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Applications of Process Capability and Process Performance Indices

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Abstract: A process is a unique combination of machines, tools, methods and personnel engaged in providing a product or service. The output of a process can be product characteristic or process output parameter. Process capability indices (C_p, C_{pk}, C_{pm}) provide a common metric to evaluate and predict the performance of processes. In this study, at the first, the process capability indices are presented. Then machine capability indices are discussed. Finally, process performance indices P_p, P_{pk}, P_{pm} are presented.

Key words: Process capability, machine capability, process performance, indices

INTRODUCTION

You determine capability by comparing the width of the process variation with the width of the specification limits. The process needs to be in control before you assess its capability; if it is not, then you will get incorrect estimates of process capability. A process is a unique combination of machines, tools, methods and personnel engaged in providing a product or service. The output of a process can be product characteristic or process output parameter. Process capability indices provide a common metric to evaluate and predict the performance of processes. You can assess process capability graphically by drawing capability histograms and capability plots. These graphics help you assess the distribution of your data and verify that the process is in control. You can also calculate capability indices, which are ratios of the specification tolerance to the natural process variation. Capability indices, or statistics, are a simple way of assessing process capability. Because they are unitless, you can use capability statistics to compare the capability of one process to another.

Process capability compares the output of an in-control process to the specification limits by using capability indices. The comparison is made by forming the ratio of the spread between the process specifications (the specification "width") to the spread of the process values, as measured by 6 process standard deviation units (the process "width"). Process capability attempts to answer the question: can we consistently meet customer requirements? The number one limitation of process capability indices is that they are meaningless if the data is not from a controlled process. The reason is simple: process capability is a prediction and you can only predict something that is stable.

Process capability indices, as a process performance measure, have become very popular in assessing the capability of manufacturing processes in practice during the past decade. More and more efforts have been devoted to studies and applications of process capability indices. For example, Rado (1989) demonstrated how imprimis technology, Inc. used the process capability indices for program planning and growth to enhance product development. The C_p and C_{pk} indices have been used in Japan and in the US automotive industry such as Ford Motor Company (Kane, 1986a, b). For more information on Process capability indices (Kotz and Johnson, 1993; Kotz *et al*., 1993).

Because the sample mean and the sample variance S^2 are not unitless, they can be cumbersome as summary statistics for the process location and variance, respectively. This is particularly true in a manufacturing process with many characteristics to be examined. Capability indices are unitless and associate the process location and variance with one-sided or two-sided specifications, with or without a target value for the process mean. These indices provide an effective means for communicating assessments of the process capability. The capability indices relate the manufacturer's specifications to the natural tolerance of six standard deviations used in US quality control literature.

In this study, at the first, a number of the process capability indices (C_p, C_{pk}, C_{pm}) are presented. Then machine capability indices are discussed. Finally, process performance indices P_p, P_{pk}, P_{pm} are presented. Also, differences between process capability and process performance indices are explained.

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PROCESS CAPABILITY INDICES

Process capability studies are used for monitoring the capability of a process. This implies that it has to be based on some sort of collection of data from the process. In order to get a fair picture of the capability of the process, it has to be stable when the data is collected. After the collection of data from a stable process, the data may be assessed in several ways. One way to do the assessment is to use process capability indices, which provide numerical measures of the capability.

C_p index: The process capability index C_p is defined to be

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL, LSL and σ denote the upper specification limit, lower specification limit and process standard deviation associated with the measurements, respectively. A process is said to be capable if the value of C_p associated with the process is at least 1.0 (Kane 1986a; Breyfogle, 1996). Since the process standard deviation is rarely known, it is estimated from a sample of n measurements X_1 ... X_n and an estimate $\hat{C}_p$ of the process capability C_p is obtained by:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

Typically, the sample standard deviation

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right]^{1/2}$$

is used to estimate σ

$$\left(\text{where } \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i\right)$$

C_{pk} index: The process capability index C_{pk} is defined as:

$$C_{pk} = Min(CPL, CPU)$$

The lower process capability index (CPL) is defined to be:

$$CPL = \frac{\mu - LSL}{3\sigma}$$

and the upper process capability index (CPU) is defined to be

$$CPU = \frac{USL - \mu}{3\sigma}$$

Estimates of CPL and CPU, respectively, are obtained by and

$$\hat{CPL} = \frac{\bar{X} - LSL}{3S} \text{ and } \hat{CPU} = \frac{USL - \bar{X}}{3S}$$

