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Mathematical Inventory Model with Decay Item under Two Levels of Trade Credit

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Abstract: The present study extends the Huang's model by considering decay item under two levels of trade credit. At first, we model the retailer's inventory system as a cost minimization problem. Then, we prove the convexity of the retailer's inventory system developed in this study. Finally, a theorem is developed to determine the retailer's optimal replenishment cycle time efficiently.

Key words: EOQ, inventory, decay item, two levels of trade credit

INTRODUCTION

In the traditional Economic Order Quantity (EOQ) model, it was tacitly assumed that the buyer must pay for the items purchased as soon as the items are received. However, in practice, the supplier frequently offers its retailer the trade credit (or permissible delay in payments) to attract retailer who considers it to be a type of price reduction.

Goyal[1] derived an EOQ model under the conditions of permissible delay in payments. But he implicitly assumed only one level of trade credit. That is, the supplier offers its retailer the trade credit but the retailer does not offer its customer the trade credit. Recently, Huang[2] modified this assumption to two levels of trade credit. That is, not only the supplier offers its retailer the trade credit but also the retailer offers its customer the trade credit. But the decay item was ignored in their models. However, many studies related to the inventory considered the decay item under the trade credit could be found[3-7].

Therefore, this study extend the Huang's model[2] by considering decay item under two levels of trade credit. Then we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal replenishment cycle time.

MATHEMATICAL FORMULATION

The following most assumptions and notation are similar to those in Huang's model[2].

Assumptions

- Demand rate, D, is known and constant.
- Shortages are not allowed.
- Time period is infinite.
- Replenishment is instantaneous.
- There is no repair or replacement of the deteriorated inventory during a given cycle.
- The constant fraction θ of on hand inventory gets deteriorated per time unit.
- $I_k \geq I_e$, $M \geq N$.
- When $T \geq M$, the account is settled at T=M and the retailer starts paying for the interest charges on the items in stock with rate I_k. When $T \leq M$, the account is settled at T=M and the retailer does not need to pay any interest charge.
- The retailer can accumulate revenue and earn interest after its customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

Notation

D = Demand rate per year
A = Ordering cost per order
c = Unit purchasing price per item
h = Unit stock holding cost per item per year excluding interest charges
I_e = Interest earned per \$ per year
I_k = Interest charged per \$ in stocks per year by the supplier

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θ = Fraction of units that deteriorate per time unit
M = The retailer's trade credit period offered by supplier in years
N = The customer's trade credit period offered by retailer in years
Q = The order quantity
T = The cycle time in years
TVC(T) = The annual total variable cost, which is a function of T
T* = The optimal replenishment cycle time which minimizes TVC(T) when T>0.

Let Q(t) denote the on-hand inventory level at time t, which is depleted by the effects of demand and deterioration, then the differential equation which describes the instantaneous states of Q(t) over (0,T) is given as:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D \qquad ,0 \le t \le T$$

then, with boundary condition Q(T) = 0. The solution of above equation is given by

$$Q(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1) \qquad ,0 \le t \le T$$

Noting that Q(0) = Q, the quantity ordered each replenishment cycle is

$$Q = \frac{D}{\theta}(e^{\theta T} - 1)$$

Furthermore, the total variable cost function per cycle consists of the ordering cost, inventory holding cost, cost of deteriorated units and capital opportunity cost. From now on, the individual cost is evaluated before they are grouped together.

- Annual ordering cost $= \frac{A}{T}$
- Annual inventory holding cost (excluding the capital opportunity cost)

$$= \frac{h}{T}\int_0^T Q(t)dt = \frac{hD}{\theta^2 T}(e^{\theta T} - \theta T - 1)$$

- Annual cost of deteriorated units

$$= \frac{c(Q - DT)}{T} = \frac{cD}{\theta T}(e^{\theta T} - \theta T - 1)$$

- From assumptions (8) and (9), there are three cases to discuss annual capital opportunity cost.

