



# Journal of Applied Sciences

ISSN 1812-5654

**science**  
alert

**ANSI***net*  
an open access publisher  
<http://ansinet.com>

## Predicting the Crack Initiation Fracture Toughness for a Crack along the Bimaterial Interface

Q.H. Shah, <sup>1</sup>M. Azram and M.H. Iliyas

Department of Manufacturing and Materials Engineering,

<sup>1</sup>Department of Science in Engineering,

International Islamic University Malaysia, Jalan Gombak, 53100 Kuala Lumpur, Malaysia

**Abstract:** Three Point Bend (3PB) bimaterial specimens of PMMA/PC were made by joining the two materials using friction welding. The specimens were tested and the crack initiation loads were recorded to evaluate the  $K_{IC}$  using ASTM standard E 1820-01. Stress field in the crack tip vicinity was studied using finite element analysis of the specimen model. Normal and shear stresses were recorded for a series of nodes ahead of the crack tip at ( $\theta=0^\circ$ ). SIF values were calculated using these stress values and it was found that the stresses at a certain distance lying outside the process zone provide a SIF value matching to that of experimental fracture toughness of the specimen under study. Based upon the experimental evidence and finite element analysis, a simple step by step procedure is proposed to calculate the fracture toughness of bimaterials for a complex stress field.

**Key words:** Bimaterial, interface, fracture toughness, process zone, finite element analysis, PMMA, PC

### INTRODUCTION

Fracture resistance of bimaterials for a crack along the interface has been a research topic pursued by many researchers because of its application in many engineering structures. Zak and Williams<sup>[1]</sup> discussed about the crack point stress singularities at a bimaterial interface. Later on Dundurs<sup>[2]</sup> made a breakthrough by introducing the well known Dundurs parameters for the bimaterial case. Dundurs parameters were used by Cook and Erdogan<sup>[3]</sup> to predict the stress intensity factor for bimaterials when the crack was lying perpendicular to the bimaterial interface. When the crack is going to propagate along the interface of two materials, it becomes a case of mode mixity depending on the angle of applied load to the crack orientation, therefore some researchers have used a Brazilian disc specimen to study the mode mixity of fracture<sup>[4-7]</sup>. As the angle of load application increases the contribution of shear stresses reduces and it may become minimum when the angle is  $90^\circ$  to the crack propagation direction. Recent literature<sup>[8-12]</sup> on the current topic reveals interesting features of bimaterial fracture.

For the bimaterial fracture toughness the relation between stresses and the stress intensity factor has been proposed as<sup>[13]</sup>:

$$\sigma_{yy} + i \sigma_{xy} = \frac{(K_1 + iK_2)r^{ie}}{\sqrt{2\pi r}} \quad (1)$$

This is the relation between the complex stress and complex stress intensity factor.  $\sigma_{yy}$  is the stress perpendicular to the crack plane while  $\sigma_{xy}$  is the shear stress.  $K_1$  and  $K_2$  are the stress intensity factors for mode I and mode II crack initiation. As can be seen from equation (1) the crack growth resistance is dependent upon a stress field and the stress singularity is represented by  $r^{-1/2+ie}$  instead of  $r^{-1/2}$ . Expression  $r^{ie}$  in equation (1) can also be written as follows:

$$\sigma_{yy} + i \sigma_{xy} = \frac{(K_1 + iK_2)e^{ie \ln r}}{\sqrt{2\pi r}} \quad (2)$$

which can be further expanded into the following expression.

$$\sigma_{yy} + i \sigma_{xy} = \frac{(K_1 + iK_2)[(\cos(\epsilon \ln r) + i \sin(\epsilon \ln r))]}{\sqrt{2\pi r}} \quad (3)$$

---

**Corresponding Author:** Dr. Qasim Hussain Shah, Department of Manufacturing and Materials Engineering,  
 International Islamic University Malaysia, Jalan Gombak, 53100 Kuala Lumpur, Malaysia  
 Tel: +60 3 2056 4472 Fax: +60 3 2056 4853 E-mail: shah915@yahoo.com; hqasim@iiu.edu.my

The r.h.s of equation (3)  $(K_1+iK_2)[(\cos(\epsilon \ln r) + i \sin(\epsilon \ln r))]$  can be multiplied and after simplification it takes the following form:

$$\sigma_{yy} + i\sigma_{xy} = \frac{[K_1 \cos(\epsilon \ln r) + iK_2 \cos(\epsilon \ln r)]}{\sqrt{2\Pi r}} \quad (4)$$

