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## Optimization of the Heat Transfer Rate in Undulated Enclosures with Multiple Partitions

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**Abstract:** In the present study, a numerical simulation of laminar natural convection in rectangular undulated enclosures with multiples horizontal partitions was carried out. The partitions have different lengths. They are placed on the undulations of the wavy hot wall. Vertical side walls were kept at constant temperature and horizontal top and bottom walls were insulated. The fluid flow and temperature fields were predicted for different configurations using computer program developed on the basis of finite volume approach. The heat transfer was seriously reduced but there is an optimum value. Effect of cavity aspect ratio, partition length and number of partitions on the heat transfer rate were also investigated.

**Key words:** Natural convection, wavy wall, partial partitions, aspect ratio

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### INTRODUCTION

Natural convection in enclosures has been receiving considerable attention due to its numerous applications such as in solar collectors, thermal design of building, nuclear reactor design and cooling of electronic equipment.

Convective flow in enclosures without partitions is very well studied. One of the first experimental and numerical studies was presented by Elder<sup>[1-3]</sup>. An overview of later results was presented by Catton<sup>[4]</sup> and Yang<sup>[5]</sup>. Results of natural convection in enclosures are also used as bench mark data for validation of various numerical methods for solving Navier-Stokes equations. A comparison of numerical results for two dimensional air filled square cavities and opposite vertical walls at different temperature was presented by De Vahl Davis and Jones<sup>[6]</sup>. The benchmark numerical results were obtained with grids up to 81×81 points at Rayleigh numbers of 10<sup>3</sup>, 10<sup>4</sup>, 10<sup>5</sup> and 10<sup>6</sup>. In many engineering applications, enclosures with vertical partitions are encountered to reduce the heat transfer by natural convection and conduction, the heat transfer rate through the system with partitions decreases with increasing number of partitions. Enclosures with single and multiple partitions are seen in solar collectors, wall bricks (building walls), cryogenic storage, nuclear reactor and others. Anderson and Bejan<sup>[7]</sup> studied the natural convection in single and double partitioned enclosures experimentally and theoretically. They found that the heat transfer rates through double partitions is 20% less than for single

partitions which is inserted in the middle of the enclosure. Nakamura *et al.*<sup>[8]</sup> performed computational and experimental studies in air filled enclosures with a single diathermal partition located vertically at the centre of the enclosure. Tong and Gerner<sup>[9]</sup> investigated numerically the effect of partition position on the heat transfer rate. They concluded that a central partition produce greatest reduction in heat transfer. In some instances, they found that due to partitions placed midway between the vertical walls, the heat transfer rate decreases by over 50%. The effect of an off centre partitions on natural convection heat transfer was also reported by Nishumura *et al.*<sup>[10]</sup> based on experimental and numerical investigations. Their experimental results for aspect ratio of 4 and 10 showed that the nusselt number for a given Rayleigh number are independent of the location of the partitions. Effects of partial and complete vertical partitions were studied numerically by Ciofalo and Karajiannis<sup>[11]</sup>, While a detailed analysis of the case with a complete vertical partitions was presented by Karajiannis *et al.*<sup>[12]</sup>, the thickness and conductivity of the enclosure were varied, as well as the aspect ration of the enclosure is from 0.1 to 16. Mamou *et al.*<sup>[13]</sup> studied analytically and numerically the influence of multiple, tick and conductive partitions inside inclined rectangular enclosures filled with air (Pr = 0.71) with a uniform heat flux at the side walls and two other walls adiabatic. The maximum average Nusselt number occurs at a lower conductivity ratio as a number of partitions increases.

