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Mathematical Inventory Model under Trade Credits Linked to Payment Time

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Abstract: This study is to extend Chou *et al.*'s model to assume that the unit selling price and unit purchasing price are not necessarily equal. Then, reformulate retailer's mathematical inventory replenishment model and obtain the optimal cycle time and optimal payment time for item so that the annual total relevant cost is minimized. One theorem is developed to efficiently determine the optimal replenishment and payment policy for the retailer.

Key words: Inventory, trade credit, payment time

INTRODUCTION

In 2005, Chou *et al.*[1] modeled the retailer's inventory system under payment delay and cash discount and developed an efficient procedure to determine the retailer's optimal ordering policy. Chou *et al.*[1] assumed that the retailer can obtain fully permissible delay in payments and cash discount if the payment is paid before the period of full delay payments permitted by the supplier. Otherwise, the retailer will just obtain partially permissible delay in payments within the period of partial delay payments permitted by the supplier. The supplier uses the credits policy linked to payment time to attract the retailer to pay the payment as soon as possible to shorten its collection period. However, they implicitly make the following assumptions:

- The unit selling price and the purchasing price per unit are assumed to be equal. However, as we know, the unit selling price for the retailer is usually significantly higher than the purchasing price per unit in order to obtain profit.
- At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold. What the above statement describes is just one arrangement of capitals of enterprises. Based on considerations of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantage.

According to the above arguments, this article will adopt the following assumptions to modify Chou *et al.*[1] model.

- The selling price per unit and the unit purchasing price are not necessarily equal to match the most practical situations.
- The retailer needs cash for business transactions. At the end of the credit period, the retailer pays off all units sold and keeps his/her profits for business transactions or other investment use. This viewpoint also can be found by Teng[2].

The main purpose of this study is to incorporate the assumptions (i) and (ii) to modify Chou *et al.*s[1] model. That is, we incorporate both Chou *et al.*[1] and Teng[2] to develop the retailer's inventory model. Then, we develop one easy-to-use theorem to efficiently determine the optimal inventory policy for the retailer.

MODEL REFORMULATION

For convenience, most notation and assumptions similar to Chou *et al.*[1] will be used in this study.

Notation:

D = Demand rate per year
A = Cost of placing one order
c = Unit purchasing price
s = Unit selling price

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h = Unit stock holding cost per year excluding interest charges

r = Cash discount rate, $0 \le r < 1$

α = The fraction of the total amount owed payable at the time of placing an order, $0 < \alpha \le 1$

I_e = Interest earned per \$ per year

I_k = Interest charges per \$ investment in inventory per year

M_1 = The period of full delay payments permitted in years

M_2 = The period of partial delay payments permitted in years, $M_1 < M_2$

T = The cycle time in years

$TRC_{11}(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T \ge M_1$

$TRC_{12}(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T < M_1$

$TRC_1(T)$ = The annual total relevant cost when payment is paid at time M_1 and $T > 0$

$$= \begin{cases} TRC_{11}(T) & \text{if } T \ge M_1 \\ TRC_{12}(T) & \text{if } T < M_1 \end{cases}$$

$TRC_{21}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T \ge M_2/\alpha$

$TRC_{22}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $M_2 \le T < M_2/\alpha$

$TRC_{23}(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T < M_2$

$TRC_2(T)$ = The annual total relevant cost when payment is paid at time M_2 and $T > 0$

$$= \begin{cases} TRC_{21}(T) & \text{if } T \ge \frac{M_2}{\alpha} \\ TRC_{22}(T) & \text{if } M_2 \le T < \frac{M_2}{\alpha} \\ TRC_{23}(T) & \text{if } T < M_2 \end{cases}$$

TRC(T) = The annual total relevant cost when $T > 0$

$$= \begin{cases} TRC_1(T) & \text{if the payment is paid at time } M_1 \\ TRC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$$

T* = The optimal cycle time of TRC(T)

Q* = The optimal order quantity = DT*.

