ_1 , Δ_2 , and Δ_3 .

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