It is necessary to study the heat and mass from an irregular surfaces because irregular surfaces are often present in many applications. It is often encountered in heat transfer devices to enhance heat transfer; for examples, flat-plate solar collectors and flat-plate condensers in refrigerators. The natural convection heat transfer from an isothermal vertical wavy surface was first studied by Yao<sup>[14,15]</sup> and using an extended Prandtl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces such a sinusoidal surface. Chou<sup>[16]</sup> studied the natural convection heat transfer along a vertical wavy surface in micro polar fluids. Chen and Wang<sup>[17,18]</sup> analysed transient forced and free convection along a wavy surface in micro fluids. Adjilout *et al.*<sup>[19]</sup> reported natural convection in an inclined cavity with hot wavy wall and cold flat wall. One of their interesting findings was the decrease of average heat transfer with the surface waviness when compared with the flat wall cavity.

In the present investigation a numerical study of the effect of the hot wavy wall with partial partitions on free convection in rectangular cavities differentially heated was undertaken. This problem is solved by using the partial differential equations which are the equations of mass, momentum and energy. Employing control volume approach, a computer program based on Simple algorithm was developed. Computations were carried out to investigate the effects of the hot wall geometry, number of partitions, partitions lengths and cavity aspect ratios on the heat transfer rate. It was found that the partitions length was an important parameter, the heat transfer was reduced with increasing partitions length; but there was an optimum value. Different configurations have been tested. The hot wall is wavy with different undulations namely 3, 5 and 7 undulations. The aspect ratio of the cavities has also been changed. While the Rayleigh number was  $10^5$  and the Prandtl number was kept constant ( $Pr = 0.71$ ). The stream functions, isotherms and the mean Nusselt number were predicted in enclosure with different aspect ratios and partitions lengths.

**MATHEMATICAL FORMULATION**

The problem can be considered to be two dimensional. Using the Boussinesq approximation to account for density variation, the steady state laminar non dimensional continuity, momentum and energy equations can be written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left[ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right] \tag{2}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left[ \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right] + Ra Pr \theta \tag{3}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left[ \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right] \tag{4}$$

Where, *u* and *v* are the dimensionless velocity components in the *x* and *y* directions, respectively. *P* and  $\theta$  are the dimensionless pressure and temperature, respectively. These dimensionless variables are defined as:

$$\begin{aligned} X &= \frac{x}{L} & Y &= \frac{y}{L} \\ U &= \frac{u}{v/L} & V &= \frac{v}{v/L} \\ P &= \frac{P + \rho g y}{\rho (v/L)^2} & \theta &= (T - T_0) / (T_h - T_c) \end{aligned}$$

with  $T_0 = (T_h + T_c)/2$ .

Equation 1-4 are subject to the following boundary conditions:

$$\begin{aligned} U = V &= 0 \text{ at all the boundaries} \\ \theta(0, Y) &= -0.5 \text{ on the cold wall} \\ \theta(1, Y) &= 0.5 \text{ on the hot wall} \end{aligned}$$

$$\frac{\partial \theta}{\partial Y} = 0 \text{ on the adiabatic walls}$$

Figure 1 shows the geometrical features of the cavities used in the present study. The grid generation calculation is based on the curvilinear co-ordinate system applied to fluid flow as described by Thompson *et al.*<sup>[20]</sup>.

Knowing that:

$0 \leq y \leq 1$  and  $0 \leq x \leq f(y)$  with:  $f(y) = [1 - Amp + Amp (\cos 2\pi ny)]$  where, *n* and *amp* are, respectively a number of undulations and amplitude, respectively.

The heat transfer rate by convection in an enclosure is obtained from the Nusselt number calculation. On the wavy wall, the local and the mean Nusselt number are expressed, respectively:

$$\begin{aligned} Nu_L &= \frac{\partial \theta}{\partial n} \\ Nu_a &= \frac{1}{s} \int_0^s \frac{\partial \theta}{\partial n} ds \end{aligned}$$