Assumptions:

- Demand rate is known and constant.
- Shortages are not allowed.
- Time horizon is infinite.
- Replenishments are instantaneous.
- $I_k \ge I_e$.
- Supplier offers a cash discount and fully delayed payment to the retailer if payment is paid within M_1, otherwise just partially delayed payment if payment is paid within M_2.
- If payment is paid within M_1, when the account is settled the retailer starts paying for the interest charges on the items in stock. If payment is paid behind M_1 but within M_2, as the order is received, the retailer must make a partial payment αcDT to the supplier. Then the retailer must pay off the remaining balance $(1-\alpha)$cDT at the end of the partially permissible delay period M_2.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When the account is settled the retailer pays off all units sold and keeps his/her profits, and starts paying for the higher interest charges on the items in stock.

The model:

- The annual total relevant cost consists of the following elements.
- Annual ordering cost = $\frac{A}{T}$.
- Annual stock holding cost (excluding interest charges) = $\frac{DTh}{2}$.
- Annual purchasing cost:
  Since the supplier offers a cash discount if payment is paid within M_1, there are two payment policies for the retailer.

Case 1: Payment is paid at time M_1, the annual purchasing cost = c(1-r)D.

Case 2: Payment is paid at time M_2, the annual purchasing cost = cD.

- Annual opportunity cost of the capital:

Case 1: Payment is paid at time M_1, according to assumptions (7) and (8); there are two sub-cases in terms of annual opportunity cost of the capital.

Case 1.1: $T \ge M_1$.

Annual opportunity cost of the capital =

$$c(1-r)I_k D(T-M_1)^2/2T - sI_e\left(\frac{DM_1^2}{2}\right)/T$$

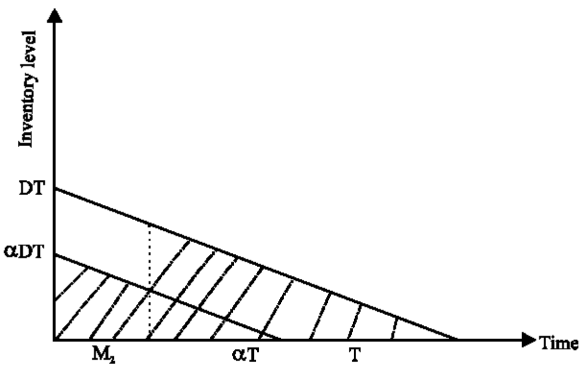

Fig. 1: The inventory level and the total amount of interest payable when $M_2/\alpha \le T$

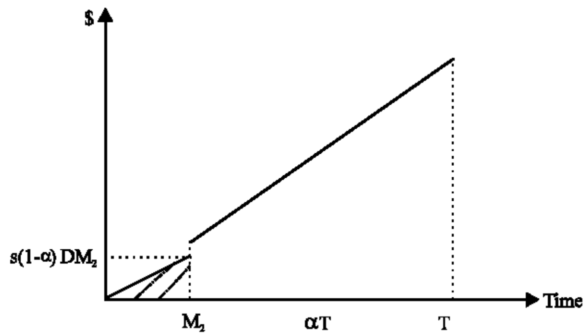

Fig. 2: The total amount of interest earned when $M_2/\alpha \le T$

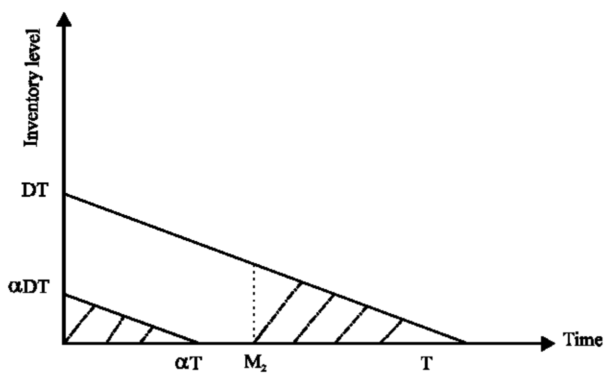

Fig. 3: The inventory level and the total amount of interest payable when $M_2 \le T < M_2\alpha$

Case 1.2: $T < M_1$

In this case, annual opportunity cost of the capital =

$$-sI_e[\frac{DT^2}{2} + DT(M_1 - T)]/T = -sI_e DT(M_1 - \frac{T}{2})/T$$

Case 2: Payment is paid at time M_2, according to assumptions (7) and (8); there are three sub-cases in terms of annual opportunity cost of the capital.

