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A Practical Implementation of the Process Capability Indices

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Abstract: The process capability indices have been used widely in the manufacturing industry to provide a numerical measure on whether a manufacturing process is capable of producing items meeting the capability requirement preset by the product designer. In this study, the process capability indices $C_p, C_p^*, C_{pk}, C_{pk}^*$ are presented. The estimators of $C_p, C_p^*, C_{pk}, C_{pk}^*$ have been proposed to provide practitioners a way to calculate indices, so that process potential and performance can be measured accurately. Finally, process capability studies are presented from a practical view in a company. Therefore, the logical conclusion was that the company's requirement now appears to be met.

Key words: Process capability indices, practical implementation, specifications limits, target value

INTRODUCTION

A process is a unique combination of machines, tools, methods and personnel engaged in providing a product or service. The output of a process can be product characteristic or process output parameter. Process capability indices provide a common metric to evaluate and predict the performance of processes. There are two commonly used process capability indices, potential process capability index (C_p) and process capability index (C_{pk}). These indices can be used for quality control. They can also be used be used as a communication tool for management as well as between the customer and producer (Kotz and Lovelace, 1998).

During the last couple of years the method process capability studies and the numerical measures of capability used, the so-called process capability indices, have received a somewhat torn reputation. Practitioners have experienced that, sometimes, the values of process capability indices they receive indicate that a process they know as capable might be incapable and vice versa. The reason for this is that estimates of process capability indices are often treated as exact measures of capability of the process and not as the estimates they actually are. Some researchers have also reported on the shortcomings of process capability indices; see, for instance, (Gunter, 1989; Dovich, 1991a,b; Pignatiello and Ranaberg, 1993). As indicated, the criticism has been focused on the specific measures of capability, the process capability indices and not primarily on the method process capability studies, as such. However, the shortcomings of process capability indices are not as severe as the misuse of them, see (Deleryd, 1996; 1999) or in other words, if process capability indices are used properly and with care, they provide valuable information about the capability of a process. The information received is most useful when improving the performance of processes, leading to lower production costs or more satisfied customers, or both (Bothe, 1997).

The process capability indices have been used widely in the manufacturing industry to provide a numerical measure on whether a manufacturing process is capable of producing items meeting the capability requirement preset by the product designer. Several estimators of indices have been proposed to provide practitioners a way to calculate them so that process potential and performance can be measured accurately. In this study, a number of process capability indices and their estimators are presented. Throughout the presentation, it is assumed that the process output is approximately normally distributed and in a state of statistical control. Also, the target value for the process mean is assumed to be the midpoint of the specification limits, unless otherwise stated. Also, a case study has presented on an audio-speaker components manufacturing process that it has developed a simple procedure using capability indices for factory engineers or supervisors to use, to determine whether a process meets the capability requirement. As a result, the capability of the adjusted process was improved significantly and the adjusted process was concluded to be satisfactory.

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PROCESS CAPABILITY INDICES

C_p index: C_p is defined as the ratio of specification width over the process spread. The specification width represents customer and/or product requirements. The process spread represents the process variations. When the process variation is large (more variation), the C_p value is small, indicating a low process capability. So, the process capability index C_p is defined to be

$$C_p = \frac{\text{Allowable process spread}}{\text{Actual process spread}} = \frac{\text{USL - LSL}}{6\sigma}$$

where, USL, LSL and σ denote the upper specification limit, lower specification limit and process standard deviation associated with the measurements, respectively. A process is said to be capable if the value of C_p associated with the process is at least 1.0 (Kane, 1986). Since the process standard deviation is rarely known, it is estimated from a sample of n measurements $X_1 \ldots X_n$ and an estimate $\hat{C}_p$ of the process capability C_p is obtained by:

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}$$

Typically, the sample standard deviation

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right]^{1/2}$$

is used to estimate σ (where, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$).

C_p^* index: The C_p^* index is the generalization of C_p to the case where the target value T for the process mean is not necessarily equal to the midpoint m of the specification limits and is defined by

$$C_p^* = \frac{\min\,(\text{T - LSL, USL - T})}{3\sigma}$$

An estimate of C_p^* can be obtained by

$$\hat{C}_p^* = \frac{\min\,(\text{T - LSL, USL - T})}{3\text{S}}$$

Note that when T = m, $C_p^* = C_p$ and $\hat{C}_p^*$.

