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An EPQ Model Under Permissible Delay in Payments and Cash Discount

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Abstract: This study tries to investigate the retailer's optimal replenishment policy under permissible delay in payments and cash discount within the EPQ framework. Mathematical models have been derived for obtaining the optimal cycle time for item so that the annual total relevant cost is minimized. Furthermore, numerical examples are given to illustrate the results developed in this study.

Key words: EPQ, permissible delay in payments, cash discount, inventory

INTRODUCTION

In the real world, the supplier often makes use of the permissible delay in payments policy to promote his/her commodities. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. From the viewpoint of the supplier, the supplier hopes that the payment is paid from retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the retailer. The retailer can obtain the cash discount when the payment is paid before cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. Many articles related to the inventory policy under permissible delay in payments and cash discount can be found in Huang^[1,2] and Huang and Chung^[3].

Recently, Chung and Huang^[4] investigated the topic of permissible delay in payments within the EPQ (economic production quantity) framework. Therefore, this study tries to incorporate both Chung and Huang^[4] and Huang and Chung^[3] to develop the retailer's inventory model. That is, we want to investigate the retailer's optimal replenishment policy under permissible delay in payments and cash discount within the EPQ framework. Then mathematical models have been derived for obtaining the optimal cycle time for item so that the annual total relevant cost is minimized.

Model formulation: For convenience, we adopt the same notation and assumptions as in Chung and Huang^[4] and Huang and Chung^[3].

The annual total relevant cost for the retailer can be expressed as:

$$TVC(T) = \text{ordering cost} + \text{stock-holding cost} + \text{purchasing cost} + \text{interest payable} - \text{interest earned.}$$

We show that the annual total relevant cost is given by:

Case 1: Payment is paid at time M_1

$$TVC_1(T) = \begin{cases} TVC_{11}(T) & \text{if } M_1 \leq PM_1/D \leq T & (1a) \\ TVC_{12}(T) & \text{if } M_1 \leq T \leq PM_1/D & (1b) \\ TVC_{13}(T) & \text{if } 0 < T \leq M_1 & (1c) \end{cases}$$

where:

$$TVC_{11}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r)D + cI_k(1-r)\rho \left(\frac{DT^2}{2} - \frac{PM_1^2}{2} \right) / T - cI_e \left(\frac{DM_1^2}{2} \right) / T, \quad (2)$$

$$TVC_{12}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r)D + cI_k(1-r) \left[\frac{D(T-M_1)^2}{2} \right] / T - cI_e \left(\frac{DM_1^2}{2} \right) / T \quad (3)$$

and

$$TVC_{13}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r) \left[D - cI_e \left(\frac{DT^2}{2} + DT(M_1 - T) \right) \right] / T. \quad (4)$$

Then, we find $TVC_{11}(PM_1/D) = TVC_{12}(PM_1/D)$ and $TVC_{12}(M_1) = TVC_{13}(M_1)$. Hence $TVC_1(T)$ is continuous and

well-defined. All $TVC_{11}(T)$, $TVC_{12}(T)$, $TVC_{13}(T)$ and $TVC_1(T)$ are defined on $T > 0$.

Case 2: Payment is paid at time M_2

$$TVC_2(T) = \begin{cases} TVC_{21}(T) & \text{if } M_2 \leq PM_2/D \leq T & (5a) \\ TVC_{22}(T) & \text{if } M_2 \leq T \leq PM_2/D & (5b) \\ TVC_{23}(T) & \text{if } 0 < T \leq M_2 & (5c) \end{cases}$$

where:

$$TVC_{21}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD + cI_k\rho \left(\frac{DT^2}{2} - \frac{PM_2^2}{2} \right) / T - cI_e \left(\frac{DM_2^2}{2} \right) / T, \quad (6)$$

$$TVC_{22}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD + cI_k \left[\frac{D(T - M_2)^2}{2} \right] / T - cI_e \left(\frac{DM_2^2}{2} \right) / T \quad (7)$$

and

$$TVC_{23}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD - cI_e \left[\frac{DT^2}{2} + DT(M_2 - T) \right] / T. \quad (8)$$

Then, we find $TVC_{21}(PM_2/D) = TVC_{22}(PM_2/D)$ and $TVC_{22}(M_2) = TVC_{23}(M_2)$. Hence $TVC_2(T)$ is continuous and well-defined. All $TVC_{21}(T)$, $TVC_{22}(T)$, $TVC_{23}(T)$ and $TVC_2(T)$ are defined on $T > 0$.

