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A Note on an EOQ Model for Deteriorating Items under Trade Credits

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Abstract: This study deals with the optimal inventory decisions for the customer under the permissible delay in payments and cash discount offered by the supplier. We develop an alternative approach to find the optimal replenishment time for the customer to improve the Ouyang *et al.*'s model. In Ouyang *et al.*'s model, they use Taylor's series approximation to obtain the explicit closed-form solution of the optimal replenishment time. We modify this approximation method to exact solution and develop an easy-finding solution theorem to help the decision-maker to decide the optimal replenishment policy.

Key words: Inventory, EOQ, permissible delay in payments, cash discount, trade credit

INTRODUCTION

Recently, Ouyang *et al.*[1] developed a model to determine the optimal replenishment time for the customer under the permissible delay in payments and cash discount offered by the supplier. They use Taylor's series approximation to obtain the explicit closed-form solution and provide an easy-to-use algorithm to find the optimal order quantity and replenishment time.

We know many researchers assume that x is sufficiently small to simplify the process of the solution procedure. They use Taylor's series approximation, let $e^x=1+x+x^2/2$, to deal with the complex solution procedure. But x is not necessarily small. At this condition, the approximation approach may cause significant errors and penalties. Therefore, this note will modify this approximation method to exact solution to find the optimal replenishment time for the customer. In addition, we develop an alternative approach to find the optimal replenishment time for the customer to improve the results of Ouyang *et al.*[1]. For convenience, we use notation and assumptions similar to Ouyang *et al.*[1]. Ouyang *et al.*[1] developed the following model for the total relevant cost per year. At first, we define the new notation:

$$Z(T)=\begin{cases}\overline{Z}(T) & \text{if the payment is paid at time } M_1\\ \overline{\overline{Z}}(T) & \text{if the payment is paid at time } M_2\end{cases}$$

When the payment is paid at time M_1, the total relevant cost per year is:

$$\overline{Z}(T)=\begin{cases}Z_1(T) & \text{if } T\ge M_1\\ Z_2(T) & \text{if } T< M_1\end{cases}$$

Where:

$$Z_1(T)=\frac{S}{T}+\frac{D[h+c\theta(1-r)]}{\theta^2 T}(e^{\theta T}-1)-\frac{hD}{\theta}+\frac{c(1-r)I_c D}{\theta^2 T}[e^{\theta(T-M_1)}-1]-\frac{c(1-r)I_c D}{\theta T}(T-M_1)-\frac{pI_d D}{2T}M_1^2 \quad (1)$$

and

$$Z_2(T)=\frac{S}{T}+\frac{D[h+c\theta(1-r)]}{\theta^2 T}(e^{\theta T}-1)-\frac{hD}{\theta}-pI_d D(M_1-\frac{T}{2}) \quad (2)$$

At $T=M_1$, we find $Z_1(M_1)=Z_2(M_1)$. Hence, $\overline{Z}(T)$ is continuous for $T>0$.

When the payment is paid at time M_2, the total relevant cost per year is:

$$\overline{\overline{Z}}(T)=\begin{cases}Z_3(T) & \text{if } T\ge M_2\\ Z_4(T) & \text{if } T< M_2\end{cases}$$

Where:

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$$Z_3(T) = \frac{S}{T} + \frac{D(h+c\theta)}{\theta^2 T}(e^{\theta T}-1) - \frac{hD}{\theta} + \frac{cI_cD}{\theta^2 T}[e^{\theta(T-M_2)}-1] - \frac{cI_cD}{\theta T}(T-M_2) - \frac{pI_dD}{2T}M_2^2 \quad (3)$$

and

$$Z_4(T) = \frac{S}{T} + \frac{D(h+c\theta)}{\theta^2 T}(e^{\theta T}-1) - \frac{hD}{\theta} - I_dD(M_2 - \frac{T}{2}) \quad (4)$$

At $T = M_2$, we find $Z_3(M_2) = Z_4(M_2)$. Hence, $\overline{\overline{Z(T)}}$ is continuous for $T > 0$.

Ouyang *et al.*[1] proved all Z_i (i = 1,2,3 and 4) are convex functions. We do not use Taylor's series approximation to find the optimal solution. We derive $Z_i'(T_i) = 0$ if $T > 0$ for all i = 1, 2, 3 and 4. According to the convexity of $Z_i(T)$(i = 1, 2, 3 and 4), the Newton's method can be used to locate T_i if T_i exists for all i = 1, 2, 3 and 4. By the convexity of Z_i (i = 1, 2, 3 and 4), we see:

$$Z_i'(T)\begin{cases} <0 & \text{if} \quad T<T_i \quad (5a) \\ =0 & \text{if} \quad T=T_i \quad (5b) \\ >0 & \text{if} \quad T>T_i \quad (5c) \end{cases}$$

Equation 5a-c imply that $Z_i(T)$ is decreasing on $(0, T_i]$ and increasing on $[T_i,\infty)$ for all i = 1, 2, 3 and 4.

