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A New Hybrid Analytical Analysis of the Magnetohydrodynamic Flow over a Rotating Disk under a Uniform Suction

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Abstract: A new hybrid analytical algorithm has been developed and used to present, in a simple analytical method, the effect of uniform suction in case of a laminar, incompressible, steady flow of an electrically conducting fluid over a rotating disk in the presence of a circular magnetic field imposed at the disc. The analytical results obtained in this work have been compared with both numerical method and other analytical method and showed to be accurate in comparison.

Key words: Analytical hybrid solution, magnetohdrodynamic, rotating disc, conducting fluid

INTRODUCTION

The flow past of a rotating disc in a viscous fluid has received considerable attention in the past few decades. Karman (1921) was the first to consider the steady laminar motion of an incompressible viscous fluid over an infinite plane disc rotating at a constant angular velocity. He derived the simplified equations that govern the flow over an infinite rotating disc and solved them by an approximate integral method. Cochran (1934) integrated the equations obtained by Karrman (1921) numerically and obtained a more accurate solution. Stuart (1959) presented a numerical solution for small values and a series solution for high values. Bodewadt (1940) solved numerically the problem of the flow produced over an infinite stationary plate in a liquid which is rotating with uniform angular velocity at an infinite distance from the plate. Kumar (1988) studied the effects of a circular magnetic field on the flow of a conducting fluid about a porous rotating disk. Gorla (1992) investigated the effect of a uniform suction in the case of a laminar incompressible steady flow of an electrically conducting fluid over a rotating disc in the presence of magnetic field, where the governing equation was reduced to a system of ordinary equation and series solutions were obtained. New methodologies to solve nonlinear differential equation analytically (Mansour and Hussein, 1990; Mansour *et al*., 1991; Mansour *et al*., 1993; Jubran *et al*., 1993; El-Asir *et al*., 1994) proved to be accurate and efficient.

In the present study, a new computerized hybrid solution to the uniform suction effects on the magnetohydrodynamic electrically conducting fluid over a rotating disc has been developed and used to solve analytically the nonlinear magnetohydrodynamic problem accurately and efficiently, for wide range values of suction parameter greater than zero, in a very simple manner. The analytical results were compared with both the series and the numerical methods.

MATERIALS AND METHODS

The disc surface is in the plane z = 0 and rotates about the z-axis with constant angular velocity (w). An axial electric current of uniform density (J_0) is imposed at the disc surface. Equivalently, a tangential magnetic field component $g = \Omega\ r$ is imposed at the disc surface. The current lines and magnetic lines are conducting fluid in an axisymmetric motion over a rotating disk as shown in Fig.1, the governing equations can be written within boundary layer approximation as:

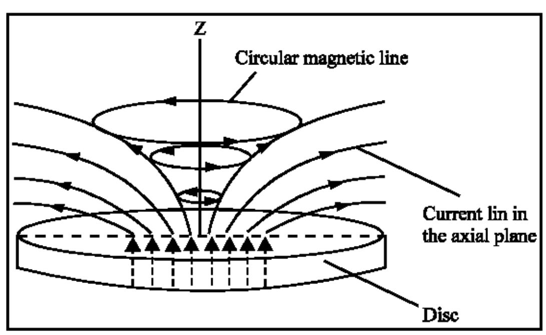

Fig. 1: Physical model of the rotating disc

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$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z}=0 \qquad (1)$$

$$\frac{\partial(rf)}{\partial r}+\frac{\partial(rh)}{\partial z}=0 \qquad (2)$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=$$
$$-\frac{1}{\rho}\frac{\partial p}{\partial r}+f\frac{\partial f}{\partial r}+h\frac{\partial f}{\partial z}-\frac{g^2}{r}+v\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right] \qquad (3)$$

$$u\frac{\partial \upsilon}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=$$
$$f\frac{\partial g}{\partial r}+h\frac{\partial g}{\partial z}+\frac{fg}{r}+v\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}-\frac{v}{r^2}\right] \qquad (4)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=$$
$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+f\frac{\partial h}{\partial r}+h\frac{\partial h}{\partial z}+v\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right] \qquad (5)$$

$$\frac{\partial(vf-ug)}{\partial r}+\frac{\partial(vh-wg)}{\partial z}+$$
$$\eta'\left[\frac{\partial^2 g}{\partial r^2}+\frac{1}{r}\frac{\partial g}{\partial r}+\frac{\partial^2 g}{\partial z^2}-\frac{g}{r^2}\right]=0 \qquad (6)$$