Decision rule of the optimal cycle time T^* : The main purpose of this section is to develop a solution procedure to determine the optimal cycle time T^* . From equations (2)-(4) and (6)-(8) yield:

$$TVC_{11}'(T) = - \left\{ \frac{2A - cM_1^2 [I_k(1-r)P\rho + I_e D]}{2T^2} \right\} + \frac{D\rho [h + cI_k(1-r)]}{2}, \quad (9)$$

$$TVC_{11}''(T) = \frac{2A - cM_1^2 [I_k(1-r)P\rho + I_e D]}{T^3} = \frac{2A - cM_1^2 P I_k (1-r) + cDM_1^2 [I_k(1-r) - I_e]}{T^3}, \quad (10)$$

$$TVC_{12}'(T) = - \left\{ \frac{2A - cDM_1^2 [I_k(1-r) - I_e]}{2T^2} \right\} + \frac{D [h\rho + cI_k(1-r)]}{2}, \quad (11)$$

$$TVC_{12}''(T) = \frac{2A + cDM_1^2 [I_k(1-r) - I_e]}{T^3}, \quad (12)$$

$$TVC_{13}'(T) = - \frac{A}{T} + \frac{D(h\rho + cI_e)}{2}, \quad (13)$$

$$TVC_{13}''(T) = \frac{2A}{T^3} > 0 \quad (14)$$

$$TVC_{21}'(T) = - \left\{ \frac{2A - cM_2^2 (I_k\rho P + I_e D)}{2T^2} \right\} + \frac{D\rho (h + cI_k)}{2}, \quad (15)$$

$$TVC_{21}''(T) = - \frac{2A - cM_2^2 (I_k\rho P + I_e D)}{T^3} = \frac{2A - cM_2^2 P I_k + cDM_2^2 (I_k - I_e)}{T^3}, \quad (16)$$

$$TVC_{22}'(T) = - \left\{ \frac{2A + cDM_2^2 (I_k - I_e)}{2T^2} \right\} + \frac{D (h\rho + cI_k)}{2}, \quad (17)$$

$$TVC_{22}''(T) = \frac{2A + cDM_2^2 (I_k - I_e)}{T^3} > 0, \quad (18)$$

$$TVC_{23}'(T) = - \frac{A}{T} + \frac{D (h\rho + cI_e)}{2} \quad (19)$$

and
$$TVC_{23}''(T) = \frac{2A}{T^3} > 0. \quad (20)$$

Equations (14), (18) and (20) imply that all $TVC_{13}(T)$, $TVC_{22}(T)$ and $TVC_{23}(T)$ are convex on $T > 0$. However, equation (10) implies that $TVC_{11}(T)$ is convex on $T > 0$ if $2A - cM_1^2 P I_k (1-r) + cDM_1^2 [I_k(1-r) - I_e] > 0$; equation (12) implies that $TVC_{12}(T)$ is convex on $T > 0$ if $2A + cDM_1^2 [I_k(1-r) - I_e] > 0$ and equation (16) implies that $TVC_{21}(T)$ is convex on $T > 0$ if $2A - cM_2^2 P I_k + cDM_2^2 (I_k - I_e) > 0$. Furthermore, we have

$$\begin{aligned} TVC_{11}'\left(\frac{PM_1}{D}\right) &= TVC_{12}'\left(\frac{PM_1}{D}\right), TVC_{12}'(M_1) \\ &= TVC_{13}'(M_1), TVC_{21}'\left(\frac{PM_2}{D}\right) = TVC_{22}'\left(\frac{PM_1}{D}\right) \\ \text{and } TVC_{22}'(M_2) &= TVC_{23}'(M_2) \end{aligned}$$

Let $TVC_{ij}'(T)=0$, for all $i=1\sim 2$ and $j=1\sim 3$. Then we can obtain

$$T_{11}^* = \sqrt{\frac{2A + cDM_1^2[I_k(1-r) - I_e] - cM_1^2PI_k(1-r)}{D\rho[h + cI_k(1-r)]}} \text{ if}$$

$$2A + cDM_1^2[I_k(1-r) - I_e] - cPM_1^2I_k(1-r) > 0, \tag{21}$$

$$T_{12}^* = \sqrt{\frac{2A + cDM_1^2[I_k(1-r) - I_e]}{D[h\rho + cI_k(1-r)]}} \text{ if}$$

$$2A + cDM_1^2[I_k(1-r) - I_e] > 0, \tag{22}$$

$$T_{13}^* = \sqrt{\frac{2A}{D(h\rho + cI_e)}}, \tag{23}$$

$$T_{21}^* = \sqrt{\frac{2A + cDM_2^2(I_k - I_e) - cM_2^2PI_k}{D\rho(h + cI_k)}} \text{ if}$$

$$2A + cDM_2^2(I_k - I_e) - cPM_2^2I_k > 0, \tag{24}$$

$$T_{22}^* = \sqrt{\frac{2A + cDM_2^2(I_k - I_e)}{D(h\rho + cI_k)}} \tag{25}$$

and

$$T_{23}^* = \sqrt{\frac{2A}{D(h\rho + cI_e)}}. \tag{26}$$

Equation (21) implies that the optimal value of T for the case of $T \geq PM_1/D$, that is $T_{11}^* \geq PM_1/D$. We substitute equation (21) into $T_{11}^* \geq PM_1/D$, then we can obtain the optimal value of T if and only if

$$-2A + \frac{M_1^2}{D}[P(P-D)h + (P^2 - D^2)cI_k(1-r) + cD^2I_e] \leq 0.$$

Similar discussion, we can obtain following results:

