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An Inventory Policy and Payment Policy under Various Credits

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Abstract: This study tries to modify Huang and Chung to develop the retailer's inventory model. We assume that the retailer can obtain the trade credit when the payment is paid within the trade credit period offered by the supplier. Otherwise, the retailer will just obtain a cash discount. In addition, we modify the assumption that the unit purchasing price and unit selling price were equal. Under these conditions, we want to investigate the retailer's optimal ordering policy and optimal payment policy within the EOQ framework. Mathematical model has been derived for obtaining the optimal cycle time and optimal payment time for item so that the annual total relevant cost is minimized. Furthermore, numerical examples are given to illustrate the results developed in this study.

Key words: EOQ, inventory, trade credit, cash discount

INTRODUCTION

Recently, the research area of trade credit has been considered. At first, Goyal[1] derived an EOQ model under the condition of trade credit. Huang and Chung[2] extended Goyal's model[1] to cash discount policy for early payment. The retailer can obtain the cash discount when the payment is paid within the cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. Huang[3] relaxed the assumption that the selling price was equal to the purchasing price in Huang and Chung[2]. Many articles related to the inventory policy under trade credit and cash discount can be found in Huang[4-6].

Earlier studies assumed that the retailer could obtain the cash discount when the payment was paid before the cash discount period offered by the supplier. Otherwise, the retailer would pay full payment within the trade credit period. What the above statement describes is just one way of various credits offered by supplier. Hence, this study wants to modify above credits offered by supplier to develop the retailer's inventory model. We assume that the retailer can obtain the trade credit when the payment is paid within the trade credit period offered by the supplier. Otherwise, the retailer will just obtain a cash discount. In addition, we modify the assumption that the unit purchasing price and unit selling price were equal. Under these conditions, we want to investigate the retailer's optimal ordering policy and optimal payment policy within the EOQ framework.

Model formulation: For convenience, we adopt the same notation and assumptions as in Huang and Chung[2].

Notation:

D	=	Annual demand
A	=	Cost of placing one order
c	=	Unit purchasing price per item
s	=	Unit selling price per item
h	=	Unit stock holding cost per item per year excluding interest charges
I_e	=	Interest which can be earned per \$ per year
I_p	=	Interest charges per \$ investment in inventory per year
r	=	Cash discount rate, $0<r<1$
M_1	=	The period of the trade credit
M_2	=	The period of cash discount, $M_1<M_2$
T	=	The cycle time
TRC(T)	=	The annual total relevant cost
T*	=	The optimal cycle time of TRC(T)
Q*	=	The optimal order quantity = DT*.

Assumptions:

1. Demand rate is known and constant.
2. Shortages are not allowed.
3. Time horizon is infinite.

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4. Replenishments are instantaneous with a known and constant lead time.
5. $I_p \geq I_e$ and $s>c$.
6. Supplier offers the trade credit if payment is paid within M_1, otherwise just offers a cash discount within M_2.
7. When the trade credit offered, a deposit is made of the unit selling price of generated sales revenue into an interest bearing account during the time the account is not settled. At the end of this period, the retailer pays off all items sold, keeps profits and starts paying for the interest charges on the items in stocks. We assume that the interest earned from profits is too small to neglect it.

The annual total relevant cost consists of the following elements.

1. Annual ordering cost $= \frac{A}{T}$.

2. Annual stock holding cost (excluding interest charges) $= \frac{DTh}{2}$.

Since the supplier offers the trade credit if payment is paid within M_1, there are two payment policies for the retailer. First, the payment is paid at time M_1 to get the trade credit, Case 1. Second, the payment is paid at time M_2 to get the cash discount, Case 2. So purchasing cost, interest payable and interest earned, we shall discuss these two cases as follows.

3. Annual purchasing cost:

Case 1: Payment is paid at time M_1, the annual purchasing cost = cD.

Case 2: Payment is paid at time M_2, the annual purchasing cost = c(1-r)D.