$$(uh-wf)-\eta'\left[\frac{\partial h}{\partial r}-\frac{\partial f}{\partial z}\right]=0 \qquad (7)$$

where, u, v and w represent the velocity components in r, θ and z directions; f,g and h represent the normalized magnetic field components in r, θ and z directions ; ν is the kinematic viscosity and η is the magnetic diffusivity, more over we define

$$ru=-\partial\psi/\partial z,\ rw=\partial\psi/\partial r,\ rf=-\partial\lambda/\partial z,$$
$$rh=-\partial\lambda/\partial r,\ \psi=-(\nu w)^{1/2}r^2H(\zeta),$$
$$\lambda=-(\nu w)^{1/2}r^2n(\zeta), v=rwG(\zeta),$$
$$g=r\Omega M(\zeta),\ \zeta=z(w/v)^{1/2} \qquad (8)$$

where w and Ω denote the angular velocity of the disk and the angular magnetic field strength, respectively. Now by assuming that a tangential magnetic field component is imposed at the disc, then $g = r\Omega$ and $\Omega=1/2J_o(\mu/\rho)^{1/2}$

where J_o is the axial electric current of uniform density imposed at the disc and the magnetic field strength components in radial and axial directions f and h are absent, then Eq. 1-7 can be reduced to:

$$H''' + 2HH'' - H'^2 - \beta^2M^2 + G^2 = 0 \qquad (9)$$

$$G'' + 2HG' - 2H'G = 0 \qquad (10)$$

$$M'' + 2\alpha HM' = 0 \qquad (11)$$

where, $\alpha = V/\eta'$ is the magnetic prandtl number, $\beta = \Omega/w$ and the primes denote differentiation with respect to ζ.

In this analysis the disc is porous and the suction occurs at the disc surface such that

$$w(0) = -2a(\nu w)^{1/2}$$

with the following boundary conditions :
1) u = 0, v = wr, g(0) at the disk surface (z = 0)
2) u = v = 0 far away from the disc ($z = \infty$)
and the boundary conditions for Eq. 9-11 are

$$H(0) = a,\ H'(0) = 0,\ H'(\infty) = 0,\ G(0) = 1,$$
$$G(\infty) = 0,\ M(0) = 1,\ M(\infty) = 0 \qquad (12)$$

where, a is the suction parameter and is equal to $\frac{w_0}{2(\nu\omega)^{1/2}}$

METHOD OF SOLUTION

The sets of equations that were derived in the foregoing section are nonlinear ordinary differential equations. To find their solution, one expects H to be nearly constant for large value of α greater than one which satisfies the boundary condition, that is $\zeta = 0$, H = a

The key point to the solution is to consider a guessed function which defines H and satisfies the boundary condition. The simplest form of the proposed guessed function is:

$$H(\zeta) = a \qquad (13)$$

In Eq. 9-11 H is replaced by the guessed function in the second term and H' is replaced by the first derivative of the guessed function. Thus, Eq. 9-11 are reduced to:

$$H''' + 2aH'' - \beta^2M^2 + G^2 = 0 \qquad (14)$$

$$G'' + 2aG' = 0 \qquad (15)$$

$$M'' + 2\alpha aM' = 0 \qquad (16)$$

The solution of the second ordinary differential Eq. 15 and 16 that satisfies the boundary conditions G(0) =1, M(0) =1, G(∞) =0, M(∞) = 0, can be written as:

$$G(\xi) = e^{-2a\xi} \tag{17}$$

$$M(\xi) = e^{-2\alpha a\xi} \tag{18}$$

Radial velocity component (H′): Upon substituting Eq. 17 and 18 into Eq.13 the following linear ordinary differential equation is obtained

$$H''' + 2aH'' = \beta^2 e^{-4\alpha a\xi} - e^{-4a\xi} \tag{19}$$

Integrating Eq. 19 twice with respect to ζ, the following first-order linear differential equation can obtain

$$H' + 2aH + \frac{1}{16a^2}\left[1 - \frac{\beta^2}{\alpha^2}e^{4a\xi(1-\alpha)}\right] e^{-2a\xi} - C_1\xi - C_2 = 0 \tag{20}$$

