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A Note on Joint Inventory and Technology Selection Decisions under Constant Demand

[1]Jyh-Rong Chang , [1]Yung-Fu Huang and [2]Hung-Fu Huang
[1]Department of Business Administration, Chaoyang University of Technology,
Taichung, Taiwan, Republic of China
[2]Department of Electrical Engineering, National Cheng Kung University, Tainan, Tainan, Republic of China

Abstract: This note deals with the optimal lot sizing decision at the technology selection stage, and modifies the optimal solution procedure in constant demand case described in Khouja (Omega 2005, 33, 47-53). This note develops an alternative approach to find the optimal lot sizing to improve the study of Khouja (Omega 2005, 33, 47-53). Finally, numerical examples are given to illustrate the result discussed in this study.

Key words: Lot sizing, inventory, EOQ

INTRODUCTION

Recently, Khouja[1] developed a model to determine the total cost per unit of time and the optimal order quantity at the technology selection stage. The cost of the technology depends on the lot size it can produce. In addition, the model investigated two different types of demand included constant demand and linearly decreasing demand. For convenience, we use notation and assumptions similar to Khouja[1]. Khouja[1] developed the following model for the total cost per unit of time over the life of a mold is:

$$TC(Q)=\begin{cases} TC_1(Q) & \text{if} \quad Q<Q_0 \quad (1a) \\ TC_2(Q) & \text{if} \quad Q\ge Q_0 \quad (1b) \end{cases}$$

where:

$$TC_1(Q)=\frac{Q}{2}h+\frac{[C_1+(C_2+C_3Q)Q+SU_0]D}{QU_0} \qquad (2)$$

and

$$TC_2(Q)=\frac{Q}{2}h+\frac{[C_1+Q(C_2+C_3Q+Su)]D}{Q^2u}. \qquad (3)$$

Since $TC_1(Q_0)=TC_2(Q_0)$ when $U_0 = uQ_0$, TC(Q) is continuous and well-defined. All $TC_1(Q)$, $TC_2(Q)$ and TC(Q) are defined on Q>0. Eq. 2 and 3 yield

$$TC_1'(Q)=\frac{-2(C_1+SU_0)D+Q^2(hU_0+2C_3D)}{2Q^2U_0}, \qquad (4)$$

$$TC_1''(Q)=\frac{2D(C_1+SU_0)}{Q^3U_0}>0, \qquad (5)$$

$$TC_2'(Q)=\frac{huQ^3-2D(C_2+Su)Q-4C_1D}{2uQ^3} \qquad (6)$$

and

$$TC_2''(Q)=\frac{2[3C_1+Q(C_2+Su)]D}{uQ^4}>0. \qquad (7)$$

Equation 5 and 7 imply that $TC_1(Q)$ and $TC_2(Q)$ are convex on Q>0. Furthermore, we have $TC_1'(Q_0)\ne TC_2'(Q_0)$. Therefore, Eq. 1a, b imply that TC(Q) is piecewise convex on Q>0.

Let $TC_i'(Q_i*)=0$ for all i= 1, 2. By the convexity of $TC_i(Q)$ (i = 1, 2), we see

$$TC_i'(Q)\begin{cases} <0 & \text{if} \quad Q<Q_i* \quad (8a) \\ =0 & \text{if} \quad Q=Q_i* \quad (8b) \\ >0 & \text{if} \quad Q>Q_i*. \quad (8c) \end{cases}$$

Equation 8a-c imply that $TC_i(Q)$ is decreasing on $(0, Q_i*]$ and increasing on $[Q_i*, \infty)$ for all i = 1, 2. Eq. 4 and 6 yield that:

$$TC_1'(Q_0)=\frac{-2(C_1+SU_0)D+Q_0^2(hU_0+2C_3D)}{2Q_0^2U_0}$$
$$=\frac{-2(C_1+SuQ_0)D+Q_0^2(huQ_0+2C_3D)}{2uQ_0^3}$$
$$\text{when } U_0=uQ_0 \qquad (9)$$

Corresponding Author: Yung-Fu Huang, No.168, Jifong E. Rd., Wufong Township, Taichung County 41349, Taiwan, Republic of China
Tel: + 886 42 47 39 477 Fax: + 886 4 247 29 772

and

$$TC_2'(Q_0) = \frac{huQ_0^3 - 2D(C_2 + Su)Q_0 - 4C_1D}{2uQ_0^3}. \quad (10)$$

Furthermore, we let

$$\Delta_1 = -2(C_1 + SuQ_0)D + Q_0^2(huQ_0 + 2C_3D) \quad (11)$$

and

$$\Delta_2 = huQ_0^3 - 2D(C_2 + Su)Q_0 - 4C_1D \quad (12)$$

Then, we can find $\Delta_1 > \Delta_2$ from Eq. 11 and 12. We can obtain optimal lot sizing Q* using following result.