Note that

$$C_p = \frac{(CPL + CPU)}{2}, \quad \hat{C}_p = \frac{\left(\hat{CPL} + \hat{CPU}\right)}{2}$$

The CPU index was developed in Japan and is utilized by a number of Japanese companies (Kane, 1986a). C_{pk} describes a distance scaled by 3σ, between the process mean and the closest specification limit. Assuming that μ is between the specification limits, let

$$k = 2|m - \mu| / (USL - LSL)$$

where, m is the midpoint of the specification limits. The ratio k is used in Japan as an index describing the amount that the process mean is off-center (Kane, 1986a). It is easily seen that if $0 \le k \le 1$, then $C_{pk} = (1-k)C_p$

From this equation it is seen that C_{pk} is C_p reduced by the factor (1-k), where k is a scaled distance between the process mean and the midpoint of the specification limits. The indices k and C_{pk} are estimated by

$$\hat{k} = 2\left|m - \bar{X}\right| / (USL - LSL), \ \hat{C}_{pk} = \min\left(\hat{CPL}, \hat{CPU}\right) = \left(1 - \hat{k}\right)\hat{C}_p,$$

respectively. Note that when μ = m, $C_{pk} = C_p$.

Now we have introduced the C_p and the C_{pk}. By studying the formulas it is easy to see that C_p only relates the tolerance interval to the process 6σ. C_{pk} also considers the targetvalue. We want both Cp and C_{pk} to be higher than 1.33. If our average is right on target, the C_p and C_{pk} are the same. The more off target we are, the bigger the difference between C_p and C_{pk}. Obviously C_{pk} can never be higher that C_p. the relation between C_p and C_{pk} is presented in Fig .1.

C_{pm} index: Chan *et al.* (1988) proposed C_{pm} as process capability index. The index C_{pm} is defined to be

$$C_{pm} = \frac{USL - LSL}{6\sigma'}$$

where USL = upper specification limit, LSL = lower specification limi, = $(E(X - T)^2)^{1/2}$ and This the target value for the process mean. Since the parameter is typically

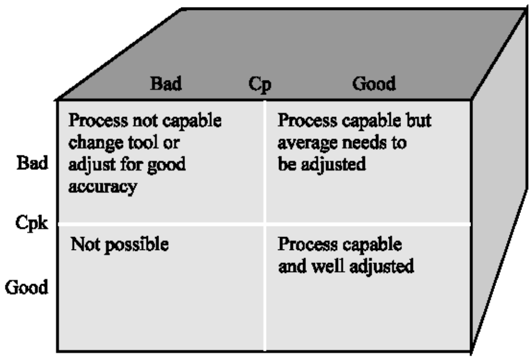

Fig. 1: The relation between C_p and C_{pk}

unknown, Chan *et al.* (1988) suggest that a sample of n measurements be used to estimate by

$$\hat{\sigma}' = \left(\sum_{i=1}^{n} \frac{(X_i - T)^2}{n-1} \right)^{1/2}$$

Consequently, an estimator for the C_{pm} index is

$$\hat{C}_{pm} = \frac{USL - LSL}{6\hat{\sigma}'}$$

WHEN IS A PROCESS CAPABLE

The question of "how good is capable?" has still not been definitively answered. Since C_p was first used, a C_p value of 1.33 has become the most commonly acceptable criterion as a lower boundary. The C_{pk} requirements vary. The most common is that C_{pk} has to be greater than 1.33. A process that has a C_{pk} lower than 1.00 is never capable.

It is very important that you understand why we use both the C_p and the C_{pk}. If we only use the C_p, we do not know whether we are on target or not. If we only use the C_{pk}, we cannot know whether a good or bad C_{pk} value is because of the centering of the process or because of the spread. So we have to use both. Together they can give us a very good indication of how well a specific tool is performing in a specific application. They are also the perfect way to compare different tools (Bissell, 1990).

MACHINE CAPABILITY INDICES

Process capability can be defined as the spread within which most of the part values within a distribution will fall, generally described as within plus or minus three standard deviations (±3σ). This baseline definition enables us to compare process capability under real manufacturing conditions with specification tolerances. Process capability is calculated over a long period of time and is influenced by the manufacturing environment. Machine capability, on the other hand, is calculated within a short period. The impact of all materials and parts used must be eliminated. This is also called "short time capability". The examination of machine capability is used to audit the quality behavior of a single machine.