Which has the solution:

$$H = \left[\frac{1}{32a^3}\left[1 + \frac{\beta^2}{\alpha^2(1-2\alpha)}e^{4a\xi(1-\alpha)}\right]e^{-2a\xi} + C_3\right] e^{-2a\xi} + \frac{1}{2a}\left[C_2 + C_1\left(\xi - \frac{1}{2a}\right)\right] \tag{21}$$

Substituting the boundary conditions given in Eq. 12 into Eq. 21 and considering H(∞) = finite value, the complete analytical solution for the axial velocity becomes

$$H = a + \frac{1}{32a^3}\left[Q\beta^2 + R\right] \tag{22}$$

and for the radial velocity H′ becomes

$$H' = \frac{1}{8a^2}S\left[\beta^2 + \frac{T}{S}\right] \tag{23}$$

Where:

$$Q = \frac{2\alpha e^{-2a\xi} - e^{-4\alpha a\xi} - 2\alpha + 1}{\alpha^2(2\alpha - 1)}, R = e^{-4a\xi} - 2e^{-2a\xi} + 1$$

$$S = \frac{e^{-4\alpha a\xi} - e^{-2a\xi}}{\alpha(2\alpha - 1)},\ T = e^{-2a\xi} - e^{-4a\xi}$$

Tangential (azimuthal) velocity component (G): To obtain the tangential velocity component (G), Eq. 10, 20 and 23 will be considered to get accurate solution for tangential velocity as follows. Rewrite Eq. 10 in the following form

$$G'' + 2HG' = 2GH' \tag{24}$$

Upon Substituting Eq. 23 for H′ and Eq. 17 for G in the right hand side of Eq. 24 and the guessed function in Eq. 13 for H in the left hand side of Eq. 24, the following second linear ordinary equation will be obtained.

$$G'' + 2aG' - \frac{1}{4a^2}\left\{\left(\frac{e^{-2a\xi(2\alpha+1)} - e^{-4a\xi}}{\alpha(2\alpha - 1)}\right)\beta^2 + e^{-4a\xi} - e^{-6a\xi}\right\} = 0 \tag{25}$$

Upon integrating Eq. 25 twice, the following solution will be obtained for tangential velocity component G

$$G = \frac{F}{96a^4}\left[\beta^2 + \frac{P}{F}\right] \tag{26}$$

where:

$$F = \frac{3}{\alpha^2(4\alpha^2 - 1)}\left[e^{-2a(2\alpha+1)\xi} - \alpha(2\alpha + 1)e^{-4a\xi} + (\alpha(2a + 1) - 1)e^{-2a\xi}\right]$$

$$P = \frac{1}{48a^4}\left[(48a^4 - 1)e^{-2a\xi} - e^{-6a\xi}\right]$$

Magnetic field strength (M): Consider Eq. 11 and rewrite it in the form.

$$M'' = -2\alpha HM' \tag{27}$$

take the derivative of Eq. 18 and rewrite it in the form

$$M' = -2\alpha a e^{-2a\xi} \tag{28}$$

substitute Eq. 22 for H into Eq. 27 we obtain

$$M'' = -2\alpha\left[a + \frac{1}{32a^3}\left(Q\beta^2 + R\right)\right]M' \tag{29}$$

rearrange Eq. 29 in the form

$$M'' + 2\alpha aM' = -\frac{\alpha}{16a^3}\left(Q\beta^2 + R\right)M' \tag{30}$$

substitute Eq. 28 to the right hand side of Eq. 30, then Eq. 30 will have the form

$$M'' + 2\alpha aM' = \frac{\alpha^2}{8a^2}\left[\frac{2\alpha e^{-2a\xi} - e^{-4\alpha a\xi} - 2\alpha + 1}{\alpha^2(2\alpha - 1)}\beta^2 + e^{-4a\xi} - 2e^{-2a\xi} - 2e^{-2a\xi} + 1\right]e^{-2\alpha a\xi} \tag{31}$$

Integrating (31) twice with respect to ζ and applying appropriate boundary conditions given in Eq. 12, the following solution for M can be obtained

$$M = C_9 e^{-6\alpha a\xi} + C_{10} e^{-2a(2+\alpha)\xi} + C_{11} e^{-2a(1+\alpha)\xi} + C_{14} e^{-2\alpha C_{12}\xi}$$