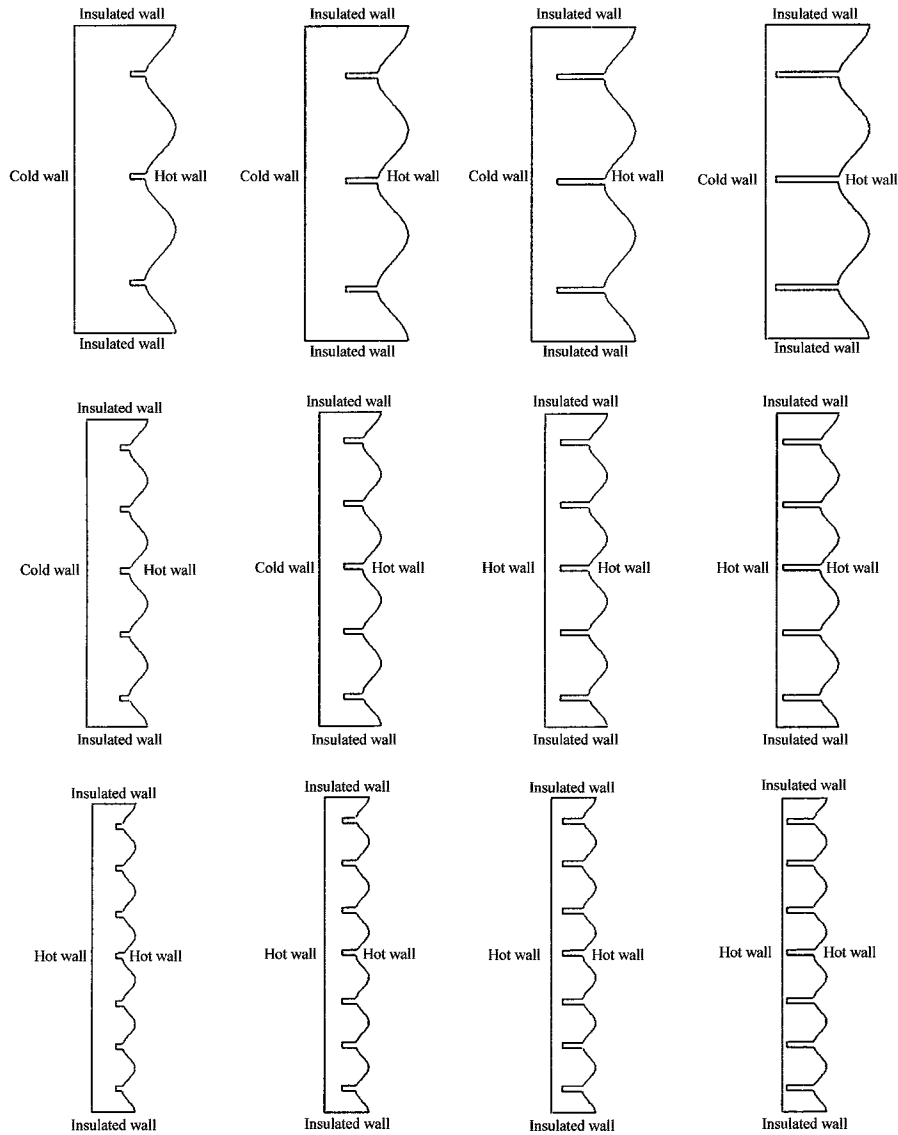


Fig. 1: The shape of the cavities used in the investigations

**NUMERICAL PROCEDURE**

The governing equations were discretized by integrating them over finite control volume. The control volume for velocity components were staggered relative to those for scalar variable. Upwind scheme was employed to discretise the convection and diffusion terms. Using SIMPLE algorithm, a computer program was developed. For the solution of algebraic equations, Gauss-seidel point by point iteration technique was employed. For all the grid points U, V, P and  $\theta$  were solved. The convergence criterion was based on the satisfaction of the continuity for each control volume in accuracy of  $10^{-5}$  [21,22]. Several grids have been tested for the different cavities. Grid refinement was applied to check the accuracy of the solution. The

Table 1: Different grids for all aspect ratio,  $Ra = 10^5$

A	3	5	7
Grid	40×121	40×201	40×281

number of grids cells used in the calculations were chosen such as that the variation of the mean Nusselt number is by less than 5%. From Sabeur-Bendehina *et al.* [23], all grids studied for different cavities are presented in Table 1.

