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Pole Assignment and Decoupling of Descriptor Systems

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Abstract: In this study, we develop a numerical algorithm for decoupling a multi-input-multi-output singular system into non-interacting subsystems using PDF (Proportional and Derivate Feedback) control laws. In this study, the bilinear transformation from continuous systems is considered. Necessary and sufficient conditions for a solution of the decoupling problem are established. When the system satisfies these conditions, the class of controllers which decouple the system is synthesized. This study presents a method for simultaneous decoupling and pole assignment of singular systems is presented. Allowing, the method is used, when systems are not statically decoupled. Finally, we give numerical examples in order to show the advantages and the simplicity of the presented approach.

Key words: Descriptor systems, decoupling, pole assignment, proportional and derivate feedback control, bilinear transformation

INTRODUCTION

It is well-known that within a multivariable control system, every input affects several (if not all) outputs resulting in a complicated input-output relationship. Decoupling control strategies have been developed for the transformation of coupled input-output systems to equivalent decoupled systems. In case that each input effects only one output, the MIMO plant can be greatly simplified into a number of SISO plants. Propelled by this idea, the decoupling controllers of linear multivariable systems retain a great deal of attention since the early work see Falb and Wolovich[1], also Howze and Pearson[2]. Since then, there have been additional important contributions on this line[3-5]. Work on decoupling in the design and synthesis of descriptor systems has been developed first by Christodoulou *et al.*[6,7]. Decoupling control strategies of descriptor systems have been further developed[8-10]. The study by Dai[5] not only emphasizes the decoupling of the closed-loop dynamics or statics, but it also ensures. In particulars decoupling descriptor system by state feedback and regular input transformation on the Matrix Fraction Descriptions (MFDs) in frequency domain, associated with a poles assignment, developed by Vafiadis and Karcanias[11] and the references therein. Recently, Duan and Zhang[12] have proposed the dynamical order assignment approach for linear descriptor systems via state derivate feedback. In this work, PDF (Proportional and Derivate Feedback) controllers were employed for simultaneous decoupling and pole assignment of descriptor systems, necessary and sufficient conditions for a solution have been established. Yet, PDF control is also used in standard systems [3]. The theory of descriptor systems has a wide variety of applications in the domains of robotics, aerodynamics, electrical networks, perturbed systems, population models in biology[8,13,14].

FORMULATION OF THE PROBLEM

In this study, an approach for the input-output decoupling of singular, generalized or descriptor systems of the form $E\dot{x}(t) = A\,x(t)+B\,u(t)$, is presented. Here, Proportional and Derivate Feedback (PDF) control laws are used. We consider the bilinear transformation[15] for the decoupling control problem of continuous descriptor systems.

Consider the linear time-invariant multivariable continuous descriptor systems described by a general state space model such that:

$$\Sigma_c : \begin{cases} E\,\dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) \end{cases} \qquad (1)$$

Where, x is the n-dimensional state vector, u is the m-dimensional input vector and y is the l-dimensional output vector. E, A, B and C are matrices of appropriate dimensions and noting that the E is a singular matrix.

For the existence of a solution to system (1), we assume that $\det(pE - A) \neq 0$ where p is the complex

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variable associated with the Laplace transformation, i.e., (pE – A) is assumed to be a regular pencil matrix.

Let the transfer function matrix of system (1) be defined as follows:

$$H(p)=C(pE-A)^{-1}B \quad (2)$$

Here, we assume that m = l, i.e., the system has an equal number of inputs and outputs. Then, it is called single-input-single-output decoupled if and only if H(p) is diagonal and nonsingular.

Many feedback laws have been used in the regular system case in order to achieve decoupling systems. Most commonly used is the static state feedback law. Also, the case with dynamic state feedback and/or dynamic precompensator is used[6]. Here, we use a PDF control law, that is:

$$u(t) = F_1 x(t) - F_2 \dot{x}(t) + G v(t) \quad (3)$$

The problem of decoupling the descriptor system (1) by PDF controller is to choose the matrices F_1, F_2 and G so that H(p) can be nonsingular and diagonal. In the next section, we first develop control laws for decoupled descriptor systems and use the basic necessary and sufficient conditions for decoupling system (1). Second, a compact procedure for computing the parameters of PDF control law is developed. Finally, we deduce a method permitting simultaneous decoupling and pole assignment for descriptor system.

