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Optimization of Stepped Spillway Dimensions and Investigation of Flow Energy Dissipation over a Physical Model

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Abstract: In this study, using the feasible direction method to determine the optimum slope and step height of a stepped spillway while the flow dissipated energy is maximized. Then the obtained optimized slope and step height were used and a wooden physical model of the Maxwill Dam with 1:25 scale has been built. Overall, eighteen experiments were carried run with several flow rates. The flow depths at up and down of the model were measured to calculate the dissipated energy of flow. Results show that the flow kinetic energy decreases by increasing the flow rate. The optimized slope and step height were applied to equation 8 to calculate the maximum flow dissipated energy. Comparing results of two physical and numerical models show a good agreement, which conform the goodness of optimized model.

Key words: Stepped spillway, dissipated energy, feasible direction method, physical model

INTRODUCTION

Spillways can be built in different shapes, based on geological and hydrological characteristics of dam sites and mostly due to its construction costs. Design and construction of spillways are important and mostly faced with cavitations phenomena. To overcome cavitations and prevent related damages, aeration of spillway face and edges can be used^[1]. High kinetic energy of flow at the toe of spillway is another difficulty, for example, the energy which should be dissipated at the toe of Tarbella dam is evaluated about 4000 MW, which is 20 times of the power plant capacity^[2].

Stepped spillway due to its special shape, can dissipate a great amount of flow kinetic energy and because of self aeration of flow passing over steps can prevent cavitations damages^[3]. So, construction of energy dissipaters especially, stilling basin at the toe of spillway or construction cost of the project reduce significantly^[4]. Stepped spillways also can be used for multipurpose systems such as river training, water quality systems and beautiful landscape of dam^[5]. Christodoulou^[6] reported that experiments on physical models of stepped spillway with the ratio of step height (h) to step length (l) equal to 0.7 show that effective parameters on dissipated energy of flow, are the ratio of flow critical depth (y_c) to the step height (h) and number of steps. He also reported that in a stepped spillway with a fixed number of steps, flow dissipated energy decreases by increasing the ratio of $y_c h^{-1}$ and for a fixed value of $y_c h^{-1}$ it is increased by

increasing the number of steps. Sorenson's experiments on the physical model of Maxwill dam with 1:25 scale resulted a flow drag coefficient (c_d) equals to 0.18. Also, equations for calculation of c_d , average velocity and flow rate per unit width of stepped spillway was introduced^[7]. Pegram *et al.*^[8] carried some experiments on two series of stepped spillway physical models with scales of 1:10 and 1:20, 30 m height, 59° slope and step heights of 0.5 to 2.00 m. They reported that at this slope, the step height does not affect flow dissipated energy, but for slopes of 40-50°, effect of step height on flow dissipated energy is significant. Also, dissipated energy of flow over a stepped spillway was evaluated about 60% more than a similar overflow spillway.

In the present study, the optimum slope and step height of a stepped spillway, while the flow dissipated energy is maximized were determined using Feasible Direction Method (FDM). Then a physical model of the Maxwill dam with 1:25 scale has been built using optimized slope and step height obtained from above optimization method. This physical model was built from wood with 21 steps, each step with 4 cm height and 30 cm width. A series of experiments has been run on this model, hydraulic parameters of flow were measured and the related dissipated energy was calculated. Finally, results of two physical and optimization models were compared.

Hydraulics of flow over stepped spillway: Two types of flow over stepped spillways, jet flow and skimming flow

has been introduced^[3]. In jet type, flow after passing a step will fall on the bottom of next step and dissipate the kinetic energy. In skimming type, a uniform flow from the crest of spillway towards the spillway's toe carries and rolling vortexes will formed on the bottom of steps. This type of flow contacts only the edge of each step. Dissipated energy of flow will increase by momentum transfer to the rolling fluid. The effective parameters which form the types of flow are flow rate (Q), step length (l) and height (h). Rajaratnam^[3] reported that at $y_c h^{-1}$ equal to 0.8, the skimming type flow will form on stepped spillways. The edges of steps on a stepped spillway produce a rough surface which dissipates energy. Sayre's^[9] experiments resulted that most of the resistance forces (about 42-98%) can be due to rough bed which is called drag force (T_b). So, for evaluation of dissipated energy, calculation of the drag coefficient (C_d), is required.