C_{pk} index: The process capability index C_{pk} is used to provide an indication of the variability associated with a process and how a process has conformed to its specifications. The index is usually used to relate the "natural tolerance" (3σ), to the specification limits. Different from C_p, C_{pk} describes how well the process fits within the specification limits, taking into account the location of the process mean. So, the process capability index C_{pk} is defined as

$$C_{pk} = \min\left(\frac{\mu - \text{LSL}}{3\sigma}, \frac{\text{USL} - \mu}{3\sigma}\right)$$

C_{pk} describes a distance scaled by 3σ, between the process mean and the closest specification limit. And

$$\hat{C}_{pk} = \min\left(\frac{\bar{X} - \text{LSL}}{3\text{S}}, \frac{\text{USL} - \bar{X}}{3\text{S}}\right)$$

C_{pk}^* index: The C_{pk}^* index is the generalization of C_p to the case where the target value T for the process mean is not necessarily equal to the midpoint m of the specification limits and is defined to be

$$C_{pk}^* = \min\left(\text{CPL}^*, \text{CPU}^*\right)$$

that

$$\text{CPL}^* = \begin{cases} \dfrac{\text{T} - \text{LSL}}{3\sigma} \cdot \left(1 - \dfrac{|\text{T} - \mu|}{\text{T} - \text{LSL}}\right), & \text{if } |\text{T} - \mu| > \text{T} - \text{LSL} \\ 0, & \text{otherwise} \end{cases}$$

and

$$\text{CPU}^* = \begin{cases} \dfrac{\text{USL} - \text{T}}{3\sigma} \cdot \left(1 - \dfrac{|\text{T} - \mu|}{\text{USL} - \text{T}}\right), & \text{if } |\text{T} - \mu| > \text{USL} - \text{T} \\ 0, & \text{otherwise} \end{cases}$$

An estimate of C_{pk}^* can be obtained by

$$\hat{C}_{pk}^* = \min\left(\widehat{\text{CPL}}^*, \widehat{\text{CPU}}^*\right)$$

that

$$\widehat{\text{CPL}}^* = \begin{cases} \dfrac{\text{T} - \text{LSL}}{3\text{S}} \cdot \left(1 - \dfrac{|\text{T} - \bar{X}|}{\text{T} - \text{LSL}}\right), & \text{if } |\text{T} - \bar{X}| > \text{T} - \text{LSL} \\ 0, & \text{otherwise} \end{cases}$$

and

$$\widehat{\text{CPU}}^* = \begin{cases} \dfrac{\text{USL} - \text{T}}{3\text{S}} \cdot \left(1 - \dfrac{|\text{T} - \bar{X}|}{\text{USL} - \text{T}}\right), & \text{if } |\text{T} - \bar{X}| > \text{USL} - \text{T} \\ 0, & \text{otherwise} \end{cases}$$

Note that when $\mu = \text{T}, C_{pk}^* = C_p^*$. Also, when $\text{T} = \text{m}, C_{pk}^* = C_{pk}$ (Ramakrishnan *et al*., 2001; Warrior, 1990).

INTERPRETING C_p, C_{pk}

C_{pk} is an index which measures how close a process is running to its specification limits, relative to the natural variability of the process. The larger the index, the less likely it is that any item will be outside the specs. If you hunt our shoot targets with bow, darts, or gun try this analogy. If your shots are falling in the same spot forming a good group this is a high C_p and when the sighting is adjusted so this tight group of shots is landing on the bullseye, you now have a high C_{pk}. C_{pk} measures how close you are to your target and how consistent you are to around your average performance. A person may be performing with minimum variation, but he can be away from his target towards one of the specification limit, which indicates lower C_{pk}, whereas C_p will be high. On the other hand, a person may be on average exactly at the target, but the variation in performance is high (but still lower than the tolerance band (i.e. specification interval). In such case also C_{pk} will be lower, but C_p will be high. C_{pk} will be higher only when you are meeting the target consistently with minimum variation.

Consider a car and a garage. The garage defines the specification limits; the car defines the output of the process. If the car is only a little bit smaller than the garage, you had better park it right in the middle of the garage (center of the specification) if you want to get all of the car in the garage. If the car is wider than the garage, it does not matter if you have it centered; it will not fit. If the car is a lot smaller than the garage (six sigma process), it doesn't matter if you park it exactly in the middle; it will fit and you have plenty of room on either side. If you have a process that is in control and with little variation, you should be able to park the car easily within the garage and thus meet customer requirements. C_{pk} tells you the relationship between the size of the car, the size of the garage and how far away from the middle of the garage you parked the car (Fieler and Loverro, 1991).