DEVELOPMENT OF THE CONTROL STRATEGY

Consider the bilinear transformation defined by:

$$\begin{cases} \dot{x}(t) = \dfrac{x((k+1)h) - x(kh)}{h} \\ x(t) = \dfrac{x((k+1)h) + x(kh)}{2} \end{cases} \quad (4)$$

for $hk \le t \le (k+1)h$,

with h assumed to be a strictly positive real parameter.

Applying this transformation to system (1), we obtain the discrete system defined by:

$$x(k+1) = \bar{A}x(k) + \bar{B}u(k) \quad (5)$$

where:

$$\bar{A} = h\left(E - \frac{h}{2}A\right)^{-1}\left(E + \frac{h}{2}A\right) \quad (6)$$

and

$$\bar{B} = h\left(E - \frac{h}{2}A\right)^{-1} B \quad (7)$$

In the following, system (5) will be said to the Discrete Bilinear Transform (DBT) of system (1).

It is seen that the parameter h/2 must be selected such that it does not match any of the eigenvalues of the matrix pencil (E, A) i.e., det (E – h/2A) ≠ 0

In the rest of the study, the parameter h is selected to satisfy det (E – h/2A) ≠ 0

In order to make this study self-contained we recall some useful results from the literature.

Proposition[14]: Consider the generalized transfer function matrix H(p) of the system (E, A, B, C) and let $\bar{H}(z) = C[zI - \bar{A}]^{-1}\bar{B}$ be the transfer function of its DBT (5), then :

$$H(p) = \bar{H}(z) \text{Where}, z = \frac{1 + \frac{h}{2}p}{1 - \frac{h}{2}p} \quad (8)$$

Theorem[1]: The system (A, B, C) is decoupled if and only if the transfer function is diagonal and not singular. Let the matrix D:

$$D = [C_1 A^{d_1} B \;\; C_2 A^{d_2} B \ldots \;\; C_m A^{d_m} B]^T \quad (9)$$

where: C_i is the i^{th} row of the matrix C

d_i:

$$d_i = \begin{cases} \min j \;:\; C_i A^{d_j} B \neq 0, j = 0, 1, \ldots, n-1, i = 0, 1, \ldots, m \\ n-1 \text{ if } C_i A^{d_j} B = 0 \text{ for all } j \end{cases}$$

there exist a pair (F, G) decouple the system (A, B, C) if only if D is not singular.

A particular solution for the pair (F,G) is given by:

$$F = -(D)^{-1} A^*$$
$$G = (D)^{-1} \Lambda$$
$$\Lambda = \text{diag}(\lambda_i) \quad \lambda_i \neq 0, \quad i \in [1,m]$$

and D is defined in (9) and A* is given by:

$$A^* = [C_1 A^{d_1+1} \;\; C_2 A^{d_2+1} \ldots C_m A^{d_m+1}]^T$$

In order to obtain H(p), the transfer function $\bar{H}(z)$ is determined using any procedure for standard linear systems (5).

Relation (8) directly implies that H(p) is decoupled if and only if $\overline{H}(z)$ is decoupled. This shows decoupling control problem for descriptor systems can be solved using classical decoupling control engineers.
In such a case, the problem of determining the necessary and sufficient conditions for the descriptor system (1) to be decoupled is reduced to determining the corresponding conditions for the DBT system (5).

Falb and Wolovich[1] have proposed a necessary and sufficient condition for the existence of a control law which decouples the DBT system (4)[16,17].

$$u(k) = Fx(k) + Gv(k) \tag{10}$$

where v(k) represents the new m-vector control and the constants matrices F and G are appropriate dimensions. The closed loop transfer function becomes:

$$\overline{H}(z) = C[zI - (\overline{A} + \overline{B}F)]^{-1}\overline{B}G \tag{11}$$

Using results of Falb and Wolovich[1] in DBT system (5), we obtain the following proposition:

Proposition: Let B^* be the m x m matrix given by:

$$B^* = [C_1\overline{A}^{d_1}\overline{B} \;\; C_2\overline{A}^{d_2}\overline{B} \ldots \;\; C_m\overline{A}^{d_m}\overline{B}]^T \tag{12}$$

where C_i is the i^{th} row of the matrix C. The superscripts d_i, i=1, 2,.., m are defined by:

$$d_i = \begin{cases} \min j : \; C_i\overline{A}^{d_j}\overline{B} \neq 0, \;\; j = \\ 0, 1, \ldots, n-1 \;, \; i = 0, 1, \ldots, m \\ n\text{-}1 \text{ if } \; C_i\overline{A}^{d_j}\overline{B} = 0 \;\; \text{for all } j \end{cases}$$

Then, there is a pair of matrices (F, G) which decouples the descriptor system (1) if and only if B* is nonsingular, i.e.,

$$\det(B^*) \neq 0 \tag{13}$$

Proof of proposition: From propositions and theorem it comes that there exists a pair (F, G) such that $\overline{H}(z)$ is diagonal and nonsingular if and only if (13) holds. In this case H(p) is also diagonal and non singular.

It should be noted that this result provides a handy tool to decouple descriptor systems since the control law can be determined using classical methods of standard systems. The problem of determining the control law for decoupled descriptor systems (1), can be reduced to constructing the matrices F and G that decouple the DBT system (5). Then, the control law which decouples system (1), provided that condition (13) holds, reduces to the following form:

$$u(t) = F_1x(t) - F_2\,\dot{x}(t) + Gv(t) \tag{14}$$

where the controller matrices F_1 and F_2 are related by the following relationship:

$$F_2 = \frac{h}{2}F_2 \tag{15}$$

and h is a scalar which satisfies:

$$\det(pE - A) \neq 0 \tag{16}$$

In (14), x is the state vector, $\dot{x}$ is the derivative of the state and v is the new input.

The following proposition suggests a set of pairs (F, G) capable of decoupling descriptor systems.

Proposition: If condition (13) holds, then the PDF con4rol law (14) is given by:

$$u(t) = F_1x(t) - F_2\,\dot{x}(t) + Gv(t)$$

where, $F_1 = F,\; F_2 = \frac{h}{2}F$ and G

$$F = -(B^*)^{-1}K \tag{17}$$

$$G = (B^*)^{-1}\Lambda \tag{18}$$

$$\Lambda = \mathrm{diag}(\lambda_i) \quad \lambda_i \neq 0, \quad i \in [1, m] \tag{19}$$

and B* is defined in (12) and K is given by:

$$K = [C_1\overline{A}^{d_1+1} \;\; C_2\overline{A}^{d_2+1} \ldots \;\; C_m\overline{A}^{d_m+1}]^T \tag{20}$$

Proof of proposition: According to Eq 8, it turnout that if $\overline{H}(z)$ is diagonal then H(p) is also diagonal.

Let us note that, the problem of decoupled DBT system is equivalent to the problem of decoupled descriptor systems.

The pairs (F, G) to decouple DBT system is established by Descusse[16], in theorem. The feedback law which decouples descriptor systems is given under the following PDF structure:

from the bilinear transformation (4), we can write:

$$x(k) = x(t) - \frac{h}{2}\dot{x}(t)$$

when replacing x(k) in expression (10), we obtain:

$$u(t) = Fx(t) - \frac{h}{2}F\dot{x}(t) + Gv(t)$$

When we apply a PDF control to system (1), then the transfer function of closed loop descriptor system becomes:

$$H(p) = C\,[p\,(E + B\,F_2) - (A + B\,F_1)]^{-1}\,B\,G \quad (21)$$

From the above considerations, it is clear that the crucial step in evaluating PDF controllers is the computation of matrices F_1, F_2 and G. The following algorithm can be used for this purpose.

Algorithm: Evaluation of PDF controllers for decoupled descriptor systems

Step 1: Compute $\bar{A}$ and $\bar{B}$, given by (6)-(7)
Step 2: Using relation (12), determine B*
Step 3: if det(B*) ≠ 0 then go to step 4
Else PDF controllers for decoupling system (1) do not exist.
Step 4: Using relations (17-19), compute F and G.
Step 5: Deduce $F_1 = F$ and $F_2 = h/2\,F$.
Step 6: Determine the closed-loop system(E_f,A_f,B G, C) from:

$$E_f = E + h/2\,B\,F \text{ and } A_f = A + B\,F$$

PDF POLE SHIFTING

The main problem here is the determination of the pole assignment of decoupled descriptor systems.