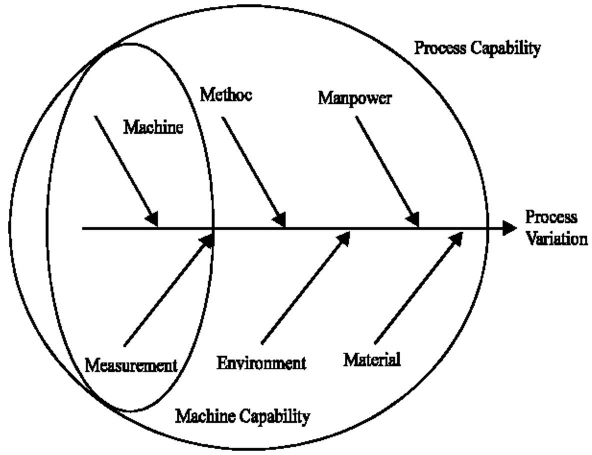

Fig. 2: Ishikawa diagram

As you now know, C_p and C_{pk} are process capability indices. Everything that affects the process affects these indices. But if we take away all variation affecting the assembly process, except the variation in the tool itself, we get what are called machine capability indices. This must be done under very controlled circumstances, preferably in a tool crib. The tests should be carried out on the same joint and by the same operator (or even better, place the tool in a fixture in order to get rid of all the operator influence). The calculations are the same for C_m as for C_p and the same for C_{mk} as for C_{pk}. So remember, C_p and C_{pk} determine whether the process is capable. The C_m and C_{mk} determine whether the machine (tool) is capable.

A process can be viewed as a series of actions or operations influenced by several elements or factors, all contributing to the eventual outcome. These elements or causes of variation can be generally broken into the following typical categories:

- Material
- Machine
- Method
- Manpower
- Environment
- Measurement

Each of these elements contributes some degree of variability to the process. Nevertheless, when we are talking about machine capability, we only determine spread and centering of machine parameters independent of the influence of other factors. This can be represented by the Ishikawa diagram (Fig. 2).

PROCESS PERFORMANCE

Process performance attempts to answer the question: does this sample from the process meet customer requirements?

P_p = Process performance index: A simple and straightforward indicator of process performance.

P_{pk} = Process performance index: Adjustment of P_p for the effect of non-centered distribution.

Process performance index basically tries to verify if the sample that you have generated from the process is capable to meet customer CTQs (requirements). It differs from process capability in that process performance only applies to a specific batch of material. Samples from the batch may need to be quite large to be representative of the variation in the batch. Process performance is only used when process control cannot be evaluated. An example of this is for a short pre-production run. Process performance generally uses sample sigma in its calculation; process capability uses the process sigma value determined from either the moving range, range, or sigma control charts (Pearn and Lin, 2004).

P_p index: P_p = Process performance index: The ratio of the tolerance (specification, or permitted amount of variation) to the variation in a sample. A value of 1 indicates the sample variation exactly equals the tolerance. Values of less than 1 indicate the allowable variation (the tolerance) is less than the sample variation. Values of more than 1 indicate that the sample variation is less than the tolerance. Use of performance indices is generally discouraged in favor of capability indices wherever possible. Your process (its natural limits) relative to its specification limits (customer requirements). Following are the graphical details and equations quantifying process capability

Normal distributions:

$$P_p = \frac{\text{HighSpec} - \text{LowSpec}}{6\sigma_X}$$

Best fit (Johnson) and True-Position (Rayleigh) distributions:

$$P_p = \frac{\text{HighSpec} - \text{LowSpec}}{\text{ordinate}_{0.99865} - \text{ordinate}_{0.00135}}$$

Folded normal distributions:

$$P_p = \frac{\text{ordinate}_{0.99865}}{3}$$

Where, x is sample sigma. $\text{Ordinate}_{0.99865}$ and $\text{ordinate}_{0.00135}$ are the z-values of the non-normal cumulative distribution curve at the 99.865% point and the 0.135% points, respectively.

P_{pk} index: P_{pk} = Process performance index: A measure of both process dispersion and its centering about the average.

$$P_{pk} = \text{Min}[\text{Ppl}, \text{Ppu}]$$

Where,

$$P_{pl} = -\frac{Z_L}{3},\ P_{pu} = -\frac{Z_U}{3}$$

Normal distributions:

$$Z_L = \frac{\bar{\bar{X}} - \text{LowSpec}}{\sigma_X}, Z_U = \frac{\text{HighSpec} - \bar{\bar{X}}}{\sigma_X}$$

Non-Normal Distributions:

$$Z_L = Z_{\text{normal, p}}, Z_U = Z_{\text{normal,1-p}}$$

Where, x-double bar is the grand average and x is sample sigma. $Z_{\text{normal, p}}$ and $Z_{\text{normal,1-p}}$ are the z-values of the normal cumulative distribution curve at the p percentage point and the 1-p percentage points, respectively.