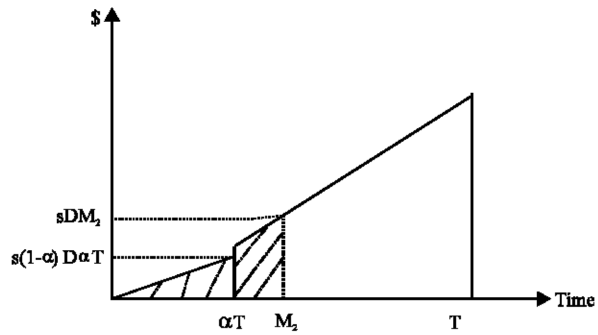

Fig. 4: The total amount of interest eared when $M_2 \le T < M_2/\alpha$

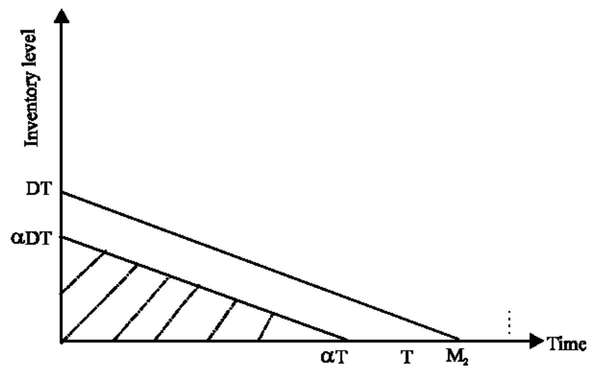

Fig. 5: The inventory level and the total amount of interes payable when $0 < T < M_2$

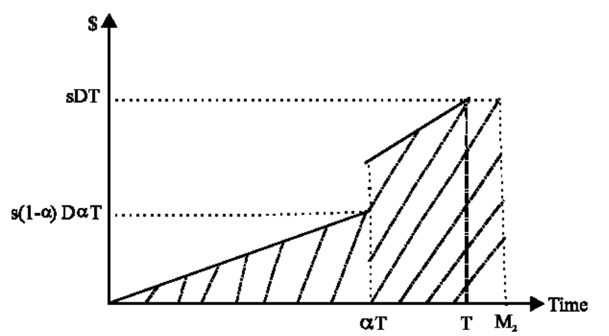

Fig. 6: The total amount of interest earned when $0 < T < M_2$

Case 2.1: $\frac{M_2}{\alpha} \le T$, (Fig. 1 and 2).

Annual opportunity cost of the capital

$$= cI_k[\frac{DT^2}{2} - (1-\alpha)DTM_2]/T - sI_e\left[\frac{(1-\alpha)DM_2^{\ 2}}{2}\right]/T$$

Case 2.2: $M_2 \le T < \frac{M_2}{\alpha}$, (Fig. 3 and 4).

Annual opportunity cost of the capital

$$= cI_k[\frac{\alpha^2DT^2}{2} + \frac{D(T-M_2)^2}{2}]/T - sI_e\left\{\frac{(1-\alpha)\alpha^2DT^2}{2} + \frac{[(1-\alpha)\alpha DT + DM_2](M_2-\alpha T)}{2}\right\}/T$$

$$= cI_k[\frac{\alpha^2DT^2}{2} + \frac{D(T-M_2)^2}{2}]/T - sI_e\left[\frac{DM_2(M_2-\alpha^2T)}{2}\right]/T$$

Case 2.3: $T < M_2$, (Fig. 5 and 6).

Annual opportunity cost of the capital

$$= cI_k(\frac{\alpha^2DT^2}{2})/T - sI_e\left\{\begin{array}{l}\frac{(1-\alpha)\alpha^2DT^2}{2} + \frac{[(1-\alpha)\alpha DT + DT](T-\alpha T)}{2}\\ +DT(M_2-T)\end{array}\right\}/T$$

$$= cI_k(\frac{\alpha^2DT^2}{2})/T - sI_eDT\left[M_2 - \frac{(1+\alpha^2)T}{2}\right]/T$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

Annual total relevant cost = ordering cost + stock-holding cost + purchasing cost + opportunity cost of the capital.