Governing equations: Effective parameters on dissipated energy of flow over stepped spillways are the napp thickness at the crest (H), depth of flow (D), step height perpendicular to the spillway main slope (h_1), average velocity (v), flow density (ρ), spillway main slope (tg (i)), flow dynamic viscosity (μ) and gravity acceleration (g), or;

$$F(E_1, E_0, E_L, H, D, h_1, v, \rho, \text{tg} (i), \mu, g) \tag{1}$$

which E_L is the dissipated energy between the flow energy at the crest (E_1) and at the toe of spillway (E_0). The percentage of dissipated energy is:

$$E_1\% = \frac{E_L}{E_1} \tag{2}$$

using dimensional analysis it can be a function^[10]

$$E_1\% = f_1 \left(\frac{H}{h_1}, \frac{D}{h_1}, \text{tg}(i), \text{Re}, \text{Fr} \right) \tag{3}$$

in which Fr is the Froude number and Re represents the Reynolds number. The other parameter which can affect the flow dissipated energy is the drag coefficient Cd can be a function of:

$$C_d = f_2 \left(\frac{D}{h_1}, \text{tg} (i), \text{Re}, \text{Fr} \right) \tag{4}$$

So, Eq. 3 can be rewritten as:

$$E_1\% = f_1 \left(\frac{H}{h_1}, C_d \right) \tag{5}$$

Yazdani^[10] by a series of experiments on a stepped spillway with slope between 40 to 55° and different step heights reported the equation of flow-dissipated energy as:

$$E_1\% = 104.33 \left(\frac{H}{h_1} \right)^{-0.015} C_d^{0.054} \tag{6}$$

and

$$C_d = 3.285 \text{Re}^{-0.013} \text{Fr}^{-2.021} \left(\frac{D}{h_1} \right)^{0.015} (\text{tg}(i))^{0.547} \tag{7}$$

Optimization model: The objective function is to maximize flow dissipated energy (Eq. 6 and 7) for appropriate slope and step height of stepped spillway or:

$$\text{Max} \{ E_1\% = 104.33 \left(\frac{H}{h_1} \right)^{-0.015} C_d^{0.054} \} \tag{8}$$

Based on the Yazdani's^[10] experiments on a stepped spillway model with step heights of 0.5-1 m and slopes (i) between 40-55°, which $\cos i = h_1/h$, (Fig. 1). The constraints of the optimization model are:

$$\cos(55^\circ) \leq h_1/h \leq \cos(40^\circ) \text{ or } 0.57 \leq h_1/h \leq 0.76 \text{ and } 0.5 \leq h \leq 1$$

The normalized above constraints can be written as:

$$g_1(x) = \frac{h_1}{0.76 h} - 1 \leq 0 \tag{9}$$

$$g_2(x) = 1 - \frac{h_1}{0.57 h} \leq 0 \tag{10}$$

$$g_3(x) = \frac{h}{0.04} - 1 \leq 0 \tag{11}$$

$$g_4(x) = 1 - \frac{h}{0.02} \leq 0 \tag{12}$$

Using the feasible direction method^[11] and applying the Dot Program in FORTRAN language for flow rates between 3.4-7.3 $\text{m}^3 \text{sec}^{-1}$ m, the optimization model (Eq. 8-12) resulted the optimum slope of 41.41° and step height of one meter for a stepped spillway. For step height between 0.5-2 m and the same flow rates, the obtained optimized slope and step height are 41.41° and 2 m, respectively. The reason for selection of flow rate about 3.4-7.3 $\text{m}^3 \text{sec}^{-1}$ m is to have skimming flow type. The dissipated energy resulted by the optimization model for

different flow rates and step heights are shown in Table 1.