P_r index: P_r = Process performance index: A measure of the percentage of the tolerance actually used by the sample. Smaller numbers are best.

$$P_r = 100 / P_p$$

Where, P_p is the process performance index.

P_{pm} index: P_{pm} = Process performance index: A measure similar to the P_{pk} index that also takes into account variation between the process average and a target value. If the process average and the target are the same value, P_{pm} will be the same as P_{pk}. If the average drifts from the target value, P_{pm} will be less than P_{pk}.

$$P_{pm} = \frac{P_p}{\sqrt{1 + \frac{(\bar{\bar{x}} - T)^2}{\sigma_X^2}}}$$

Where, T is the process target, x-doublebar is the grand average and x is sample sigma.

DIFFERENCES BETWEEN C_{PK} AND P_{PK}

It differs from process capability in that process performance only applies to a specific batch of material. Samples from the batch may need to be quite large to be representative of the variation in the batch. C_{pk} is for short term, P_{pk} is for long term. P_{pk} produces an index number (like 1.33) for the process variation. C_{pk} references the variation to your specification limits. If you just want to know how much variation the process exhibits, a P_{pk} measurement is fine. If you want to know how that variation will affect the ability of your process to meet customer requirements (CTQ's), you should use C_{pk}. It could be argued that the use of P_{pk} and C_{pk} (with sufficient sample size) are far more valid estimates of long and short term capability of processes since the 1.5 sigma shift has a shaky statistical foundation. C_{pk} tells you what the process is CAPABLE of doing in future, assuming it remains in a state of statistical control. P_{pk} tells you how the process has performed in the past. You cannot use it predict the future, like with C_{pk}, because the process is not in a state of control. The values for C_{pk} and P_{pk} will converge to almost the same value when the process is in statistical control. That is because Sigma and the sample standard deviation will be identical (at least as can be distinguished by an F-test). When out of control, the values will be distinctly different, perhaps by a very wide margin.

C_p and C_{pk} are for computing the index with respect to the sub grouping of your data (different shifts, machines, operators, etc.), while P_p and P_{pk} are for the whole process (no sub grouping). For both P_{pk} and C_{pk} the 'k' stands for "centralizing facteur"- it assumes the index takes into consideration the fact that your data is maybe not centered (and hence, your index shall be smaller). It is more realistic to use P_p and P_{pk} than C_p or C_{pk} as the process variation cannot be tempered with by inappropriate sub grouping. However, C_p and C_{pk} can be very useful in order to know if, under the best conditions, the process is capable of fitting into the specs or not. It basically gives you the best case scenario for the existing process. C_p should always be greater than 2.0 for a good process which is under statistical control. For a good process under statistical control, C_{pk} should be greater than 1.5. As for P_{pk}/C_{pk}, they mean one or the other and you will find people confusing the definitions and you will find books defining them versa and vice versa. You will have to ask the definition the person is using that you are talking to. I just finished up a meeting with a vendor and we had a nice discussion of C_{pk} vs. P_{pk}. We had the definitions exactly reversed between us. The outcome was to standardize on definitions and move forward from there. My suggestion to others is that each company has a procedure or document (we do not) which has the definitions of C_{pk} and P_{pk} in it. This provides everyone a standard to refer to for when we forgot or get confused.

CONCLUSIONS

Process capability can be defined as the spread within which most of the part values within a distribution will fall, generally described as within plus or minus three standard deviations ($\pm 3\sigma$). Capability indices help to prevent production of items which do not meet the specification requirements and help to continuously monitor product improvement. Finally, indices provide a common language for successful communication of information about the process capability and location. In order to estimate process capability, you must know the location, spread and shape of the process distribution. These parameters are, by definition, changing in an out of control process. Therefore, only use process capability indices if the process is in control for an extended period. Machine capability, on the other hand, is calculated within a short period. The impact of all materials and parts used must be eliminated. This is also called "short time capability". The examination of machine capability is used to audit the quality behavior of a single machine. Machine capability, on the other hand, is calculated within a short period. The impact of all materials and parts used must be eliminated. This is also called "short time capability". The examination of machine capability is used to audit the quality behavior of a single machine. Process performance is only used when process control cannot be evaluated.

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