Where:

$$C_9 = -\frac{\beta^2}{3\alpha^2(2\alpha-1)(\beta^2+64a^4-1)}$$

$$C_{10} = \frac{\alpha^3}{(\alpha+2)(64\alpha a^4-\alpha^2+\beta^2)}$$

$$C_{11} = \frac{2\alpha^2(\beta^2-\alpha(2\alpha-1))}{(32\alpha a^4-\alpha^2+\beta^2)(\alpha+1)(2\alpha-1)}$$

$$C_{12} = \frac{\alpha^2(32a^4+1)-\beta^2}{32\alpha^2 a^3}$$

$$C_{14} = 1-C_9-C_{10}-C_{11}$$

RESULTS AND DISCUSSION

In this study, the numerical results obtained from the presented new hybrid analytical solution to solve the problem of a uniform suction in case of a laminar, incompressible, steady flow of an electrically conducting fluid over a rotating disk in the presence of a circular magnetic field imposed at the disc are computed graphically and shown in Fig. 2-4, where the radial velocity, tangential velocity and magnetic strength against ζ, respectively are considered for β = 1, α = 10 and a = 1, 1.5, 2, 2.5, 3. These obtainable curves are similar to those obtained by Grola (1992). The effect of the magnetic prandtl number α, the axial coordinate ζ, the suction parameter α andβon the radial velocity H′, the tangential velocity G and the magnetic strength M, are shown in Fig. 5 and 6 where increasing β results in decreasing the radial velocity and in increasing both the tangential velocity and the magnetic strength, but as shown, this decrease or increase is very small no matter what the values of ζ and the suction parameter a are. On the other hand, the magnetic strength is very sensitive to any change in the value of the suction parameter a or ζ values, while the radial velocity and tangential velocity are not highly sensitive to the suction parameter for small values of ζ and become more sensitive as ζ value is increased. Results of radial velocity, tangential velocity and magnetic strength obtained from both, the series solution by Gorla and the new presented analytical method are shown in Fig. 7-9 with respect to the numerical

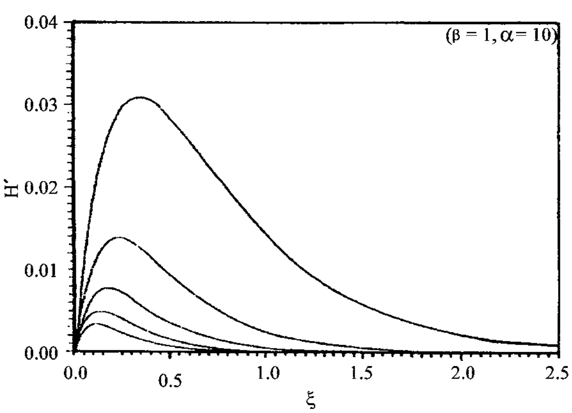

Fig. 2: Radial velocity distribution H′ versus axial coordinate ζ for suction parameter a =1, 1.5, 2, 2.5, 3 (top to bottom)

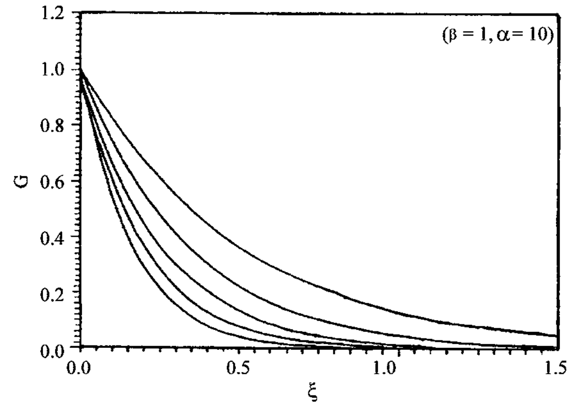

Fig. 3: Tangential velocity distribution G versus axial coordinate ζ for suction parameter a = 1, 1.5, 2, 2.5, 3 (top to bottom)