Physical model: To control the results of optimization model, a physical model of Manxwill Dam stepped spillway with scale of 1:25 was built. The characteristics of the prototype are as follow:

The crest and toe elevations are 106.325 and 85.3 m, respectively, the width of spillway is 61m, the maximum flow rate per unit width is 61 m and the design head is 2.6 m.

Scale of the model was selected based on the general rule of scaled hydraulic models^[12] which specifies that the 1:60 should be the minimum scale of physical models^[13]. This model was built from wood. The surface of steps was painted to form roughness which comparable to the roughness of concrete in prototype. The scale of model was selected equal to 1:25, so, the ratio of the model dimensions to the prototype dimensions is:

$$v_r = \frac{v_m}{v_p} = (1_r)^{1.5} = 0.008$$

and the flow parameters are:

$$q_r = \frac{q_m}{q_p} = (1_r)^{1.5} = 0.008$$

$$Q_r = \frac{Q_m}{Q_p} = (1_r)^{2.5} = 0.00032$$

Based on the USBR recommendation, the minimum width of spillway in a two dimensional model is 15 cm, so, the model width equal to 30 cm was selected. The height of model equals to 0.84 m. The maximum flow rate per unit width of model equals to 0.048 m³ sec⁻¹ m and the maximum flow rate over the spillway is 0.0144 m³ sec⁻¹ which is equal 14.4 L sec⁻¹. Based on the optimization model results, the model slope is 41.41°, step height equals to 4 cm, the model length (L) become 95 cm and step bottom equals to 4.5 cm (Fig. 1).

A 90° V-notch weir used to measure the flow rate. This weir was installed at the inlet canal and the discharge coefficient of weir was obtained by calibration, which is equal to 0.578, so, the weir equation in this model is:

$$Q = 1.378 * h^{5/2} \tag{13}$$

Which h represents depth of water upstream of the weir (Fig. 2a). To supply water for experiments, a concrete storage tank with 7.2 m³ capacities was used. To store water upstream the model and simulate a reservoir, a steel tank with 1 m³ capacity was installed upstream the model. Two pumps each with 5 L sec⁻¹ capacity transfer water through a 2 inches pipe from the concrete storage tank to

Table 1: Percentage of flow energy dissipation obtained from optimization model for slope of 41.41°

Discharge (q) m ³ sec ⁻¹ m	Step height (m)					
	0.50	0.75	1.00	1.50	1.75	2.00
3.40	85.30	89.10	95.30	95.70	97.30	98.60
4.20	84.02	78.73	94.10	94.30	95.85	97.00
4.58	83.60	87.30	93.60	93.85	95.40	96.70
5.00	83.40	87.00	93.20	93.80	95.30	96.00
5.30	82.40	86.10	92.20	92.57	94.10	95.40
5.70	81.90	85.50	91.80	92.00	93.50	94.80
6.20	81.40	84.90	90.60	91.36	92.80	94.00
6.75	81.20	84.70	89.70	91.12	92.60	93.90
7.30	80.20	83.70	88.30	90.00	91.50	92.80

Table 2: Measured flow parameters on physical model and calculated the related flow rates

Upstream canal depth H (cm)	Downstream canal depth D (cm)	Head of water above triangular weir (cm)	Q (m ³ sec ⁻¹)
1.40	1.10	7.60	0.00219
1.60	1.15	8.00	0.00249
1.80	1.30	8.40	0.00282
2.20	1.50	8.90	0.00325
2.30	1.65	9.20	0.00354
2.50	1.80	9.50	0.00383
2.80	2.00	9.90	0.00425
2.90	2.10	10.30	0.00469
3.00	2.20	10.50	0.00492
3.30	2.40	11.20	0.00578
3.40	2.50	11.70	0.00645
3.50	2.70	12.30	0.00731
3.55	2.80	12.50	0.00777
3.58	2.90	12.90	0.00824
3.70	3.10	13.20	0.00872
3.75	3.30	13.50	0.00923
3.76	3.41	14.00	0.01000
3.82	3.52	14.50	0.01111