Let us first note that, the transfer function of closed loop DBT system is defined by Descusse[16].

$$H(z) = C\,(zI - \bar{A} - \bar{B}F)^{-1}\,\bar{B}G = \mathrm{diag}\left(\frac{\lambda_i}{z^{d_i+1}}\right) \quad (22)$$

From above, we note that the closed loop poles are located at the origin.

The pole assignment of the decoupled descriptor system is specified by the next corollary.

Corollary: If system (1) satisfies the condition ″ B^* regular″ then the transfer function of the decoupled descriptor system is given by the following:

$$H(p) = \mathrm{diag}\left\{\frac{N_1(p)}{D_1(p)}, \ldots, \frac{D_m(p)}{D_m(p)}\right\} \quad (23)$$

where, the polynomials $N_i(p)$ and $D_i(p)$ given by:

$$N_i(p) = \lambda_i\left(1 - \frac{h}{2}p\right)^{d_i} \text{ and } D_i(p) = \left(1 + \frac{h}{2}p\right)^{d_i+1}$$

and the parameter λ_i are those defined in theorem, with $i \in [1, m]$.

Proof of corollary: The transfer function of the decoupled descriptor system can be written as:

$$H(p) = C\left[p(E + \frac{h}{2}BF) - (A + BF)\right]^{-1} BG \quad (24)$$

and since:

$$p\,(E + \frac{h}{2}BF) - (A + BF) = \frac{1}{h}\,[\,(1 + \frac{h}{2}p)\,(E - \frac{h}{2}A) - (1 - \frac{h}{2}p)\,(E + \frac{h}{2}A) - h\,(1 - \frac{h}{2}p)\,BF\,]$$

then:

$$H(p) = \frac{C}{(1 - \frac{h}{2}p)}\,[\,\frac{(1 + \frac{h}{2}p)}{(1 - \frac{h}{2}p)}\,(E - \frac{h}{2}A) - (E + \frac{h}{2}A) - h\,BF\,]^{-1} h\,B\,G \quad (25)$$

Let

$$H_2(p) = C\,[\,\frac{(1 + \frac{h}{2}p)}{(1 - \frac{h}{2}p)}\,(E - \frac{h}{2}A) - (E + \frac{h}{2}A) - h\,B\,F\,]^{-1}\,h\,B\,G \quad (26)$$

When replacing $z = \dfrac{1+\frac{h}{2}p}{1-\frac{h}{2}p}$ in expression (26), we obtain:

$$H_2(z) = C\begin{bmatrix} z\,(E-\frac{h}{2}A) - (E-\frac{h}{2}A) \\ -h\,B\,F \end{bmatrix}^{-1} hB\,G \qquad (27)$$

Factorizing, (27) becomes:

$$H_2(z) = C\,[\,(E-\frac{h}{2}A)\,(z\,I - (E-\frac{h}{2}A)^{-1} (E-\frac{h}{2}A) - (E-\frac{h}{2}A)^{-1}hBF)\,]^{-1}\,h\,B\,G$$

$$H_2(z) = C\,[\,(z\,I - (E-\frac{h}{2}A)^{-1}(E-\frac{h}{2}A) - (E-\frac{h}{2}A)^{-1}hBF)\,]^{-1}\,h\,(E-\frac{h}{2}A)^{-1}\,BG \qquad (28)$$

Using relations (6) and (7) for $\bar{A}$ and $\bar{B}$ expression (28) becomes:

$$H_2(z) = C(zI - \bar{A} - \bar{B}F)^{-1}\bar{B}\,G$$

and with (22) is mind we get:

$$H_2(z) = \mathrm{diag}\left(\frac{\lambda_i}{z^{d_i+1}}\right)$$

Going back to the p-domain we get:

$$H_2(p) = \mathrm{diag}\left(\frac{\lambda_i(1-\frac{h}{2}p)^{d_i+1}}{(1+\frac{h}{2}p)^{d_i+1}}\right)$$

Taking for the expression above in H(p) one can deduce easily that H(p) is given by (23).