4. Annual cost of interest charges for the items kept in stock:

Case 1: Payment is paid at time M_1

Case 1.1: $T \geq M_1$

Annual cost of interest charges for the items kept in stock $= \frac{D(T-M_1)^2 cI_p}{2T}$.

Case 1.2: $T \leq M_1$

In this case, annual cost of interest charges for the items kept in stock = 0.

Case 2 : Payment is paid at time M_2

In this case, the retailer will not obtain the trade credit. Hence, annual cost of interest charges for the items kept in stock $= \frac{c(1-r)I_p DT^2}{2T}$.

5. Annual interest earned:

Case 1: Payment is paid at time M_1

Case 1.1: $T \geq M_1$

Annual interest earned $= \frac{DM_1^2 sI_e}{2T}$.

Case 1.2: $T \leq M_1$

$$\text{Annual interest earned} = \frac{DsI_e[(\frac{T^2}{2} + T(M_1 - T)]}{T} = DsI_e(M_1 - \frac{T}{2}).$$

Case 2: Payment is paid at time M_2

In this case, the retailer will not obtain the trade credit. Hence, annual interest earned = 0.

From the above arguments, the annual total relevant cost for the retailer can be expressed as: TRC(T) = ordering cost + stock-holding cost + purchasing cost + interest payable - interest earned.

We show that the annual total relevant cost is given by

Case 1: Payment is paid at time M_1

$$TRC_1(T) = \begin{cases} \frac{A}{T} + \frac{DTh}{2} + cD + \frac{cI_p D(T-M_1)^2}{2T} - \frac{sI_e DM_1^2}{2T} \\ \text{if } M_1 \leq T & \text{(1a)} \\ \frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_1 - \frac{T}{2}) \\ \text{if } 0 < T \leq M_1 & \text{(1b)} \end{cases}$$

Let

$$TRC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + \frac{cI_p D(T-M_1)^2}{2T} - \frac{sI_e DM_1^2}{2T} \quad (2)$$

and

$$TRC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + cD - DsI_e(M_1 - \frac{T}{2}) \quad (3)$$

At $T = M_1$, we find $TRC_{11}(M_1) = TRC_{12}(M_1)$. Hence $TRC_1(T)$ is continuous and well-defined. All $TRC_{11}(T)$, $TRC_{12}(T)$ and $TRC_1(T)$ are defined on $T>0$.

Case 2: Payment is paid at time M_2

$$TRC_2(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + \frac{c(1-r)I_pDT^2}{2T} \tag{4}$$

Decision rule of the optimal cycle time T*: The main purpose of this section is to develop a solution procedure to determine the optimal cycle time T* and optimal payment time (M_1 or M_2).
From Eq. 2-4 yield

$$TRC_{11}'(T) = \frac{-[2A + DM_1^2(cI_p - sI_e)]}{2T^2} + \frac{D[h + cI_p]}{2}, \tag{5}$$

$$TRC_{11}''(T) = \frac{2A + DM_1^2(cI_p - sI_e)}{T^3}, \tag{6}$$

$$TRC_{12}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2}, \tag{7}$$

$$TRC_{12}''(T) = \frac{2A}{T^3} > 0, \tag{8}$$

$$TRC_2'(T) = -\frac{A}{T^2} + \frac{D[h + c(1-r)I_p]}{2} \tag{9}$$

and

$$TRC_2''(T) = \frac{2A}{T^3} > 0. \tag{10}$$

Equation 8 and 10 imply that all $TRC_{12}(T)$ and $TRC_2(T)$ are convex on T>0. However, Eq. 6 implies that $TRC_{11}(T)$ is convex on T>0 if $2A + DM_1^2(cI_p - sI_e) > 0$.