**RESULTS AND DISCUSSION**

In the present investigation, the tests were performed for three rectangular enclosures with the aspect ration namely 3, 5 and 7 with the same number of undulations for each cavity, respectively and for four partitions length;

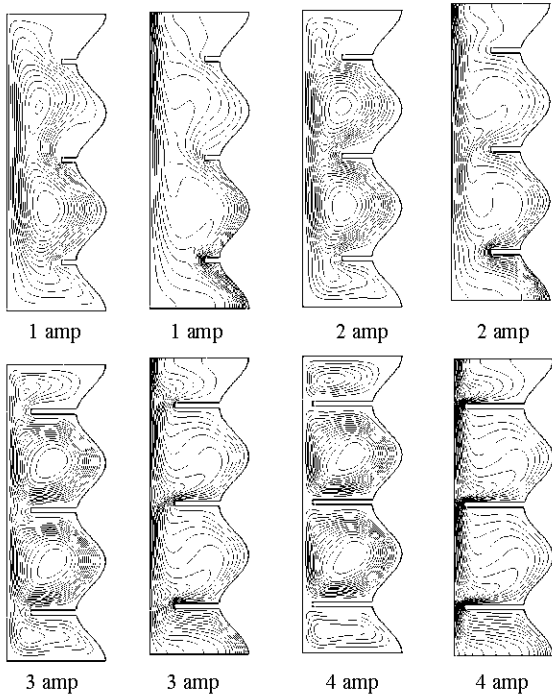


Fig. 2: Streamlines and isotherms for different configurations ( $A = 3$ )

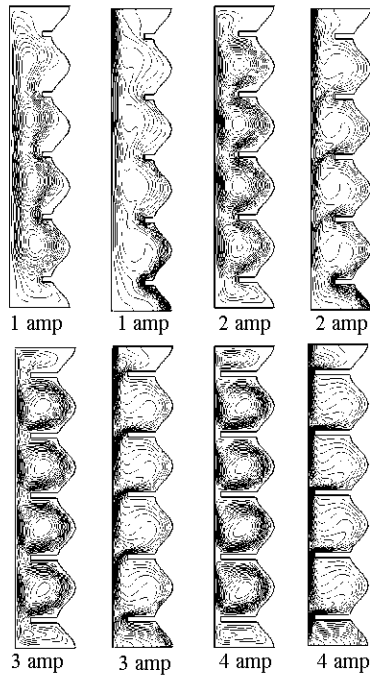


Fig. 3: Streamlines and isotherms for different configurations ( $A = 5$ )

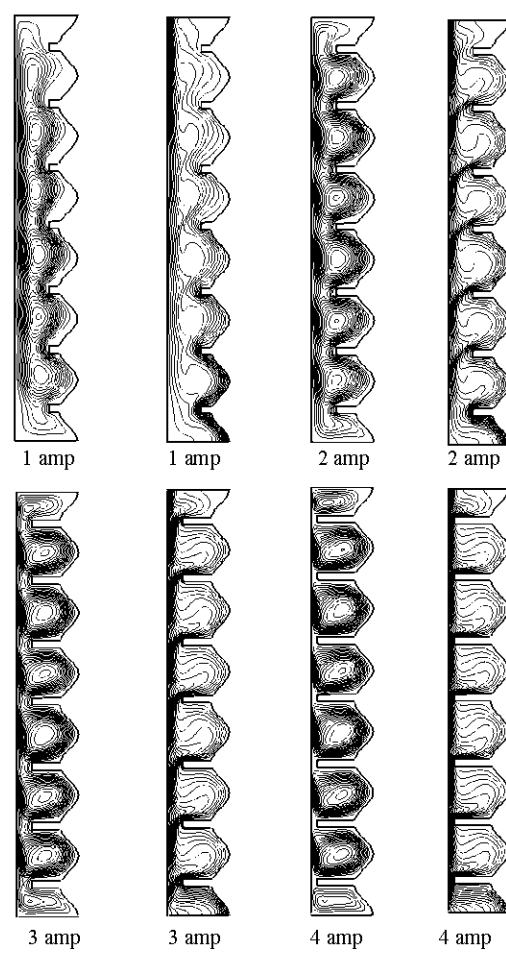


Fig. 4: Streamlines and isotherms for different configurations ( $A = 7$ )