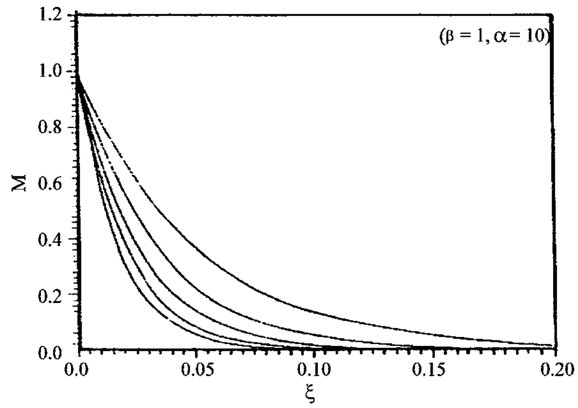

Fig. 4: Magnetic field strength M’ versus aial coordinate ζ for suction parameter a = 1,1.5, 2, 2.5, 3 (top to bottom)

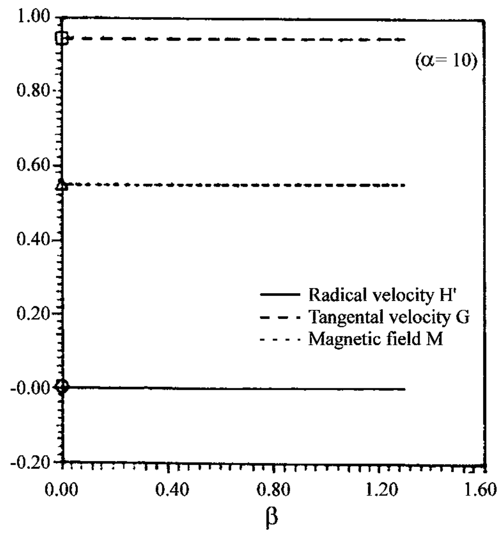

Fig. 5a: Radial velocity H′, Tangential velocity G and Magnetic field strength M versus β for $\zeta = 0.01$ and a = 3

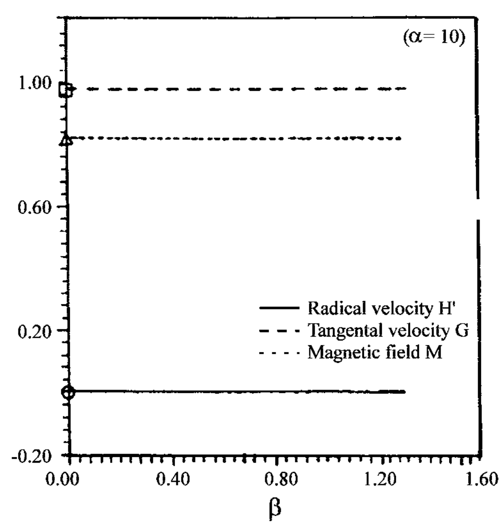

Fig. 5b: Radial velocity H′, Tangential velocity G and Magnetic field strength M versus β for $\zeta = 0.01$ and a =1

integration method results, where the solid line represents the results of the new hybrid method, the dashed line represents the results of numerical integration method and dotted line represents series method (Gorla solution). According to Fig. 7, one can see that for a small value of suction parameter (a = 1), the new hybrid method provides exact solution for the radial velocity in the range $2.3<\zeta<0.1$ and provides a reasonable solution else where, while for higher suction parameter (a = 3), the presented work provides exact solution for the entire range of ζ. Figure 8, shows the tangential velocity

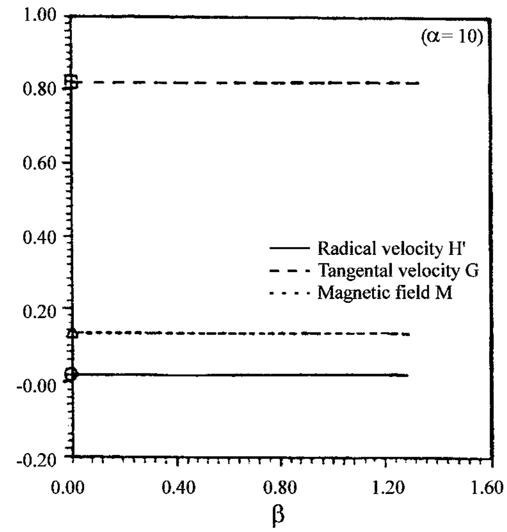

Fig. 5c: Radial velocity H′, Tangential velocity G and Magnetic field strength M versus β for $\zeta = 0.1$ and a = 1