Let $TRC_{ij}'(T) = 0$, for all i = 1~2 and j = 1~2. Then we can obtain

$$T_{11}^* = \sqrt{\frac{2A + DM_1^2(cI_p - sI_e)}{D(h + cI_p)}} \text{ if } 2A + DM_1^2(cI_p - sI_e) > 0, \tag{11}$$

$$T_{12}^* = \sqrt{\frac{2A}{D(h + sI_e)}} \tag{12}$$

and

$$T_2^* = \sqrt{\frac{2A}{D[h + c(1-r)I_p]}} \tag{13}$$

Equation 11 implies that the optimal value of T for the case of $T \geq M_1$, that is $T_{11}^* \geq M_1$. We substitute Eq. 11 into $T_{11}^* \geq M_1$, then we can obtain the optimal value of T

if and only if $-2A + DM_1^2(h + sI_e) \leq 0$.

Similarly, Eq. 12 implies that the optimal value of T for the case of $T \leq M_1$, that is $T_{12}^* \leq M_1$. We substitute Eq. 12 into $T_{12}^* \leq M_1$, then we can obtain the optimal value of T

if and only if $-2A + DM_1^2(h + sI_e) \geq 0$.

Furthermore, we let

$$\Delta = -2A + DM_1^2(h + sI_e). \tag{14}$$

From above arguments, the optimal cycle time T* and optimal payment time (M_1 or M_2) can be obtained as follows.

Theorem 1

A. If $\Delta>0$, then $TRC(T^*) = \min\{TRC_1(T_{12}^*), TRC_2(T_2^*)\}$. Hence T* is T_{12}^* or T_2^* and optimal payment time is M_1 or M_2 associated with the least cost.
B. If $\Delta<0$, then $TRC(T^*) = \min\{TRC_1(T_{11}^*), TRC_2(T_2^*)\}$. Hence T* is T_{11}^* or $T_2^*$and optimal payment time is M_1 or M_2 associated with the least cost.
C. If $\Delta = 0$, then $TRC(T^*) = \min\{TRC_1(M_1), TRC_2(T_2^*)\}$. Hence T* is M_1 or T_2^* and optimal payment time is M_1 or M_2 associated with the least cost.

Theorem 1 immediately determines the optimal cycle time T* and optimal payment time (M_1 or M_2) after computing the number Δ. Theorem 1 is really very simple.

Numerical examples: To illustrate the results, let us apply the proposed method to solve the following numerical examples. The optimal cycle time and optimal payment time are summarized in Table 1.

Table 1: Optimal cycle time and optimal payment time with various r
Let A = \$30/order, D = 1000 units/year, c = \$10/unit, s = \$30/unit, h = \$5/unit/year, I_e = \$0.13/\$/year, I_p = \$0.15/\$/yaer, M_1 = 0.15 year, M_2= 0.2 year

r	Δ	Theorem 1	T*	Q*	Optimal payment time	TRC (T*)
0.02	>0	A	T_{12}^*=0.08211	82.1	M_1	10145.8
0.03	>0	A	T_{12}^*=0.08211	82.1	M_1	10145.8
0.04	>0	A	T_{12}^*=0.08211	82.1	M_1	10145.8
0.05	>0	A	T_2^*=0.09664	96.6	M_2	10120.9
0.06	>0	A	T_2^*=0.09675	96.8	M_2	10020.2
0.07	>0	A	T_2^*=0.09686	96.9	M_2	9919.4
0.08	>0	A	T_2^*=0.09698	97.0	M_2	9818.7

CONCLUSIONS

The supplier offers the trade credit and the cash discount policy to stimulate the demand of the retailer. The supplier can use the marketing alternatives of the trade credit or the cash discount to promote his/her products. However, the supplier can also use these alternatives to increase the retailer's payment flexibility. The retailer can choice an advantage alternative to him/her to pay the payment of the amount of purchasing cost. This study investigates the retailer's inventory policy and payment policy under trade credit or cash discount linked to payment time within the EOQ framework and provides a very efficient solution procedure to determine the optimal cycle time T* and the optimal payment time.

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