$$M_1 \leq T_{12}^* \leq PM_1/D$$

if and only if

$$-2A + \frac{M_1^2}{D}[P(P-D)h + (P^2 - D^2)cI_k(1-r) + cD^2I_e] \geq 0.$$

and if and only if $-2A + DM_1^2(h\rho + cI_e) \leq 0$.
 $T_{13}^* \leq M_1$ if and only if $-2A + DM_1^2(h\rho + cI_e) \geq 0$.
 $T_{21}^* \geq PM_2/D$ if and only if

$$-2A + \frac{M_2^2}{D}[P(P-D)h + (P^2 - D^2)cI_k + cD^2I_e] \leq 0.$$

$M_2 \leq T_{22}^* \leq PM_2/D$
 if and only if

$$-2A + \frac{M_2^2}{D}[P(P-D)h + (P^2 - D^2)cI_k + cD^2I_e] \geq 0$$

and if and only if $-2A + DM_2^2(h\rho + cI_e) \leq 0$.
 $T_{23}^* \leq M_2$ if and only if $-2A + DM_2^2(h\rho + cI_e) \geq 0$.
 Let

$$\Delta_1 = -2A + \frac{M_1^2}{D}[P(P-D)h + (P^2 - D^2)cI_k(1-r) + cD^2I_e], \tag{27}$$

$$\Delta_2 = -2A + DM_1^2(h\rho + cI_e), \tag{28}$$

$$\Delta_3 = -2A + \frac{M_2^2}{D}[P(P-D)h + (P^2 - D^2)cI_k + cD^2I_e] \tag{29}$$

and

$$\Delta_4 = -2A + DM_2^2(h\rho + cI_e). \tag{30}$$

From equations (27)-(30), we can obtain $\Delta_3 > \Delta_1 > \Delta_2$ and $\Delta_3 > \Delta_4 > \Delta_2$. Summarized above arguments, we can obtain following results.

Theorem 1:

- (A) If $\Delta_2 \geq 0$, then $TVC(T^*) = \min\{TVC(T_{11}^*), TVC_2(T_{23}^*)\}$. Hence T^* is T_{11}^* or T_{23}^* associated with the least cost.
- (B) If $\Delta_1 \geq 0$, $\Delta_2 < 0$ and $\Delta_4 \geq 0$, then $TVC(T^*) = \min\{TVC_1(T_{12}^*), TVC_2(T_{23}^*)\}$. Hence T^* is T_{12}^* or T_{23}^* associated with the least cost.
- (C) If $\Delta_1 \geq 0$, $\Delta_2 < 0$ and $\Delta_4 < 0$, then $TVC(T^*) = \min\{TVC_1(T_{12}^*), TVC_2(T_{22}^*)\}$. Hence T^* is T_{12}^* or T_{22}^* associated with the least cost.
- (D) If $\Delta_1 < 0$ and $\Delta_4 \geq 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(T_{23}^*)\}$. Hence T^* is T_{11}^* or T_{23}^* associated with the least cost.
- (E) If $\Delta_1 < 0$, $\Delta_3 > 0$ and $\Delta_4 < 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(T_{22}^*)\}$. Hence T^* is T_{11}^* or T_{22}^* associated with the least cost.
- (F) If $\Delta_3 \leq 0$, then $TVC(T^*) = \min\{TVC_1(T_{11}^*), TVC_2(T_{21}^*)\}$. Hence T^* is T_{11}^* or T_{21}^* associated with the least cost.

Table 1: Optimal cycle time with various r

Let A=\$100/order, D=1000 units/year, c=\$60/unit, P=1350 units/year, h=\$5/unit/year, I_c=\$0.15/\$/year, I_r=\$0.25/\$/yaer, M₁=0.1 year, M₂=0.15 year

r	Δ_1	Δ_2	Δ_3	Δ_4	Theorem 1	T*	TVC (T*)
0.1	>0	<0	>0	>0	B	T ₁₂ *=0.129	54554
0.15	>0	<0	>0	>0	B	T ₁₂ *=0.130	51551
0.2	>0	<0	>0	>0	B	T ₁₂ *=0.132	48549
0.25	>0	<0	>0	>0	B	T ₁₂ *=0.133	45546
0.3	<0	<0	>0	>0	D	T ₁₁ *=0.135	42542
0.35	<0	<0	>0	>0	D	T ₁₁ *=0.140	39539
0.4	<0	<0	>0	>0	D	T ₁₁ *=0.147	36534

Theorem 1 immediately determines the optimal cycle time T* after computing the numbers Δ_1 , Δ_2 , Δ_3 and Δ_4 . Theorem 1 is really very simple.

Numerical examples: To illustrate the results, let us apply the proposed method to solve the following numerical examples. The optimal cycle time is summarized in Table 1.

CONCLUSION

The supplier offers the permissible delay in payments policy to stimulate the demand of the retailer. However, the supplier can also use the cash discount policy to attract retailer to pay the full payment of the amount of purchasing cost to shorten the collection period. This study investigates the retailer’s replenishment policy under permissible delay in payments and cash discount within the EPQ framework and provides a very efficient solution procedure to determine the optimal cycle time T*.

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