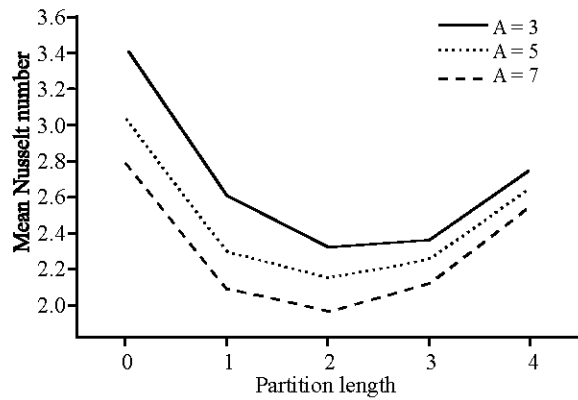


Fig. 5: Variation of the Nusselt number with partitions lengths for all cavities

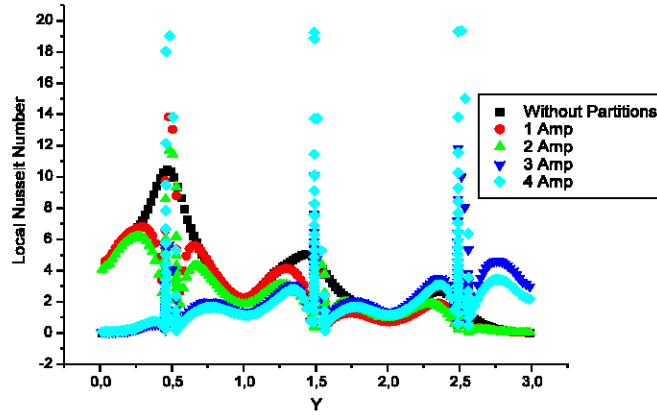


Fig. 6: Local Nusselt number for the cavity with aspect ratio 3 , three undulations and three partitions

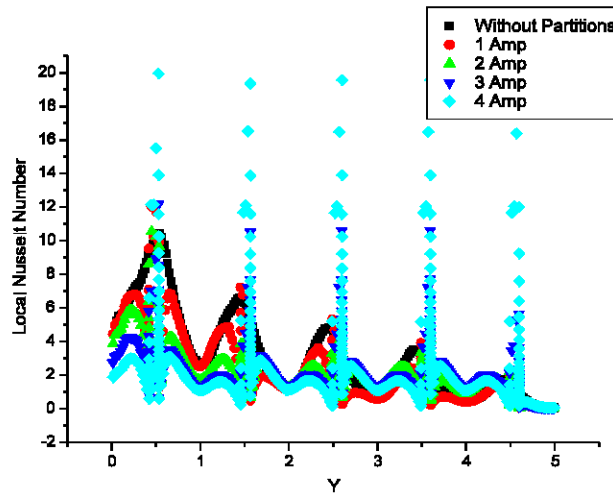


Fig. 7: Local Nusselt number for the cavity with aspect ratio 5, five undulations and five partitions

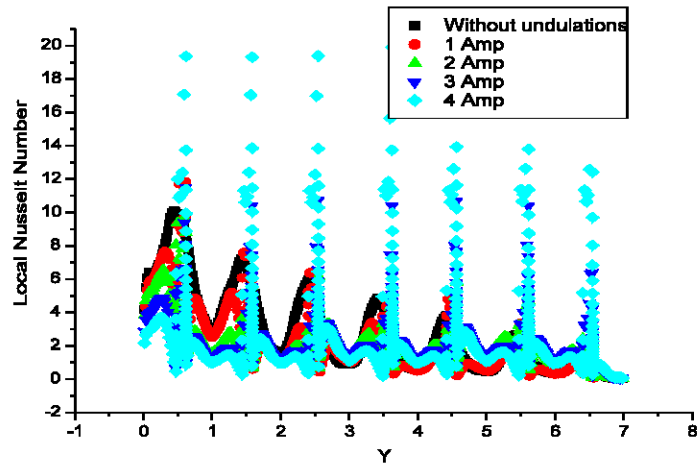


Fig. 8: Local Nusselt number for the cavity with aspect ratio 7, seven undulations and seven partitions

while the thickness was kept constant and equal to the half of the undulation amplitude which was equal to 0.15.

