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A Study of Moving Hydraulic Jump in Rectangular Channels

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Abstract: The occurrence of unsteady super and subcritical flow regimes is considered as a special case in unsteady flow in a channel reach which can clearly be exemplified by a moving hydraulic jump. Some complexity is inherent in the numerical analysis of such flow condition and published experimental data regarding this kind of flow are scarce. In this research an experimental setup was prepared to compile the hydraulic behavior of a moving hydraulic jump in a rectangular flume. The flume was equipped with a sluice gate at the upstream boundary. A temporal water stages were generated at the upstream side of the gate in order to provide unsteady supercritical flow regime at its downstream side and moving hydraulic jumps along the channel for the different downstream end boundary conditions. Based on the recorded data, some flow parameters such as flow depth, pressure head and energy head were determined. The results reveal the presence of relationships between the discharge and moving hydraulic jump parameters. Also, by applying proper assumptions and using steady state momentum and energy equations, simple and time independent relationships were obtained, that reasonably determines the pressure head in subcritical region of unsteady mixed flow. Consequently, having the discharge variation as a boundary condition, the moving hydraulic jump parameter could reliably be determined based on time independent relationships.

Key words: Jump parameters, subcritical region, temporal water stages

INTRODUCTION

Because of numerous inherent advantages, numerical simulations have been extensively used for studying the flow behavior and predicting flow conditions in vast relevant area of hydraulic and water resources engineering; yet there are still many issues of great practical interest that have not been fully covered by these powerful means. In such cases physical models provide simple and practical solutions. They also grant reliable data which could be used along with field measurements to verify numerical models. Several numerical algorithms were presented so far for studying unsteady flow in open channel networks. These algorithms should be able to deal with all kind of flow conditions that could likely occur in field conditions. However, one of the well known cases that still require further investigations and studies is the moving hydraulic jump in which changes between supercritical and subcritical flow regimes occur. Consequently, the availability of sufficient data regard the behavior of this kind flow in different circumstances would provide insight understanding of the phenomenon and make it possible to develop robust numerical schemes or at least improves current schemes.

The occurrence of moving hydraulic jump in field conditions is quite often. There are verity of hydraulic structures such as control gates, weirs and culverts in an irrigation network that generate, accommodate or convey moving hydraulic jump during their normal operation or at a given discharge range. For instance, Fig. 1 shows a series of control structures in a channel reach that produce this kind of flow. In this condition changes in the discharge at the channel head or at the lateral offtakes, which is a common practice due to water delivery schedule, affect the water surface profile and force the transcritical flow front to move. Also, the incidence of moving hydraulic jump during a flood event is most likely to occur in natural waterways especially where obstructing structures such as bridges are placed along the channel. Changes in the channel longitudinal profile or variations in boundary roughness might also lead to the presence of unsteady mixed flow regimes. Moreover the concordance of subcritical and supper critical flow is usually observed in a channel reach after a dam failure. These cases clearly demonstrate the frequent occurrence of mixed flow regimes and their practical importance in field conditions and illuminate the needs for further studies to obtain the required knowledge to deal with them.

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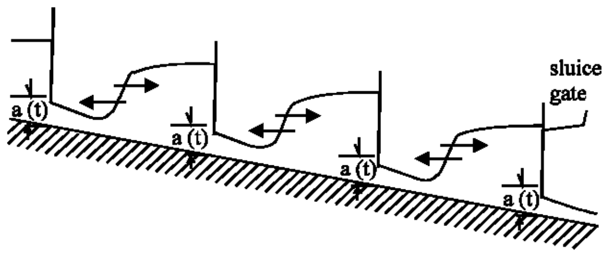

Fig. 1: Operation of consecutive sluice gates in a channel reach (Meselhe, 1994)