From equation (4) mode I and Mode II can be separated because it is a complex number where the real part on the l.h.s. of equation is equal to the real part on the r.h.s. Therefore the real part can be expressed as follows:

$$\sigma_{yy} = \frac{K_1 \cos(\epsilon \ln r)}{\sqrt{2\Pi r}} \quad (5)$$

and the imaginary part would follow as:

$$\sigma_{xy} = \frac{K_2 \cos(\epsilon \ln r)}{\sqrt{2\Pi r}} \quad (6)$$

From equation (5) and (6)  $K_1$  and  $K_2$  can be evaluated as follows:

$$K_1 = \frac{\sigma_{yy} \sqrt{2\Pi r}}{\cos(\epsilon \ln r)} \quad (7)$$

$$K_2 = \frac{\sigma_{xy} \sqrt{2\Pi r}}{\cos(\epsilon \ln r)} \quad (8)$$

In the above equations the value can be found by using:

$$\epsilon = \frac{1}{2\Pi} \ln \frac{1-\beta}{1+\beta} \quad (9)$$

or

$$\epsilon = \begin{bmatrix} \left( \frac{K_1 + 1}{\mu_1 \mu_2} \right) \\ \left( \frac{K_2 + 1}{\mu_2 \mu_1} \right) \end{bmatrix} \quad (10)$$

$\beta$  in (9) is one of the Dundurs material parameters expressed as:

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \quad (11)$$

where,  $\kappa_i = (3-4\nu_i)$  for plane strain and  $\kappa_i = \frac{(3-\nu_i)}{(1+\nu_i)}$

for plane stress, while  $\mu_i = \frac{E_i}{2(1+\nu_i)}$  being the shear modulus of the material.

### MATERIALS AND METHODS

3PB specimens from PMMA, PC and bimaterial PMMA/PC were made as shown in Fig. 1. A 3 mm wide notch was machined into the specimen. The triangular notch tip is located at 9.3 mm from the bottom edge where a 0.7 mm pre-crack was introduced. The specimen thickness is 10 mm. Fracture tests were conducted according to ASTM standard E 1820-01. Thee 3PB specimens were tested for three cases as depicted in Fig.1. In case 1 and 2, monolithic material specimens of PMMA and PC were tested while in case 3 the bimaterial PMMA/PC specimens were tested.

Tensile tests on the PMMA and PC were conducted and the mechanical properties were found as given in Table 1.

3PB fracture tests on monolithic material specimens of PMMA, PC and bimaterial specimens of PMMA/PC were conducted and the fracture loads (crack initiation loads) were recorded for each case.

Table 1: Mechanical properties of PMMA and PC

Material	ratio	Elastic modulus (Gpa)	Yield strength (MPa)	Elongation at break (%)	Hardness (HRB)	Fracture toughness (MPa $\sqrt{mm}$ )
PMMA	0.35	3.3	87	4	24.58	48.48
PC	0.37	2.2	52	100~150	12.45	93.28

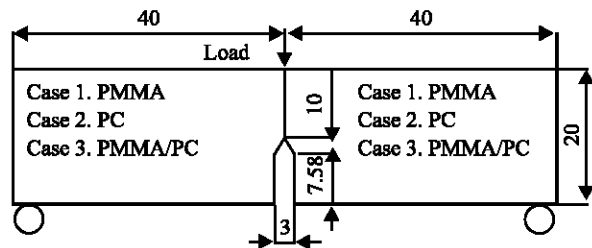


Fig. 1: The sketch showing the 3PB specimen used for fracture tests of monolithic PMMA, PC and bimaterial PMMA/PC

Using the ASTM formula for 3PB specimen the  $K_{IC}$  values for each case were calculated.

**Finite element stress analysis:** Monolithic and bimaterial specimens were modeled using finite element software ANSYS. A full crack specimen model was used. The model was discretized into 19,534 six node isoparametric elements with a total of 39, 523 nodes for plane strain conditions.

The stresses perpendicular to the crack plane at distances of 0.10, 0.15, 0.202, 0.257, 0.315, 0.375 mm along the crack line ( $\theta=0^\circ$ ) are recorded in Table 2 and Fig. 2. These stress values are substituted in equations (7) and (8) to evaluate the SIF values for varying distance from the crack tip. It is found that for PMMA specimen the stress values at 0.15~0.202 give a SIF value that matches the fracture toughness recorded experimentally. For PC specimens this distance is from 0.202~0.257 mm from the crack tip. On further investigation it was found that this particular distance points to a location outside the process zone (or plastic zone).

The SIF values for the stresses ahead of the crack tip are shown in Fig. 2. In Fig. 2 it can be seen that the experimental fracture toughness values are same as the SIF values calculated for stresses in the bimaterial specimen. Note that for bimaterial the fracture toughness value obtained from finite element analysis was 32 MPvmm while the experimental value was about 29.50. But if the complex part of  $K_{IC}$  is also included, it

Table 2: Stresses  $\sigma_{yy}$  and  $\sigma_{xy}$  at particular distances from the crack tip

Distance (mm)	PMMA ( $\sigma_{yy}$ ) MPa	PC ( $\sigma_{yy}$ ) MPa	BM ( $\sigma_{yy}$ ) MPa	BM ( $\sigma_{xy}$ ) MPa
0.10	52.06	98.28	32.53	1.0400
0.15	48.60	98.90	32.78	0.9700
0.202	42.62	85.18	28.27	0.7519
0.257	37.91	75.25	24.96	0.5900
0.315	33.91	67.13	22.26	0.4900
0.375	30.76	60.28	20.00	0.4000