We show that the annual total relevant cost is given by

Case 1: Payment is paid at time M_1

$$TRC_1(T) = \begin{cases} TRC_{11}(T) & \text{if} \quad T \ge M_1 & (1a)\\ TRC_{12}(T) & \text{if} \quad 0 < T < M_1 & (1b)\end{cases}$$

where

$$TRC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + \frac{c(1-r)I_kD(T-M_1)^2}{2T} - \frac{sI_eDM_1^2}{2T} \quad (2)$$

and

$$TRC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DsI_e(M_1 - \frac{T}{2}) \quad (3)$$

At $T = M_1$, we find $TRC_{11}(M_1) = TRC_{12}(M_1)$. Hence $TRC_1(T)$ is continuous and well-defined. All $TRC_{11}(T)$, $TRC_{12}(T)$ and $TRC_1(T)$ are defined on $T > 0$.

Case 2: Payment is paid at time M_2

$$TRC_2(T) = \begin{cases} TRC_{21}(T) & \text{if} \quad T \ge \frac{M_2}{\alpha} & (4a)\\ TRC_{22}(T) & \text{if} \quad M_2 \le T < \frac{M_2}{\alpha} & (4b)\\ TRC_{23}(T) & \text{if} \quad 0 < T < M_2 & (4c)\end{cases}$$

where

$$TRC_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_k[\frac{DT^2}{2} - (1-\alpha)DTM_2]/T - sI_e\left[\frac{(1-\alpha)DM_2^2}{2}\right]/T \quad (5)$$

$$TRC_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_k[\frac{\alpha^2DT^2}{2} + \frac{D(T-M_2)^2}{2}]/T - sI_e\left[\frac{DM_2(M_2-\alpha^2T)}{2}\right]/T \quad (6)$$

and

$$TRC_{23}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + cI_k(\frac{\alpha^2DT^2}{2})/T - sI_eDT\left[M_2 - \frac{(1+\alpha^2)T}{2}\right]/T \quad (7)$$

Since $TRC_{21}(\frac{M_2}{\alpha}) = TRC_{22}(\frac{M_2}{\alpha})$ and $TRC_{22}(M_2) = TRC_{23}(M_2)$, $TRC(T)$ is continuous and well-defined. All $TRC_{21}(T)$, $TRC_{22}(T)$, $TRC_{23}(T)$ and $TRC_2(T)$ are defined on $T > 0$.

Optimality conditions: From Eq. 2, 3, 5-7 yield

$$TRC_{11}'(T) = \frac{-\{2A + DM_1^2[c(1-r)I_k - sI_e]\}}{2T^2} + \frac{D[h + c(1-r)I_k]}{2}, \quad (8)$$

$$TRC_{11}''(T) = \frac{2A + DM_1^2[c(1-r)I_k - sI_e]}{T^3}, \quad (9)$$

$$TRC_{12}'(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2}, \quad (10)$$

$$TRC_{12}''(T) = \frac{2A}{T^3} > 0, \quad (11)$$

$$TRC_{21}'(T) = \frac{-[2A - s(1-\alpha)DM_2^2I_e]}{2T^2} + \frac{D(h + cI_k)}{2}, \quad (12)$$

$$TRC_{21}''(T)=\frac{2A-s(1-\alpha)DM_2^{\,2}I_e}{T^3}, \quad (13)$$

$$TRC_{22}'(T)=\frac{-[2A+DM_2^{\,2}(cI_k-sI_e)]}{2T^2}+\frac{D[h+cI_k(1+\alpha)^2]}{2}, \quad (14)$$

$$TRC_{22}''(T)=\frac{2A+DM_2^{\,2}(cI_k-sI_e)}{T^3}, \quad (15)$$

$$TRC_{23}'(T)=-\frac{A}{T^2}+\frac{D[h+c\alpha^2I_k+s(1+\alpha^2)I_e]}{2} \quad (16)$$

and

$$TRC_{23}''(T)=\frac{2A}{T^3}>0. \quad (17)$$

Equation 11 and 17 imply that all $TRC_{12}(T)$ and $TRC_{23}(T)$ are convex on $T > 0$. Equation 9 implies $TRC_{11}(T)$ is convex on $T>0$ if $2A+DM_1^{\,2}[c(1-r)I_k-sI_e]>0$. Equation 13 implies $TRC_{21}(T)$ is convex on $T>0$ if $2A-s(1-\alpha)DM_2^{\,2}I_e>0$. Equation 15 implies $TRC_{22}(T)$ is convex on $T>0$ if $2A+DM_2^{\,2}(cI_k-sI_e)>0$.