Case T≥M: The annual capital opportunity cost

$$= \frac{cI_k \int_M^T Q(t)dt - cI_e \int_N^M Dtdt}{T}$$

$$= \frac{cI_k D}{\theta^2 T}[e^{\theta(T-M)} - \theta(T-M) - 1] - \frac{cI_e D(M^2 - N^2)}{2T}$$

Case N≤T<M: The annual capital opportunity cost

$$= -\frac{cI_e[\int_N^T Dtdt + DT(M-T)]}{T}$$

$$= -\frac{cI_e D}{2T}[2MT - N^2 - T^2]$$

Case T<N: The annual capital opportunity cost

$$= -\frac{cI_e \int_N^M DTdt}{T} = cI_e D(M-N)$$

According to the above arguments, we have

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if} \quad M \le T & (1a) \\ TVC_2(T) & \text{if} \quad N \le T < M & (1b) \\ TVC_3(T) & \text{if} \quad 0 < T < N & (1c) \end{cases}$$

where:

$$TVC_1(T) = \frac{A}{T} + \frac{D}{\theta^2 T}(c\theta + h)(e^{\theta T} - \theta T - 1) + \frac{cI_k D}{\theta^2 T}[e^{\theta(T-M)} - \theta(T-M) - 1] - \frac{cI_e D(M^2 - N^2)}{2T} \quad \text{if } T > 0 \qquad (2)$$

$$TVC_2(T) = \frac{A}{T} + \frac{D}{\theta^2 T}(c\theta + h)(e^{\theta T} - \theta T - 1) - \frac{cI_e D}{2T}(2MT - N^2 - T^2) \quad \text{if} \quad T > 0 \qquad (3)$$

and

$$TVC_3(T) = \frac{A}{T} + \frac{D}{\theta^2 T}(c\theta + h)(e^{\theta T} - \theta T - 1) - cI_e D(M - N) \text{ if } T>0 \qquad (4)$$

Since $TVC_1(M) = TVC_2(M)$ and $TVC_2(N) = TVC_3(N)$, TVC(T) is continuous and well defined.

THE CONVEXITY

Here, we shall show that three inventory functions described as above section are convex on their appropriate domains.

Theorem 1

- $TVC_1(T)$ is convex on $[M, \infty)$.
- $TVC_2(T)$ is convex on $(0, \infty)$.
- $TVC_3(T)$ is convex on $(0, \infty)$.
- $TVC(T)$ is convex on $(0, \infty)$.

Before proving Theorem 1, we need the following lemma.

Lemma 1

$$e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M - \frac{\theta^2(M^2 - N^2)}{2} > 0 \quad \text{if} \quad T \geq M$$

Proof

Let $g(T) = e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M - \frac{\theta^2(M^2 - N^2)}{2}$, then we have $g'(T) = \frac{\theta^3 T^2}{2} e^{\theta(T-M)}$.

So $g(T)$ is increasing on (M, ∞) and $g(T) > g(M) = \frac{\theta^2 N^2}{2} > 0$ if $T > M$. Consequently, $e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M - \frac{\theta^2(M^2 - N^2)}{2} > 0$ if $T \geq M$. This completes the proof.

The proof of Theorem 1

(1) Form Eq. 2 yields

$$TVC_1'(T) = -\frac{A}{T^2} + \frac{D}{\theta^2 T^2}(c\theta + h)(\theta T e^{\theta T} - e^{\theta T} + 1) + \frac{cI_k D}{\theta^2 T^2}[\theta T e^{\theta(T-M)} - e^{\theta(T-M)} + 1 - \theta M] + \frac{cI_e D(M^2 - N^2)}{2T^2} \tag{5}$$

and

$$\begin{aligned} TVC_1''(T) &= \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[e^{\theta T}(1 - \theta T + \frac{\theta^2 T^2}{2}) - 1] + \frac{2cI_k D}{\theta^2 T^3}[e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M] - \frac{cI_e D(M^2 - N^2)}{T^3} \\ &\geq \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[(e^{\theta T} \cdot e^{-\theta T}) - 1] + \frac{2cI_k D}{\theta^2 T^3}[e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M - \frac{\theta^2(M^2 - N^2)}{2}] \\ &= \frac{2A}{T^3} + \frac{2cI_k D}{\theta^2 T^3}[e^{\theta(T-M)} - 1 - \theta T e^{\theta(T-M)} + \frac{\theta^2 T^2}{2} e^{\theta(T-M)} + \theta M - \frac{\theta^2(M^2 - N^2)}{2}]. \end{aligned} \tag{6}$$

Lemma 1 imply that $\frac{d^2 TVC_1(T)}{dT^2} > 0$ if $T \geq M$.

Therefore, $TVC_1(T)$ is convex on $[M, \infty)$.