The main feature of mixed flow regimes, which should be paid a special attention when modeling this kind of flow, is the different directions of wave propagation in sub and suppercritical flows. An appropriate numerical solution to simulate such flow cases should be able to capture the shock and discontinuity of the flow, to take into account the direction of the wave propagations and to consider suitable boundary condition for each flow regime. A number of finite difference shock capturing schemes have been presented for solving the system of governing equations in aerodynamics (Kim and Han, 2000). Because of the similarity between shallow water equations and the Naver-Stockes equations for compressible flow, attempts have been done to apply those schemes for solving the Saint-Venant equations and capturing the shock (Martin and Zovne, 1971; Fennema and Chaudhry, 1986, 1990; Garcia-Navaro and Saviron, 1992; Meselhe *et al.*, 1997; Jin and Fread, 1997; Tseng, 1999; Yost and Rao, 2001; Tseng *et al.*, 2001). Although, the Preissmann numerical scheme is considered a standard one for solving one dimensional open channel flow and applicable to sub and supper critical flow regimes individually; It could not be employed for mixed flow conditions (Meselhe and Holly, 1997). In addition to numerical difficulties associated with the presence of both sub and supercritical flows, their experimental investigations also require special laboratory treatments to compile the required temporal data.

The one dimensional flow governing equations are also applicable to flow having hydraulic discontinuity and hydraulic jumps. The conservation form of these equations for horizontal channel having negligible friction is usually presented as follows (Cunge *et al.*, 1980):

$$\frac{\partial U}{\partial t} + A'\frac{\partial U}{\partial x} = 0.0 \qquad (1)$$

Where $U = \begin{bmatrix} A \\ Q \end{bmatrix}; A' = \begin{bmatrix} 0 & 1 \\ c^2 - V^2 & 2V \end{bmatrix}$; A = flow cross section; Q = Q(x,t) = discharge; V = flow velocity; $c = (gA/B)^{0.5}$ = wave celerity and B = channel width at the flow surface. The eigenvalues of this matrix are λ_1 = V+c and λ_2 = V-c, which take different signs based on the Froude number value, $Fr = V/(gA/B)^{0.5}$, in sub and supercritical flow conditions.

Basically, if a discontinuity in the flow state variables is encountered in a flow case, the differential form of the governing equation could not be used to describe the flow condition (Meselhe and Holly, 1997). While the integral form of these equations could be employed whether or not a discontinuity presents in the solution domain. However, some numerical difficulties prevented the presence of proper solution in many commercial flood routing models (Jin and Fread, 1997; Zhang and Summer, 1999). Recording the required data for this kind of flow in field conditions is extremely difficult. Therefore, compiling relevant and reliable data by means of experimental setups to study this phenomenon and to test and verify numerical simulations is very essential. However, the presence of such data was clamed very scarce and bears limitations (Gharangik and Chaudhry, 1991; Zhang and Summer, 1999; Tseng, 1999). Therefore, attempts in this regards will improve the understanding of the phenomenon and seem necessary. In this research an experimental setup was constructed and used to investigate unsteady mixed flow regimes, i.e. moving hydraulic jumps. Analyzed Data revealed the presence of specific relationships between steady and unsteady states variables in this flow condition.

MATERIALS AND METHODS

The experimental setup consisted of a rectangular tilting flume 9 m long, 0.25 m wide and 0.5 m deep. The sidewalls of the flume were constructed of clear glass and the flume invert was made of steel and equipped with manometers along its centerline. At the downstream end of the flume a rotating flap tail gate was installed to control the flow depth along the flume. From a large constant head reservoir water was supplied to the flume entrance by a pipe equipped with a control valve. From the downstream end of the flume water entered an underground reservoir and then it was circulated by means of a pump to the constant head reservoir. Within the flow circulation path a rectangular weir was installed and used for flow measurement and calibration. A sluice gate was installed inside the flume at 0.7 m distance from its entrance to provide the required condition for generating moving hydraulic jump at its downstream side. The water surface elevation at the upstream side of the sluice gate was recorded by means of low head pressure transducer which was connected to a data acquisition system designed for this purpose. To eliminate the