About the existence of T_i (i = 2 and 4), since $\lim_{T\to 0^+} Z_i'(T) = -\infty$ and $\lim_{T\to\infty} Z_i'(T) = \infty$, the Intermediate Value Theorem[2] implies that T_i^* exists for i = 2 and 4. On the other hand, since $\lim_{T\to\infty} Z_1'(T) = \infty$, there are two cases to occur:

(1) If $Z_1'(M_1) \le 0$, then T_1 exists.
(2) If $Z_1'(M_1) > 0$, we can not make sure whether $\lim_{T\to 0^+} Z_1'(T)$ is less than 0. Therefore, we do not know whether T_1 exists. Although it is so, the convexity of $Z_1(T)$ on $[M_1,\infty)$ implies that $Z_1(T)$ is increasing on $[M_1,\infty)$ if $Z_1'(M_1) > 0$.

Likewise, since $\lim_{T\to\infty} Z_3'(T) = \infty$, there are two cases to occur:

(1) If $Z_3'(M_2) \le 0$, then T_3 exists.
(2) If $Z_i'(M_3) > 0$, we can not make sure whether $\lim_{T\to 0^+} Z_3'(T)$ is less than 0. Therefore, we do not know whether T_3 exists. Although it is so, the convexity of $Z_3(T)$ on $[M_2,\infty)$ implies that $Z_3(T)$ is increasing on $[M_2,\infty)$ if $Z_3'(M_2) > 0$.

Equation A1, A7, A10 and A14 in Ouyang *et al.*[1] yield that:

$$Z_1'(M_1) = Z_2'(M_1) = \frac{-2\theta^2 S + 2D[h + c\theta(1-r)]\left(\theta M_1 e^{\theta M_1} - e^{\theta M_1} + 1\right) + (\theta M_1)^2 pI_dD}{2(\theta M_1)^2} \quad (6)$$

and

$$Z_3'(M_2) = Z_4'(M_2) = \frac{-2\theta^2 S + 2D(h + c\theta)(\theta M_2 e^{\theta M_2} - e^{\theta M_2} + 1) + (\theta M_2)^2 pI_dD}{2(\theta M_2)^2} \quad (7)$$

Furthermore, we let:

$$\Delta_1 = -2\theta^2 S + 2D[h + c\theta(1-r)]\left(\theta M_1 e^{\theta M_1} - e^{\theta M_1} + 1\right) + (\theta M_1)^2 pI_dD \quad (8)$$

and

$$\Delta_2 = -2\theta^2 S + 2D(h + c\theta)\left(\theta M_2 e^{\theta M_2} - e^{\theta M_2} + 1\right) + (\theta M_2)^2 pI_dD \quad (9)$$

Then, the optimal cycle time T* and optimal payment time (M_1 or M_2) can be obtained as following theorem.

Theorem 1:

(A) If $\Delta_1 > 0$ and $\Delta_2 > 0$, then $Z(T^*) = \min\{Z_2(T_2), Z_4(T_4)\}$. Hence T^* is T_2 or T_4 and optimal payment time is M_1 or M_2 associated with the least cost.

(B) If $\Delta_1 \le 0$ and $\Delta_2 \le 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_3(T_3)\}$. Hence T^* is T_1 or T_3 and optimal payment time is M_1 or M_2 associated with the least cost.

(C) If $\Delta_1 \le 0$ and $\Delta_2 > 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_4(T_4)\}$. Hence T^* is T_1 or T_4 and optimal payment time is M_1 or M_2 associated with the least cost.

(D) If $\Delta_1 > 0$ and $\Delta_2 \le 0$, then $Z(T^*) = \min\{Z_2(T_2), Z_3(T_3)\}$. Hence T^* is T_2 or T_3 and optimal payment time is M_1 or M_2 associated with the least cost.

Proof:

(A) If $\Delta_1>0$ and $\Delta_2>0$, then we have $Z_1'(M_1) = Z_2'(M_1)>0$ and $Z_3'(M_2) = Z_4'(M_2)> 0$. Equation 5a-c imply that:

(i) $Z_1(T)$ is increasing on $[M_1,\infty)$.
(ii) $Z_2(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, M_1)$.
(iii) $Z_3(T)$ is increasing on $[M_2,\infty)$.
(iv) $Z_4(T)$ is decreasing on $(0, T_4]$ and increasing on $[T_4,M_2)$.

Combining (i)-(iv), we have that $\bar{Z}(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, \infty)$ and $\bar{\bar{Z}}(T)$ is decreasing on $(0, T_4]$ and increasing on $[T_4, \infty)$. Therefore, $Z(T^*) = \min\{Z_2(T_2), Z_4(T_4)\}$. Consequently, T^* is T_2 or T_4 and optimal payment time is M_1 or M_2 associated with the least cost.

(B) If $\Delta_1\le 0$ and $\Delta_2\le 0$, then we have $Z_1'(M_1) = Z_2'(M_1)\le 0$ and $Z_3'(M_2) = Z_4'(M_2)\le 0$. Equation 5a-c imply that:

(i) $Z_1(T)$ is decreasing on $[M_1,T_1]$ and increasing on $[T_1,\infty)$.
(ii) $Z_2(T)$ is decreasing on $(0, M_1)$.
(iii) $Z_3(T)$ is decreasing on $[M_2,T_3]$ and increasing on $[T_3,\infty)$.
(iv) $Z_4(T)$ is decreasing on $(0, M_2)$.