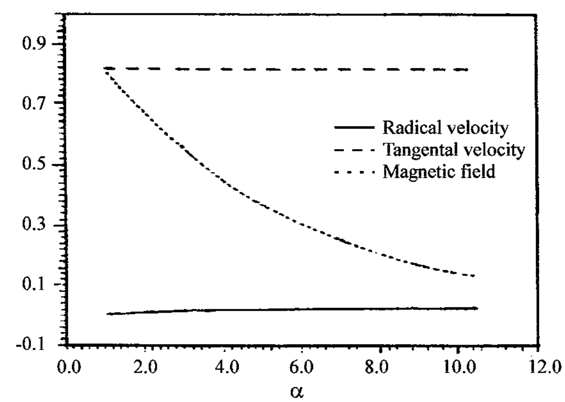

Fig. 6a: Radial velocity H′, Tangential velocity G and Magnetic field strength M versus Magnetic Prandtl number α for $\zeta = 0.1$, a = 1 and β = 1

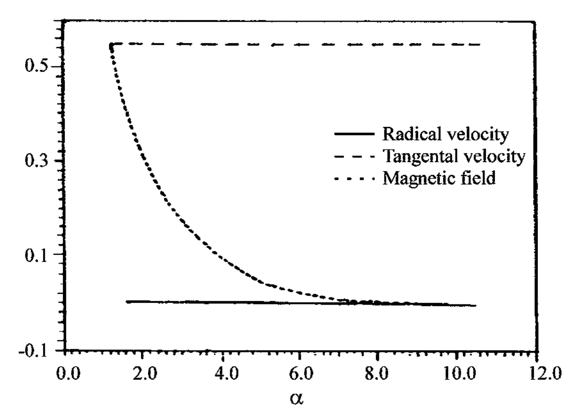

Fig. 6b: Radial velocity H′, Tangential velocity G and Magnetic field strength versus Magnetic Prandtl number α for $\zeta = 0.1$, a = 3 and β = 1

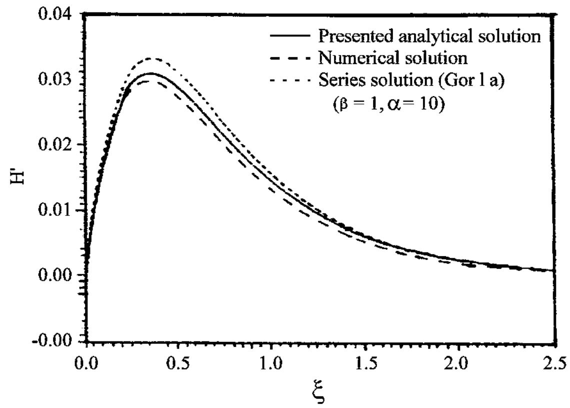

Fig. 7a: Radial velocity distribution H′ versus axial coordinate ζ for suction parameter a = 1

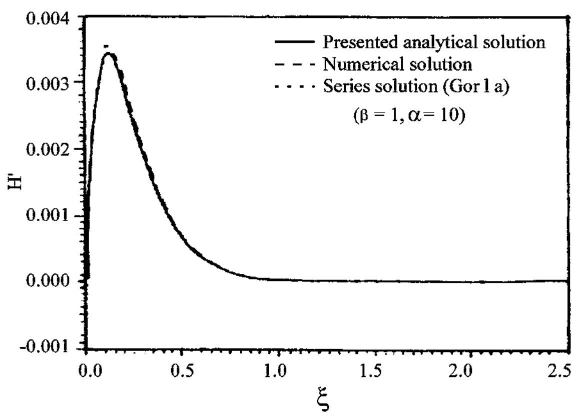

Fig. 7b: Radial velocity distribution H′ versus axial coordinate ζ for suction parameter a =3

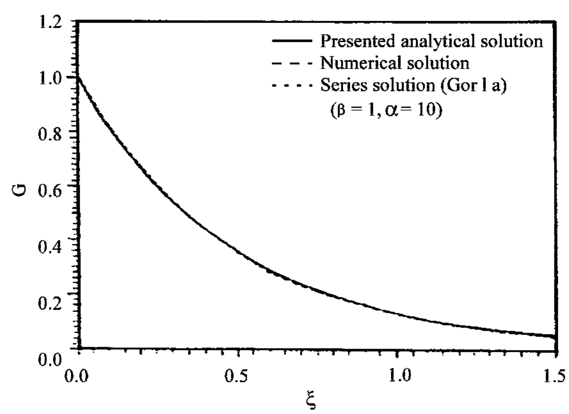

Fig. 8a: Tangential velocity distribution G versus axial coordinate ζ for suction parameter a = 1