Theorem 1

(A) If $\Delta_2>0$, then $TC(Q^*)= TC_1(Q_1^*)$ and $Q^* = Q_1^*$.
(B) If $\Delta_1>0$ and $\Delta_2\leq 0$, then $TC(Q^*)= \min\{TC_1(Q_1^*), TC_2(Q_2^*)\}$. Hence, Q* is Q_1^* or Q_2^* associated with the least cost.
(C) If $\Delta_1\leq 0$, then $TC(Q^*)= TC_2(Q_2^*)$ and $Q^* = Q_2^*$.

Proof

(A) If $\Delta_2>0$ then $\Delta_1>0$. We have $TC_1'(Q_0)>0$ and $TC_2'(Q_0)>0$. Eq. 8a-c imply that
   (i) $TC_1(Q)$ is decreasing on $(0, Q_1^*]$ and increasing on $[Q_1^*, Q_0)$.
   (ii) $TC_2(Q)$ is increasing on $[Q_0, \infty)$.

Combining (i), (ii) and Eq. 1a and b, we have that TC(Q) is decreasing on $(0, Q_1^*]$ and increasing on $[Q_1^*, \infty)$. Consequently, $Q^* = Q_1^*$.

(B) If $\Delta_1>0$ and $\Delta_2\leq 0$. We have $TC_1'(Q_0)>0$ and $TC_2'(Q_0)\leq 0$. Eq. 8a-c imply that
   (i) $TC_1(Q)$ is decreasing on $(0, Q_1^*]$ and increasing on $[Q_1^*, Q_0)$.
   (ii) $TC_2(Q)$ is decreasing on $[Q_0, Q_2^*]$ and increasing on $[Q_2^*, \infty)$.

Combining (i), (ii) and Eq. 1a and b, we find that
(iii) TC(Q) is decreasing on $(0, Q_1^*]$.
(iv) TC(Q) is increasing on $[Q_1^*, Q_0]$.
(v) TC(Q) is decreasing on $[Q_0, Q_2^*]$.
(vi) TC(Q) is increasing on $[Q_2^*, \infty)$.

Hence $TC(Q^*)= \min\{TC_1(Q_1^*), TC_2(Q_2^*)\}$. Consequently, Q* is Q_1^* or Q_2^* associated with the least cost.

(C) If $\Delta_1\leq 0$ then $\Delta_2\leq 0$. We have $TC_1'(Q_0)\leq 0$ and $TC_2'(Q_0)\leq 0$. Eq. 8a-c imply that
(i) $TC_1(Q)$ is decreasing on $(0, Q_0)$.
(ii) $TC_2(Q)$ is decreasing on $[Q_0, Q_2^*]$ and increasing on $[Q_2^*, \infty)$.

Combining (i), (ii) and Eq. 1a and b, we have that TC(Q) is decreasing on $(0, Q_2^*]$ and increasing on $[Q_2^*, \infty)$. Consequently, $Q^* = Q_2^*$.

Incorporating the above arguments, we have completed the proof of Theorem 1.

Above Theorem 1 developed in this note is an alternative approach to determine the optimal lot sizing under minimizing the total cost per unit of time. However, Khouja[1] also developed a procedure to find the optimal solution in this situation. Khouja[1] suggested four cases to find the optimal solution. But we find case (d) can be deleted. Since $Q_1^*\geq Q_0$, we can easily obtain the sufficient condition for optimality of $TC_2(Q)$ is negative. That is, the Eq. 11 in Khouja[1] does not exist when $Q_1^*\geq Q_0$. It implies that Q_2^* does not exist. Therefore, case (d) in Khouja's[1] optimal solution procedure does not exist. Theorem 1 developed in this note explains that after computing the numbers Δ_1 and Δ_2, we can immediately determine which one of Q_1^* or Q_2^* is optimal. Theorem 1 essentially modifies the solution procedure described in Khouja[1].

NUMERICAL EXAMPLES

To illustrate the results, let us apply the proposed method to solve the same numerical examples as Khouja[1]. Let h=\$15/unit/year, $g(Q)=10+8Q+7Q^2$ (i.e. $C_1= 10$, $C_2= 8$ and $C_3= 7$) and U= 12 if Q<10; otherwise U= 1.2Q (i.e. $U_0= 12$).

Example 1: When S=\$15/setup and D=30 units/year. Then, we have $\Delta_2>0$. Using Theorem 1-(A), we get $Q^* = Q_1^*= 4$. $TC(Q^*)=TC_1(Q_1^*)=$ \$238.75/year.

Example 2: When S=\$30/setup and D=72 units/year. Then, we have $\Delta_1>0$, $\Delta_2<0$, $Q_1^*= 7$ and $Q_2^*= 19$. Using Theorem 1-(B), we can find $TC_1(Q_1^*)$ = \$711.64/year>$TC_2(Q_2^*)$ = \$703.11/year. Therefore, $Q^* = Q_2^*= 19$ and $TC(Q^*)=TC_2(Q_2^*)=$ \$703.11/year. These results are different from the numerical example 1 in Khouja[1] under same value of all parameters.

Example 3: When S=\$70/setup and D=72 units/year. Then, we have $\Delta_1<0$. Using Theorem 1-(C), we get $Q^* = Q_2^*= 27$. $TC(Q^*)=TC_2(Q_2^*)=$ \$827.77/year.

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