Combining (i)-(iv), we have that $\bar{Z}(T)$ is decreasing on $(0, T_1]$ and increasing on $[T_2, \infty)$ and $\bar{\bar{Z}}(T)$ is decreasing on $(0, T_3]$ and increasing on $[T_3, \infty)$. Therefore, $Z(T^*) = \min\{Z_1(T_1), Z_3(T_3)\}$. Consequently, T^* is T_1 or T_3 and optimal payment time is M_1 or M_2 associated with the least cost.

(C) If $\Delta_1\le 0$ and $\Delta_2>0$, then we have $Z_1'(M_1) = Z_2'(M_1)\le 0$ and $Z_3'(M_2) = Z'(M_2)> 0$. Equation 5a-c imply that:

- $Z_1(T)$ is decreasing on $[M_1, T_1]$ and increasing on $[T_1, \infty)$.
- $Z_2(T)$ is decreasing on $(0, M_1)$.
- $Z_3(T)$ is increasing on $[M_2, \infty)$.
- $Z_4(T)$ is decreasing on $(0, T_4]$ and increasing on $[T_4, M_2)$.

Combining (i)-(iv), we have that $\bar{Z}(T)$ is decreasing on $(0, T_1]$ and increasing on $[T_2, \infty)$ and $\bar{\bar{Z}}(T)$ is decreasing on $(0, T_4]$ and increasing on $[T_4, \infty)$. Therefore, $Z(T^*) = \min\{Z_1(T_1), Z_4(T_4)\}$. Consequently, T^* is T_1 or T_4 and optimal payment time is M_1 or M_2 associated with the least cost.

(D) If $\Delta_1>0$ and $\Delta_2\le 0$, then we have $Z_1'(M_1) = Z_2'(M_1)>0$ and $Z_3'(M_2) = Z_4'(M_2)\le 0$. Equations 5a-c imply that:

(i) $Z_1(T)$ is increasing on $[M_1, \infty)$.
(ii) $Z_2(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, M_1)$.
(iii) $Z_3(T)$ is decreasing on $[M_2, T_3]$ and increasing on $[T_3, \infty)$.
(iv) $Z_4(T)$ is decreasing on $(0, M_2)$.

Combining (i)-(iv), we have that $\bar{Z}(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, \infty)$ and $\bar{\bar{Z}}(T)$ is decreasing on $(0, T_3]$ and increasing on $[T_3, \infty)$. Therefore, $Z(T^*) = \min\{Z_2(T_2), Z_3(T_3)\}$. Consequently, T^* is T_2 or T_3 and optimal payment time is M_1 or M_2 associated with the least cost.

Incorporating the above arguments, we have completed the proof of Theorem 1.

Theorem 1 immediately determines the optimal cycle time T^* and optimal payment time (M_1 or M_2) after computing the numbers Δ_1 and Δ_2. Theorem 1 is really very simple.

A special case: Here, we want to deduce one previously published model as a special case.

Huang and Chung's[3] model: Suppose that the selling price per unit is equal to the unit purchasing price and the deterioration is ignored. Let p = c, we have:

$$\lim_{\theta\to 0^+} Z_1(T) = \frac{S}{T} + \frac{DTh}{2} + c(1-r)D + \frac{c(1-r)I_c D(T-M_1)^2}{2T} - \frac{cI_d D M_1^2}{2T} \quad (10)$$

$$\lim_{\theta\to 0^+} Z_2(T) = \frac{S}{T} + \frac{DTh}{2} + c(1-r)D - DcI_d\left(M_1 - \frac{T}{2}\right), \quad (11)$$

$$\lim_{\theta\to 0^+} Z_3(T) = \frac{S}{T} + \frac{DTh}{2} + cD + \frac{cI_c D(T-M_2)^2}{2T} - \frac{cI_d DM_2{}^2}{2T} \quad (12)$$

and

$$\lim_{\theta\to 0^+} Z_4(T) = \frac{S}{T} + \frac{DTh}{2} + cD - DcI_d(M_2 - \frac{T}{2}) \quad (13)$$

Equations 10-13 will be consistent with Eq. 1(a,b) and 4(a,b) in Huang and Chung[3], respectively. Hence, Huang and Chung[3] will be a special case of Ouyang *et al.*[1] model.

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REFERENCES

1. Ouyang, L.Y., C.T. Chang and J.T. Teng, 2005. An EOQ model for deteriorating items under trade credits. J. Oper. Res. Soc., 56: 719-726.
2. Purcell, E.J. and D. Varberg, 1987. Calculus with Analytic Geometry. Prentice-Hall, Inc. Englewood Cliffs, New Jersey.
3. Huang, Y.F. and K.J. Chung, 2003. Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. Asia-Pacific J. Oper. Res., 20: 177-190.