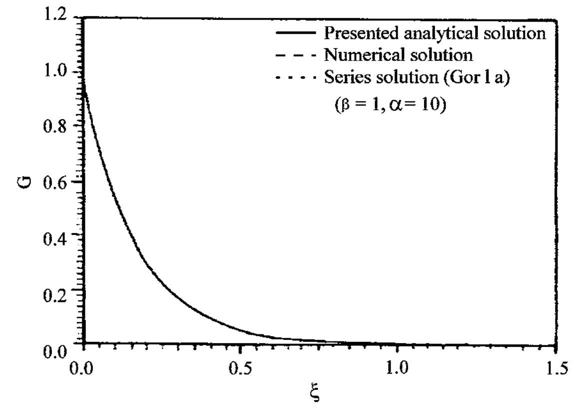

Fig. 8b: Tangential velocity distribution G versus axial coordinate ζ for suction parameter a = 3

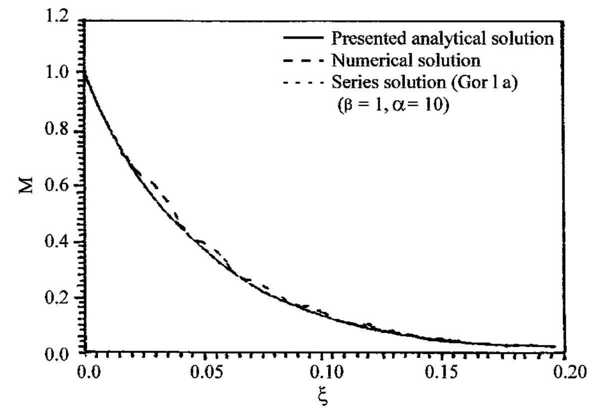

Fig. 9a: Magnetic field strength M versus axial coordinate ζ for suction parameter a = 1

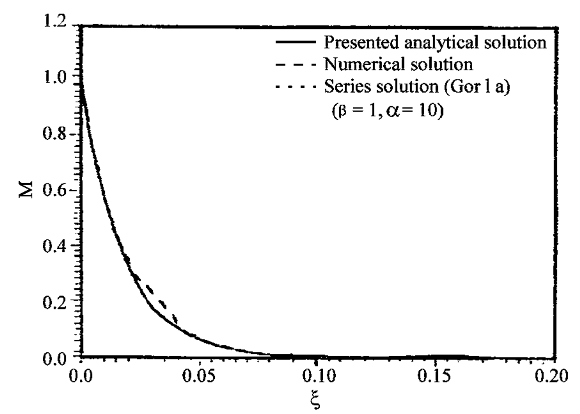

Fig. 9b: Magnetic field strength M versus axial coordinate ζ for suction parameter a = 3

versus the axial coordinate ζ. It is obvious that the presented method provides exact solution in the entire range of ζ for a greater than zero. Figure 9 shows the magnetic strength, where it is shown that the new method provides almost an exact solution for both small and large suction parameter in the entire range of ζ.

To assess the effectiveness of the presented new hybrid analytical solution, the results of the new method and the series solution method by Gorla (1992) are compared with the numerical integration method. The results of the new method prove to be closer and more accurate as shown in Fig. 7-9. In addition, the new method is very simple and straight forward in terms of mathematical process. In conclusion, the new hybrid analytical numerical method presented here has been successfully applied to obtain simple, accurate and reliable solution and is shown to be better in comparison with the series solution.

NOTATION

a	Suction parameter
G	Azimuthal velocity component
H	Axial velocity component
f, g, h	Components of magnetic field strength in radial, azimuthal and axial directions, respectively
J_0	Current density
M	Magnetic field strength
p	Pressure
r	Radial direction
u v w	Radial, azimuthal and axial velocity components
W_0	Uniform suction at the disc surface axial direction
z	Axial direction
Greek Symbols	
ρ	Density
σ	Electrical conductivity
α	Magnetic Prandtl number
β	Ω/ω
ν	Kinematic viscosity
ξ	Dimensionless axial coordinate
η	A parameter (a ξ)
η	Magnetic diffusivity
w	Angular velocity of the disc
Ω	Angular magnetic field strength

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