It should be noted that this result provides a method for simultaneous decoupling and pole assignment to "-2/h" of descriptor systems.

To illustrate these theoretical results, let us consider two examples.

ILLUSTRATIVE EXAMPLES

Example 1: DAI,[8]: Let a continuous system {E, A, B, C} be descriptor and given by:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where, n=3, m=2 and rank $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = 5 = n+m$.

there are two design ways :

Thus, by Dai,[8], there exists a proportional feedback:

$$u(t) = F\,x(t) + G\,v(t)$$

such that the closed-loop is statically decoupled.

In the next Λ is chosen as: $\Lambda = I_2$ λ_i=1, i∈[1, 2], where, I_2: identity matrix

System (1) may be decoupled via PDF controllers, here, h is chosen to be 2. For the above system, we find $d_1 = d_2 = 0$ and from (7), we obtain: $B^* = \begin{bmatrix} 2 & 2 \\ 0 & -2 \end{bmatrix}$ which is nonsingular and, therefore, the descriptor system can be decoupled. The matrices F and G are then:

$$F = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & -1 \end{bmatrix} \text{ and } G = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

The transfer function of the closed-loop system is:

$$H(p) = \frac{1}{p+1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note that the resulting closed-loop system is descriptor:

$$Ef = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix},\ Af = \frac{1}{2}\begin{bmatrix} -1 & 2 & 1 \\ 0 & -2 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$Bf = B\,G = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

Where, h is chosen to be 4, the matrices F and G which decoupled systems are:

$$F = \frac{1}{4}\begin{bmatrix} -1 & 0 & 2 \\ 0 & -2 & -1 \end{bmatrix} \text{ and } G = \frac{1}{4}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

then the transfer function of the closed-loop system is:

$$H(p)=\frac{1}{2p+1}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

we can easy verify corollary, the decoupled transfer function in two case of h.

Example 2: Consider the following descriptor system, described by: DAI,[8]

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 0\end{bmatrix}\dot{x}(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}x(t)+\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}x(t)$$

This system cannot be statically decoupled because, DAI,[8]:

$$\text{rank}\begin{bmatrix}A & B\\ C & 0\end{bmatrix}=4<5=n+m$$

For all value of h, the matrix Λ is chosen as: $\Lambda = I_2$
$\lambda_i = 1, i \in [1, 2]$

Here, we choose h=2, we find $d_1 = d_2 = 0$ and from steps 3 and 4, we obtain:

$$F=\frac{1}{2}\begin{bmatrix}-1 & -2 & -1\\ 0 & 0 & 1\end{bmatrix} \text{ and } G=\frac{1}{2}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

The transfer function of the closed loop system is:

$$H(p)=\frac{1}{p+1}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

If h is chosen to be 3, the matrices F and G which decoupled systems are:

$$F=\frac{1}{3}\begin{bmatrix}-1 & -3 & -2.25\\ 0 & 0 & 1.5\end{bmatrix} \text{ and } G=\frac{1}{3}\begin{bmatrix}1 & -2.25\\ 0 & 1.5\end{bmatrix}$$

then the closed loop decoupled system has the following transfer function :

$$H(p)=\frac{\frac{2}{3}}{p+\frac{2}{3}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

We have verified for other values of h the transfer function of the closed loop decoupled system can be written as:

$$H(p)=\frac{2/h}{\Delta(p)}I_2 \text{ with: } (p)=p+\frac{2}{h}$$ and

The pole of the decoupled system is assigned to "-2/h".

CONCLUSIONS

A strategy for decoupling a linear descriptor system using PDF control laws was proposed. Necessary and sufficient conditions for decoupling were given on the basis that the matrix B*defined in (11) is non-singular. The class of PDF controllers which decouple a descriptor system were shown to be in the form of (h/2 F, F). Also the method is used, when systems are not statically decoupled. The closed loop decoupled system was realized with an assigned pole at "2/h" A method for simultaneous decoupling and pole assignment was presented. It was verified that the proposed decoupling strategy can be easily applied to discrete-time descriptor systems.

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