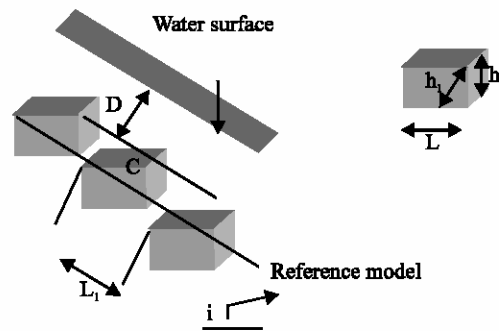


Fig. 1: Longitudinal section of a step with their parameters

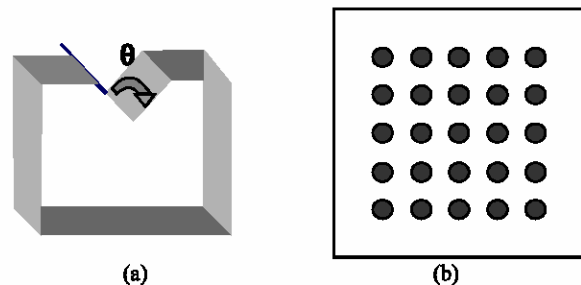


Fig. 2: Plan view of a) v-notch weir and b) plate with holes

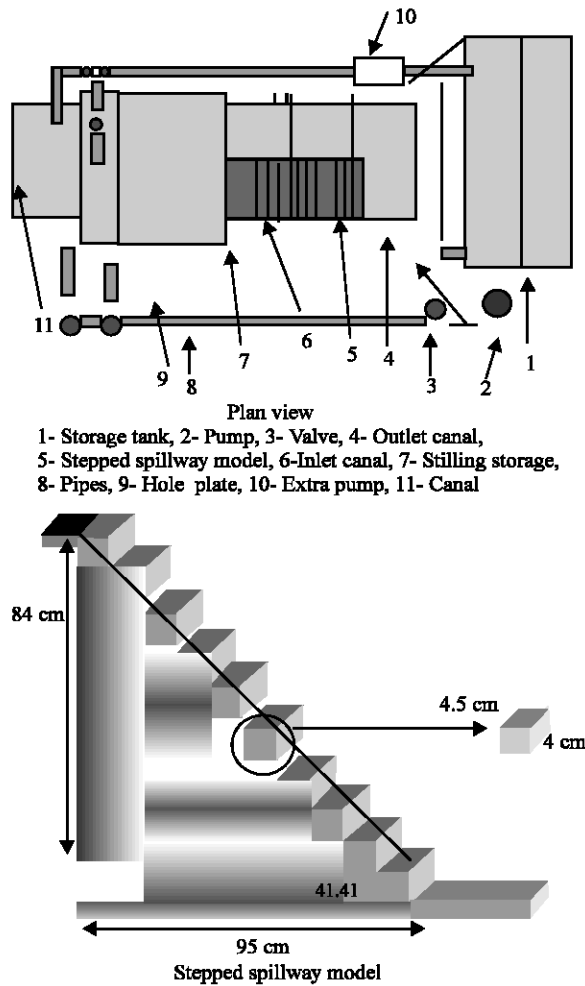


Fig. 3: Plan view of the stepped pillway model

the steel tank. To prevent flow turbulence, a 1x1 m steel plate with 25 holes each hole with 3 cm diameter was installed inside the steel tank at 25 cm distance from the inlet (Fig. 2b). A rectangular canal with 30 cm width, 15 cm height and 82 cm length transfer water from the steel tank to upstream the model. A same canal, with 40 cm length, returned water from downstream of the model to the concrete storage tank (Fig. 3).