Furthermore, we have $TRC_{11}'(M_1)=TRC_{12}'(M_1)$, $TRC_{21}'(\frac{M_2}{\alpha})\neq TRC_{22}'(\frac{M_2}{\alpha})$ and $TRC_{22}'(M_2)\neq TRC_{23}'(M_2)$

Therefore, Eq. 1a,b imply that $TRC_1(T)$ is convex on $T>0$ if if $2A+DM_1^{\,2}[c(1-r)I_k-sI_e]>0$ and Eq. 4a-c imply that $TRC_2(T)$ is piecewise convex but not convex in general on $T>0$ if $2A-s(1-\alpha)DM_2^{\,2}I_e>0$.

DECISION RULE OF THE OPTIMAL CYCLE TIME T* AND OPTIMAL PAYMENT TIME

Let $TRC_{11}'(T_{11}^*) = TRC_{12}'(T_{12}^*) = TRC_{21}'(T_{21}^*) = TRC_{22}'(T_{22}^*) = TRC_{23}'(T_{23}^*) = 0$. We can obtain

$$T_{11}^*=\sqrt{\frac{2A+DM_1^{\,2}[c(1-r)I_k-sI_e]}{D[h+c(1-r)I_k]}} \quad \text{if } 2A+DM_1^{\,2}[c(1-r)I_k-sI_e]>0 \quad (18)$$

$$T_{12}^*=\sqrt{\frac{2A}{D(h+sI_e)}}, \quad (19)$$

$$T_{21}^*=\sqrt{\frac{2A-s(1-\alpha)DM_2^{\,2}I_e}{D(h+cI_k)}} \quad \text{if } 2A-s(1-\alpha)DM_2^{\,2}I_e>0 \quad (20)$$

$$T_{22}^*=\sqrt{\frac{2A+DM_2^{\,2}(cI_k-sI_e)}{D[h+cI_k(1+\alpha^2)]}} \quad \text{if } 2A+DM_2^{\,2}(cI_k-sI_e)>0 \quad (21)$$

and

$$T_{23}^*=\sqrt{\frac{2A}{D[h+c\alpha^2I_k+s(1+\alpha^2)I_e)]}}. \quad (22)$$

Equation 18 implies that the optimal value of T for the case of $T \ge M_1$, that is $T_{11}^* \ge M_1$. We substitute Equation 18 into $T_{11}^* \ge M_1$, then we can obtain the optimal value of T

if and only if $-2A+DM_1^{\,2}(h+sI_e)\le 0$.

Likewise, Eq. 19 implies that the optimal value of T for the case of $T < M_1$, that is $T_{12}^* < M_1$. We substitute Eq. 19 into $T_{12}^* < M_1$, then we can obtain the optimal value of T

if and only if $-2A+DM_1^{\,2}(h+sI_e)>0$.

Similar disscussion, we can obain following results:

$M_2/\alpha \le T_{21}^*$ if and only if

$-2A+DM_2^{\,2}[s(1-\alpha)I_e+\frac{h+cI_k}{\alpha^2}]\le 0$.

$M_2 \le T_{22}^* < M_2/\alpha$ if and only if

$-2A+DM_2^{\,2}(sI_e+\frac{h+cI_k}{\alpha^2})>0$ and

if and only if

$-2A+DM_2^{\,2}(h+c\alpha^2I_k+sI_e)\le 0$

$T_{23}^* < M_2$ if and only if

$-2A+DM_2^{\,2}[h+c\alpha^2I_k+s(1+\alpha^2)I_e)]>0$.