The Fig. 2 shows the streamlines and the isotherms distribution for a cavity with aspect ratio 3, three undulations and different partition length; the flow is multi cellular, it is observed that the partitions length affects the flow pattern in the cavity, it is noticed that an increasing in length partitions leads to stretched cells in the cavity. Due to this behaviour of the flow pattern, the heat transfer is also strongly affected by the partitions lengths. It is clearly seen that near the wavy wall, the thermal boundary layer thickness increases in the flow direction resulting in a decrease in the local heat transfer along this wall compared with the undulated wall without partitions, the thermal boundary layer thickness seems to increase with the presence of partitions. Each partial partitions contribute to the thickening of the thermal boundary layer. The same remarks are valid for cavity aspect ratio 5 and 7 with five and seven undulations respectively as shown in Fig. 3 and 4.

Figure 6-8 represent the local Nusselt number distribution, respectively for undulated and undulated with partitions cavities. The wavy trend of the local Nusselt number is characterising the cavity with undulations. The presence of partitions results in peaks in the local Nusselt number curve. The number of peaks corresponds to the partitions number. It is noticed that the mean Nusselt number first decreases with the increasing of partitions length; however, further increasing in the partitions length causes an increase in the mean Nusselt number for all configurations tested. It is observed that the variation of the mean Nusselt number with the partitions lengths has an optimum for all cavities as shown in Fig. 5.

The mean Nusselt number monotonically decreases with 1 and 2 amplitude partitions lengths; but when partitions length reaches to 3 amplitudes the average Nusselt number increases; it is also seen that an increasing in aspect ratio of the cavities results in a decrease of the mean Nusselt number; this remark is also in agreements with the results of Sabeur-Bendehina *et al.*<sup>[23]</sup>. Therefore, it can be concluded that there is an optimum of partition length to minimize the heat transfer rate through a fluid layer contained in enclosure.

### CONCLUSIONS

In this study, the natural convection heat transfer in rectangular enclosure with wavy hot wall and partial partitions attached to the hot wall was numerically

analysed. Using the control volume approach and SIMPLE algorithm; the simulations were performed to investigate the effects of partitions lengths on the heat transfer rate. The partition lengths is an important parameter for the optimisation of the heat transfer rate. This heat transfer diminution is interesting in solar application. Indeed, in solar collector for example, the exchange by radiation increases with the increase of exchange area.

### NOMENCLATURE

A	Aspect ratio of enclosure
C <sub>p</sub>	Specific heat of fluid
G	Gravitational acceleration
K	Thermal diffusivity
H	Width of the enclosure
L	Length of the enclosure
Nu	Nusselt number
P, p	Dimensionless and dimensional pressure
Pr	Prandtl number = $\mu C_p/k$
Ra	Rayleigh number = $\frac{g \beta (T_h - T_c) L^3}{\nu k}$
T, $\theta$	Dimensionless and dimensional temperature
U, u	Dimensionless and dimensional horizontal velocity component
V, v	Dimensionless and dimensional vertical velocity component
X, x	Dimensionless and dimensional coordinate along the horizontal direction
Y, y	Dimensionless and dimensional coordinate along the vertical direction
Greek symbols	
$\beta$	Thermal dilatation coefficient
$\nu$	Kinematic viscosity
$\theta$	Dimensionless temperature
$\rho$	Density
$\mu$	Viscosity
Subscripts	
a	Average
L	Local
h, c	Hot wall and cold wall

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