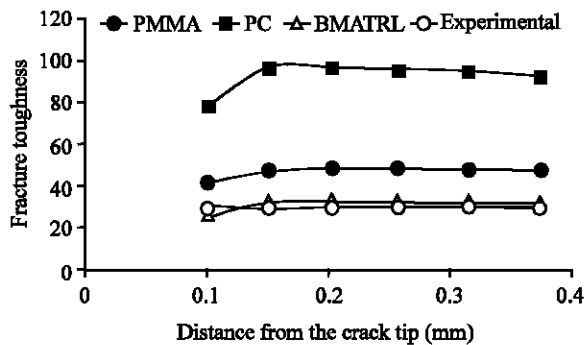


Fig. 2: SIF values calculated using equations (7). The SIF units are  $MPa \sqrt{m}$  ie

would add up to 30.35 which is very near to the value obtained from the experiment. As the shear stress values were ignorable because the load application angle was maximum ( $90^\circ$ ) to the crack plane, therefore suppressing the SIF values corresponding to the shear stress, it was assumed that maximum contribution was of  $\sigma_{yy}$  instead of  $\sigma_{xy}$ . SIF values were calculated using equation (7). No need was felt to utilize equation (8) as the shear stress  $\sigma_{xy}$  values were very small when compared to  $\sigma_{yy}$  ( of an order of 2~3 %).

**Step by step chart of activities to get  $K_{IC}$  for bimetals**

1. Conduct fracture tests.
2. Record the crack initiation load.
3. Perform Finite Element Analysis.
4. Record the  $\sigma_{yy}$  and  $\sigma_{xy}$  values ahead of the crack tip at ( $\theta=0^\circ$ ) outside process zone.
5. Using complex stress field equations (7) and (8) calculate  $K_{IC}$  values for bimetals.

**CONCLUSIONS**

Fracture tests were conducted on monolithic 3PB specimens of PMMA, PC and bimaterial PMMA/PC.  $K_{IC}$  values were established using ASTM standard E 1820-01. Finite element analysis of the full specimen was performed using ANSYS and the stresses were studied in the specimens in detail. Using  $\sigma_{yy}$  and  $\sigma_{xy}$  ahead of the crack tip at certain distance from the crack tip SIF values were calculated. It was found that the stress values ahead of the process zone gave the matching fracture toughness values in corresponding cases. It was also found that the  $K_{IC}$  values for the bimetals are far smaller than any of the two monolithic materials<sup>[10]</sup>.

A step by step procedure to calculate the fracture toughness values has been proposed.

**REFERENCES**

1. Zak, A.R. and M.L. Williams, 1963. Crack point stress singularities at a bimaterial interface. J. Applied Mechanics-Transactions ASME., pp: 142-143.
2. Dundurs, J., 1969. Discussion of edge-bonded dissimilar orthogonal elastic wedges under normal and shear loading. J. Applied Mechanics, 36: 650-652.
3. Cook, T.S. and F. Erdogan, 1972. Stresses in bonded materials with a crack perpendicular to the interface. Intl. J. Eng. Sci., 10: 677-697.
4. Akisanya, A.R., Fleck, N.A., Brittle fracture of adhesive joints. Intl. J. Fracture, 58: 93-114.

5. Atkinson, C., R.E. Smelser and J. Sanchez, 1982. Combined mode fracture via the cracked Brazilian disk test. *Intl. J. Fracture*, 18: 279-291.
6. Leslie, B.S., N. Travitzky and D. Ashkenazi, 2000. Interface fracture properties of a bimaterial ceramic composite. *Mechanics of Materials*, 32: 711-722.
7. Jorge, B.S., 1998. Bimaterial Brazilian specimen for determining interfacial fracture toughness. *Engineering Fracture Mechanics*, 59: 57-71.
8. Dong Kil Shin and Jung Ju Lee, 2001. Fracture parameters of interfacial crack of bimaterial under the impact loading. *Intl. J. Solids and Structures*, 38: 5303-5322.
9. Christina, B. and C. Persson, 2001. A numerical method for calculating stress intensity factors for interface cracks in bimetals. *Engineering Fracture Mechanics*, 68: 235-246.
10. Jiang, F., K. Zhao and J. Sun, 2003. Evaluation of interfacial crack growth in bimaterial metallic joints loaded by symmetric three-point bending. *Intl. J. Pressure Vessels and Piping*, 80: 129-137.
11. Bernd, L. and T. Schuller, 2001. Essential work of interfacial fracture: A method to characterize adhesion at polymer-polymer interfaces. *Intl. J. Adhesion and Adhesives*, 21: 55-58.
12. Ikeda, T. and N. Miyazaki, 1998. Mixed mode fracture criterion of interface crack between dissimilar materials. *Engineering Fracture Mechanics*, 59: 725-735.
13. Dollfer, J., W. Beckert, B. Lauke and K. Schneider, 2000. Fracture mechanical characterization of mixed-mode toughness of thermoplast/glass interfaces. *Computational Materials Sci.*, 19: 223-228.