MATERIALS AND METHODS

In this study, 18 experiments on the stepped spillway model with different flow rates were run, this model was made from wood, in civil engineering laboratory, shahid Bahonar University at year 2003. Flow parameters such as water depths upstream and downstream of the model (H and D) were measured. Depths of water were measured using piezometric tubes.

Depth of water above the v-notch weir was measured to calculate flow rate (Q). Table 2 shows the measured and calculated parameters of flow. Calculated flow rates and measured flow depths were used to calculate average flow velocity. Flow kinetic energy up and down the model was calculated, to obtain dissipated energy of flow over the model (Table 3).

RESULTS AND DISCUSSION

Flow dissipated energy obtained by applying Eq. 8-12 to the feasible direction method for Maxwell dam with different flow rates and step heights. Table 1 showed that dissipated energy of flow decreases by increasing flow rates. Also, results of physical model (Table 3)

Table 3: Percentage of calculated dissipated energy and $y_c h^{-1}$, using the measured data of physical model

Energy upstream of the physical model (cm)	Energy downstream of the physical model (cm)	Percentage of dissipated energy (%)	q (m ³ sec ⁻¹ m)	y _c (m)	y _c h ⁻¹
0.854	0.0334	96.00	0.00730	0.0175	0.439
0.856	0.0380	95.56	0.00830	0.0190	0.479
0.858	0.0369	95.38	0.00940	0.0208	0.520
0.862	0.0414	95.20	0.00100	0.0228	0.571
0.863	0.0425	95.00	0.00118	0.0242	0.605
0.865	0.0436	94.96	0.00127	0.0255	0.638
0.868	0.0456	94.70	0.00142	0.0273	0.684
0.869	0.0489	94.37	0.00156	0.0292	0.730
0.870	0.0500	94.20	0.00164	0.0300	0.754
0.873	0.0570	93.47	0.00190	0.0335	0.839
0.874	0.0630	92.79	0.00215	0.0361	0.903
0.875	0.0685	92.10	0.00243	0.0392	0.982
0.875	0.0716	91.80	0.00259	0.0409	1.022
0.876	0.0727	91.20	0.00274	0.0425	1.063
0.877	0.0758	90.40	0.00291	0.0441	1.104
0.877	0.0771	90.00	0.00307	0.0458	1.147
0.878	0.0810	89.50	0.00333	0.0483	1.210
0.878	0.0890	89.30	0.00367	0.0516	1.290

Table 4: Results of optimization and physical models of stepped spillway

q (m ³ sec ⁻¹ m)	H (cm)	D (cm)	V (m sec ⁻¹)	E _t % Optimidation model	E _t % physical model	Percentage of error (%)
3.40	3.85	2.91	0.92	95.3	91.2	4.50
4.20	3.72	3.47	0.952	94.1	90.4	4.10
4.58	3.75	3.50	1.02	93.6	89.5	4.60
5.00	3.82	3.60	1.04	93.2	89.0	4.72
5.30	3.90	3.72	1.09	92.8	87.9	5.57

confirm the above statement. Also, flow dissipated energy decreases by increasing the ratio of $y_c h^{-1}$; it is serious for $y_c h^{-1}$ more than 0.75. Decreasing energy loss as a function of increasing the ratio of $y_c h^{-1}$ is the same results which were obtained by Cristodoulou^[6]. High energy loss for $y_c h^{-1}$ less than 0.75 can be due to napp flow type and for $y_c h^{-1}$ more than 0.75 the flow is skimming type. The dissipated energy obtained by two physical and numerical models are shown in Table 4. Comparison of dissipated energy obtained by these two models shows differences rages from 4.6 to 5.57%, which can be a reasonable range. Also, the data of Table 4 show that the dissipated energy decreases by increasing the flow rate.

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