(2) and (3) Eq. 3 and 4 yield

$$TVC_2'(T) = -\frac{A}{T^2} + \frac{D(c\theta + h)}{\theta^2 T^2}(\theta T e^{\theta T} - e^{\theta T} + 1) - \frac{cI_e D}{2T^2}(N^2 - T^2), \tag{7}$$

$$\begin{aligned} TVC_2''(T) &= \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[e^{\theta T}(1 - \theta T + \frac{1}{2}\theta^2 T^2) - 1] + \frac{cI_e D N^2}{T^3} \\ &> \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[e^{\theta T} \cdot e^{-\theta T} - 1] + \frac{cI_e D N^2}{T^3} = \frac{2A}{T^3} + \frac{cI_e D N^2}{T^3} > 0, \end{aligned} \tag{8}$$

$$TVC_3'(T) = -\frac{A}{T^2} + \frac{D(c\theta + h)}{\theta^2 T^2}(\theta T e^{\theta T} - e^{\theta T} + 1) \tag{9}$$

and

$$\begin{aligned} TVC_3''(T) &= \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[e^{\theta T}(1 - \theta T + \frac{\theta^2 T^2}{2}) - 1] \\ &> \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3}[e^{\theta T} \cdot e^{-\theta T} - 1] = \frac{2A}{T^3} > 0. \end{aligned} \tag{10}$$

Therefore, $TVC_2(T)$ and $TVC_3(T)$ is convex on $(0, \infty)$, respectively.

(4) Case (1) implies that $TVC'_1(T)$ is increasing on $[M,\infty)$. Cases (2) and (3) implies that $TVC'_2(T)$ and $TVC'_3(T)$ is increasing on (0,M]. Since $TVC'_1(M) = TVC'_2(M)$ and $TVC'_2(N) = TVC'_3(N)$, then $TVC'(T)$ is increasing on T>0. Consequently TVC(C) is convex on T>0. Combining the above arguments, we have completed the proof.

DETERMINATION OF THE OPTIMAL REPLENISHMENT CYCLE TIME T*

Consider the following equations:

$$TVC'_1(T) = 0, \quad (11)$$

$$TVC'_2(T) = 0 \quad (12)$$

and

$$TVC'_3(T) = 0. \quad (13)$$

If the root of Eq. 11, 12 or 13 exists, then it is unique. For convenience, let T_i^* (i = 1, 2, 3) denote the root of Eq. 11, 12 and 13, respectively. By the convexity of $TVC_i(T)$ (i = 1,2,3), we see

$$TVC_1'(T)\begin{cases} <0 & \text{if} \quad M \le T < T_1^* \quad (14a) \\ =0 & \text{if} \quad T = T_1^* \quad (14b) \\ >0 & \text{if} \quad T > T_1^* \quad (14c) \end{cases}$$

$$TVC_2'(T)\begin{cases} <0 & \text{if} \quad N \le T < T_2^* \quad (15a) \\ =0 & \text{if} \quad T = T_2^* \quad (15b) \\ >0 & \text{if} \quad T > T_2^* \quad (15c) \end{cases}$$

and

$$TVC_3'(T)\begin{cases} <0 & \text{if} \quad 0 < T < T_3^* \quad (16a) \\ =0 & \text{if} \quad T = T_3^* \quad (16b) \\ >0 & \text{if} \quad T > T_3^* \quad (16c) \end{cases}$$

Although $\lim_{T\to\infty} TVC_1'(T) = \infty$, we can not make sure that whether $\lim_{T\to 0^+} TVC_1'(T)$ is less than 0, therefore, one of the following results will be occurred. One is that if $TVC'_1(M)\le 0$, then T_1^* exists and $T_1^* \ge M$, the other is that if $TVC'_1(M)>0$, then the convexity of $TVC'_1(T)$ on $[M, \infty)$ implies that $TVC'_1(T)$ is increasing on $[M, \infty)$. On the other hand, it is needless to say that Eq. 15a-c and 16a-c implies that $TVC_i(T)$ is decreasing on $(0,T_i^*]$ and increasing on $[T_i^*,\infty)$ for i = 2,3. In addition, $\lim_{T\to 0^+} TVC_i'(T) = -\infty$ and $\lim_{T\to\infty} TVC_i'(T) = \infty$ the Intermediate Value Theorem[8] implies that T_2^* and T_3^* are exist.