Furthermore, we let

$$\Delta_{11}=-2A+DM_1^{\,2}(h+sI_e), \quad (23)$$

$$\Delta_{21}=-2A+DM_2^{\,2}[s(1-\alpha)I_e+\frac{h+cI_k}{\alpha^2}], \quad (24)$$

$$\Delta_{22} = -2A + DM_2^2(sI_e + \frac{h + cI_k}{\alpha^2}), \quad (25)$$

$$\Delta_{23} = -2A + DM_2^2(h + c\alpha^2 I_k + sI_e), \quad (26)$$

and

$$\Delta_{24} = -2A + DM_2^2[h + c\alpha^2 I_k + s(1+\alpha^2)I_e)]. \quad (27)$$

Since $M_1 < M_2$, we can get $\Delta_{22} > \Delta_{21}$, $\Delta_{22} \geq \Delta_{23} > \Delta_{11}$ and $\Delta_{24} \geq \Delta_{23} > \Delta_{11}$ from Eq. 23-27. Summarized above arguments, the optimal cycle time T* and optimal payment time (M_1 or M_2) can be obtained as follows.

Theorem 1:

(A) If $\Delta_{11} \leq 0$, $\Delta_{21} > 0$, $\Delta_{22} > 0$, $\Delta_{23} > 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{$TRC_{11}(T_{11}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{11}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(B) If $\Delta_{11} \leq 0$, $\Delta_{21} > 0$, $\Delta_{22} > 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{22}(T_{22}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{11}^*, T_{22}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(C) If $\Delta_{11} \leq 0$, $\Delta_{21} > 0$, $\Delta_{22} > 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} \leq 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{22}(T_{22}^*)$ }. Hence T* is T_{11}^* or T_{22}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(D) If $\Delta_{11} \leq 0$, $\Delta_{21} \leq 0$, $\Delta_{22} > 0$, $\Delta_{23} > 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{$TRC_{11}(T_{11}^*)$, $TRC_{21}(T_{21}^*)$, $TRC_{23}(T_{23}^*)$}. Hence T* is T_{11}^*, T_{21}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(E) If $\Delta_{11} \leq 0$, $\Delta_{21} \leq 0$, $\Delta_{22} > 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{21}(T_{21}^*)$, $TRC_{22}(T_{22}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{11}^*, T_{21}^*, T_{22}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(F) If $\Delta_{11} \leq 0$, $\Delta_{21} \leq 0$, $\Delta_{22} > 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} \leq 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{21}(T_{21}^*)$, $TRC_{22}(T_{22}^*)$ }. Hence T* is T_{11}^*, T_{21}^* or T_{22}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(G) If $\Delta_{11} \leq 0$, $\Delta_{21} \leq 0$, $\Delta_{22} \leq 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{21}(T_{21}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{11}^*, T_{21}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(H) If $\Delta_{11} \leq 0$, $\Delta_{21} \leq 0$, $\Delta_{22} \leq 0$, $\Delta_{23} \leq 0$ and $\Delta_{24} \leq 0$, then TRC(T*) = min{ $TRC_{11}(T_{11}^*)$, $TRC_{21}(T_{21}^*)$ }. Hence T* is T_{11}^* or T_{21}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(I) If $\Delta_{11} > 0$, $\Delta_{21} > 0$, $\Delta_{22} > 0$, $\Delta_{23} > 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{$TRC_{12}(T_{12}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{12}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

(J) If $\Delta_{11} > 0$, $\Delta_{21} \leq 0$, $\Delta_{22} > 0$, $\Delta_{23} > 0$ and $\Delta_{24} > 0$, then TRC(T*) = min{ $TRC_{12}(T_{12}^*)$, $TRC_{21}(T_{21}^*)$, $TRC_{23}(T_{23}^*)$ }. Hence T* is T_{12}^*, T_{21}^* or T_{23}^*, optimal payment time is M_1 or M_2 associated with the least cost.

CONCLUSIONS

This study extends Chou *et al.*'s[1] model to assume that the unit selling price and unit purchasing price are not necessarily equal. This article reinvestigates the retailer's replenishment and payment policy under supplier offered cash discount and permissible delay in payments linked to retailer payment time. Then, we reformulate the retailer's mathematical inventory replenishment model and provide a very efficient solution procedure to determine the optimal cycle time and optimal payment time for the retailer.

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