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Algebraic Improvement on Effects of Random Defective Rate and Imperfect Rework Process on Economic Production Quantity Model

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Abstract: The present note studied the effect of random defective rate and imperfect rework process on the Economic Production Quantity (EPQ) model. They demonstrate that the optimal lot size can be solved algebraically and the expected inventory cost can be derived immediately as well. In this note, we will offer a simple algebraic approach to replace their algebraic skill to find the optimal solution under the expected annual cost minimized.

Key words: Imperfect rework process, algebraic derivation, lot size, EPQ

INTRODUCTION

The EPQ (Economic Production Quantity) model is widely used by the manufacturing sector as a decision-making tool for the control of the optimal production lot size. However, the classical EPQ model assumes that the manufacturing process will always produce perfect quality in a production lot. This is debatable sometimes in the most real-life situations.

In previous several papers, the EOQ and EPQ formulae have been derived using differential calculus and the need to prove optimality conditions with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. But Ronald *et al.* (2004) thought that their algebraic procedure is too sophisticated to be absorbed by ordinary readers. Hence, Ronald *et al.* (2004) derived a procedure to transform a two-variable problem into two steps and then, in each step, they solve a one-variable problem using only the algebraic method without referring to calculus. Recently, Chang *et al.* (2005) rewrote the objective function of Ronald *et al.* (2004) such that the usual skill of completing the square can handle the problem without using their sophisticated method.

Recently, Chiu *et al.* (2004) studied the effects of random defective rate and imperfect rework process on the EPQ model by using the traditional methodology. Then, Chiu and Chiu (2005) reconsidered Chiu *et al.* (2004) model to take the reworking of random defective items using the algebraic approach. Although, Chiu and Chiu (2005) used the algebraic approach to find the optimal solution. However, their method still is difficult to many younger students in the process of completing the square. Therefore, in this note, we will offer a more simple algebraic skill than Chiu and Chiu (2005) to complete the process of square. We think this method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus as Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) stated.

ALGEBRAIC IMPROVEMENT IN THE CHIU AND CHIU'S MODEL

For convenience, we adopt the same notation and assumptions as Chiu and Chiu (2005) in this note.

From Eq. 9 in Chiu and Chiu (2005), we know the expected annual cost, E[TCU(Q)] can be expressed as:

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$$E[TCU(Q)] = \frac{\lambda}{1-\varphi E[x]}\left[C + C_R(1-\theta)E[x] + C_s\varphi E[x]\right] + \frac{1}{1-\varphi E[x]}$$
$$\left\{\frac{K\lambda}{Q} + \frac{hQ}{2}\left(1-\frac{\lambda}{P}\right) + \frac{\lambda Q(1\text{-}\theta)^2}{2P_1}[h_1 - h(1-\theta_1)]E[x^2] - hQ\varphi\left(1-\frac{\lambda}{P}\right)E[x] + \frac{hQ\varphi^2}{2}E[x^2]\right\} \tag{1}$$

Our goal is to find the minimum solution of E[TCU(Q)] by algebraic approach. Then, Chiu and Chiu (2005) completed the square by adding and subtracting an additional term described as Eq. 11 and 12 in Chiu and Chiu (2005). We think that their method still is difficult to many younger students in the process of completing the square. We directly divide Eq. 1 into three terms, one is independent on Q; another is dependent on 1/Q and the other is dependent on Q. Rearranging Eq. 1, we have:

$$E[TCU(Q)] = \frac{\lambda}{1-\varphi E[x]}\left[C + C_R(1-\theta)E[x] + C_s\varphi E[x]\right] + \frac{1}{1-\varphi E[x]}$$
$$\left\{(K\lambda)\frac{1}{Q} + \left[\frac{h}{2}\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{2P_1}[h_1 - h(1-\theta_1)]E[x^2] - h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + \frac{h\varphi^2}{2}E[x^2]\right]Q\right\} \tag{2}$$

From Eq. 2, we can obtain:

$$E[TCU(Q)] = \frac{\lambda}{1-\varphi E[x]}\left[C + C_R(1-\theta)E[x] + C_s\varphi E[x]\right] + \frac{1}{1-\varphi E[x]}$$
$$\left\{\sqrt{(K\lambda)\frac{1}{Q}} - \sqrt{\left[\frac{h}{2}\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{2P_1}[h_1 - h(1-\theta_1)]E[x^2] - h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + \frac{h\varphi^2}{2}E[x^2]\right]Q}\right\}^2$$
$$+ \frac{1}{1-\varphi E[x]}\sqrt{2K\lambda\left[h\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{P_1}[h_1 - h(1-\theta_1)]E[x^2] - 2h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + h\varphi^2E[x^2]\right]} \tag{3}$$

It implies that when

$$\sqrt{(K\lambda)\frac{1}{Q}} = \sqrt{\left[\frac{h}{2}\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{2P_1}[h_1 - h(1-\theta_1)]E[x^2] - h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + \frac{h\varphi^2}{2}E[x^2]\right]Q} \tag{4}$$

Then, we can get the minimum value of E[TCU(Q)] as follows:

$$E[TCU(Q^*)] = \frac{\lambda}{1-\varphi E[x]}\left[C + C_R(1-\theta)E[x] + C_s\varphi E[x]\right] + \frac{1}{1-\varphi E[x]}$$
$$\sqrt{2K\lambda\left[h\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{P_1}[h_1 - h(1-\theta_1)]E[x^2] - 2h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + h\varphi^2E[x^2]\right]} \tag{5}$$

Equation 5, in this note, is the same as Eq. 16 in Chiu and Chiu (2005). Reconsidering Eq. 4, in this note, implies that

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right) + \frac{\lambda(1\text{-}\theta)^2}{P_1}[h_1 - h(1-\theta_1)]E[x^2] - 2h\varphi\left(1-\frac{\lambda}{P}\right)E[x] + h\varphi^2E[x^2]}} \tag{6}$$

Eq. 6, in this note, is the same as Eq. 14 in Chiu and Chiu (2005).

Our procedure avoids the complicated process to complete the square. We think this procedure can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

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