From Eq. 5, 7 and 9 yield that

$$TVC_1'(M) = TVC_2'(M) = -\frac{A}{M^2} + \frac{D}{\theta^2 M^2}(c\theta + h)(\theta M e^{\theta M} - e^{\theta M} + 1) + \frac{cI_e D(M^2 - N^2)}{2M^2} \quad (17)$$

and

$$TVC_2'(N) = TVC_3'(N) = -\frac{A}{N^2} + \frac{D(c\theta + h)}{\theta^2 N^2}(\theta N e^{\theta N} - e^{\theta N} + 1) \quad (18)$$

For convenience, we define

$$\Delta_1 = -\frac{A}{M^2} + \frac{D}{\theta^2 M^2}(c\theta + h)(\theta M e^{\theta M} - e^{\theta M} + 1) + \frac{cI_e D(M^2 - N^2)}{2M^2} \quad (19)$$

and

$$\Delta_2 = -\frac{A}{N^2} + \frac{D(c\theta + h)}{\theta^2 N^2}(\theta N e^{\theta N} - e^{\theta N} + 1). \quad (20)$$

Since $TVC_2(T)$ is convex on T>0 which implies that $TVC'_2(T)$ is increasing on T>0, so we have $\boldsymbol{\Delta}_1 = TVC'_2(M) > TVC'_2(N) = \boldsymbol{\Delta}_2$.

Then, we have the following results:

Theorem 2

1. If $\Delta_1 \le 0$, then $TVC(T^*) = TVC_1(T_1^*)$. Hence T^* is T_1^*.
2. If $\Delta_2 \ge 0$, then $TVC(T^*) = TVC_3(T_3^*)$. Hence T^* is T_3^*.
3. If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC_2(T_2^*)$. Hence T^* is T_2^*.

Proof

1. If $\Delta_1 \le 0$, then $\Delta_2 < 0$ which implies that $TVC'_1(M) = TVC'_2(M) \le 0$ and $TVC'_2(N) = TVC'_3(N) < 0$. Equation 14a-c-16a-c imply that

(i) $TVC_1(T)$ is decreasing on $[M, T_1^*]$ and increasing on $[T_1^*, \infty)$.
(ii) $TVC_2(T)$ is decreasing on [N,M).
(iii) $TVC_3(T)$ is decreasing on (0,N).

Combining (i), (ii) and (iii), we conclude that TVC(T) has the minimum value at $T = T_1^*$ on $(0, \infty)$. Hence, we conclude that $TVC(T^*) = TVC_1(T_1^*)$. Consequently, T^* is T_1^*.

2. If $\Delta_2 \geq 0$, then $\Delta_1 > 0$ which implies that $TVC'_1(M) = TVC'_2(M) > 0$ and $TVC'_2(N) = TVC'_3(N) \geq 0$. Equation 14a-c-16a-c imply that

(i) $TVC_1(T)$ is increasing on $[M,\infty)$.
(ii) $TVC_2(T)$ is increasing on $[N,M)$.
(iii) $TVC_3(T)$ is decreasing on $(0, T_3^*]$ and increasing on $[T_3^*, N)$.

Combining (i), (ii) and (iii), we conclude that TVC(T) has the minimum value at $T = T_3^*$ on $(0, \infty)$. Hence, we conclude that $TVC(T^*) = TVC_3(T_3^*)$. Consequently, T^* is T_3^*.

3. If $\Delta_1 > 0$ and $\Delta_2 < 0$ which implies that $TVC'_1(M) = TVC'_2(M) > 0$ and $TVC'_2(N) = TVC'_3(N) < 0$. Equation 14a-c-16a-c imply that

(i) $TVC_1(T)$ is increasing on $[M,\infty)$.
(ii) $TVC_2(T)$ is decreasing on $[N,T_2^*]$ and increasing on $[T_2^*, M)$.
(iii) $TVC_3(T)$ is decreasing on $(0,N)$.

Combining (i), (ii) and (iii), we conclude that TVC(T) has the minimum value at $T = T_2^*$ on $(0,\infty)$. Hence, we conclude that $TVC(T^*) = TVC_2(T_2^*)$. Consequently, T^* is T_2^*.

Combining the above arguments, we have completed the proof.

CONCLUSIONS

This study modifies Huang's model[2] by considering decay item to find the retailer's optimal replenishment cycle time under two levels of trade credit. In addition, we develop an easy-to-use procedure to find the optimal replenishment cycle time for the retailer. This procedure is the main contribution of this study.

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