The value itself can be thought of as the amount the process (car) can widen before hitting the nearest spec limit (garage door edge).

C_{pk} = 1/2 means you've crunched nearest the door edge
C_{pk} = 1 means you're just touching the nearest edge
C_{pk} = 2 means your width can grow 2 times before touching
C_{pk} = 3 means your width can grow 3 times before touching

The general rule of thumb states that if a C_{pk} value of a process is less than 1.33, then the process is incapable of producing a repeatable part.

CASE STUDY: PROCESS CAPABILITY INDICES

As a discussion example, we consider the process capability and process yield study. This study involved a manufacturer and supplier of audio-speaker components. The weight of the rubber edge has studied, which is one of the key components that reflect the sound quality of driver units. The data on rubber edge are given in Table 1.

The USL, LSL and target value were 8.94, 8.46 and 8.70, respectively, the target value being set equal to the midpoint between the specification limits. The company had been using C_p and C_{pk} as process capability indicators.

Computing

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}},$$

with

$$\hat{\sigma} = \frac{s}{c_4}$$

and

$$c_4 = \frac{4(n-1)}{(n-3)}$$

used to approximate the value of c_4 for n = 8, we obtain $\hat{C}_p = 1.53$. If we used the estimator of C_p given in section 2, we would obtain 1.52.

Table 1: Process capability data

The weight of the rubber edge (g)			
8.63	8.65	8.57	8.73
8.65	8.58	8.64	8.70
8.57	8.65	8.63	8.65
8.57	8.67	8.57	8.56
8.54	8.67	8.61	8.66
8.69	8.65	8.59	8.65
8.63	8.69	8.56	8.66
8.64	8.66	8.71	8.68
8.59	8.62	8.53	8.62
8.61	8.63	8.51	8.54
8.60	8.59	8.72	8.67
8.66	8.65	8.58	8.62
8.65	8.64	8.64	8.54
8.50	8.64	8.69	8.62
8.61	8.52	8.64	8.66
8.61	8.69	8.75	8.56
8.63	8.66	8.59	8.60
8.67	8.66	8.61	8.62
8.54	8.61	8.58	8.61
8.62	8.55	8.65	8.66

We will use s/c_4 e , but since $c_4 = 0.997$ when n = 80. It can be shown that $\bar{x} = 8.62$ and s = 0.05. Using this values, we can $C_{pk} = 1.04$.

The company's requirement is $1.00 \le C_{pk} \le 1.33$. Even though $\hat{C}_{pk}$ exceeds the lower bound on acceptable values of C_{pk}, the lower bound is just barely exceeded, so we know there is a substantial probability that the true value of C_{pk} is less than 1.0.

Machine settings were subsequently adjusted and when a new sample of 80 observations was obtained, $\bar{x} = 8.67$ and s = 0.05. Since the new sample mean is much closer to the midpoint between the specification limits than was the pervious sample mean, the value of $\hat{C}_{pk}$ will be considerably greater. It can be shown that $\hat{C}_{pk} = 1.4$. Since this exceeds the upper bound on acceptable values of C_{pk}, we would expect to conclude that the true value is probably greater than the lower bound. Therefore, the logical conclusion is that the company's requirement now appears to be met. The proposed procedure is applied to the case of the manufacturing process which we studied. Problems causing the process failing to meet the capability requirement were identified. Quality improvement activities were conducted and machine settings were adjusted. As a result, the capability of the adjusted process was improved significantly and the adjusted process was concluded to be satisfactory.

CONCLUSIONS

Process capability index C_{pk} has been the most popular index used in the manufacturing industry as a process performance measure. Process capability indices, C_p and C_{pk} are useful, so long as the fundamental underlying assumptions, such as normal distribution, stable process and variable data, are known. If two processes are identical except one with higher C_{pk} than another, then the process yield with higher C_{pk} will be higher than the process with lower C_{pk}. Since C_p does not take the process mean into account and reflect the non-conformance rejects, the conclusion does not apply to C_p. However C_{pk} cannot replace yield, unless all other conditions and processes are the same.

When the C_p and C_{pk} indices are examined jointly, some critical information about the process capability is revealed. Since C_{pk} is indirectly related to the percentage of non-conforming products, it may be used as an estimation of the process capability to produce products that conform to specifications in terms of the specific process characteristics. C_p, when shown together with C_{pk}, gives a quick estimate of how far the actual process is from its target and thus, how much work is needed to bring the process to its potential. Finally, process capability studies are presented from a practical view in a company. Therefore, the logical conclusion was that the company's requirement now appears to be met.

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