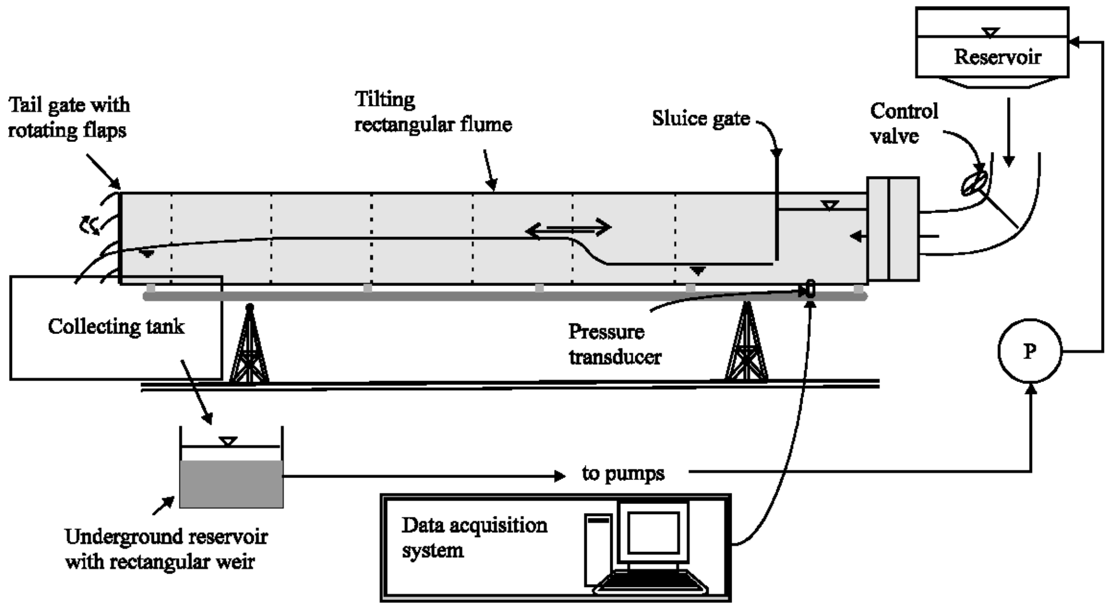

Fig. 2: A general presentation of the experimental setup

vibration effects on the transducer records, it was connected to a manometer tube installed at a safe distance from the flume and calibrated in place prior to its use. The sluice gate was calibrated during the first stage of the experimental program and its stage-discharge curve was obtained. An average value of 0.62 for the contraction coefficient of the gate was obtained which concords with that reported in the literature (Henderson, 1966; Bos, 1989). The moving hydraulic jump was recorded by a video camera and the required data -including the spatial and temporal water surface profile, and flow depth-were determined by digitizing the records. Figure 2 presents an overall sketch of the experimental setup employed in the current study.

Since the sluice gate was calibrated in place it facilitated recording flow hydrographs generated at its upstream side. Based to the minimum flow rate, the gate opening was chosen so that a hydraulic jump formed at the gate vicinity. Accordingly, there would be a sufficient length in the downstream direction to accommodate the moving hydraulic jump during the rising stage of the hydrographs.

The flow hydrograph of each run was produced by continues and gradual movements of the control valve mounted on the upstream supplying pipe. The valve was gradually opened to a maximum value in order to generate the rising stage of each hydrograph; during this stage the jump traveled downward and then moved back towards the upstream sluice gate during the falling stage of the hydrograph. The employed experimental setup facilitated complete records of the required unsteady flow data.

Data analysis: For each run a complete set of data was recorded. The data for instants at which the flow parameters showed apparent changes in their values due to changes in the boundary conditions, were considered and tabulated for further analysis. In addition to the measured unsteady flow parameters such as discharge at the gate position, Q_g, super and subcritical flow depths directly upstream and downstream of the jump front, y_{1U} and y_{2U}, which were tabulated for specified moments, the steady conjugate depth, y_{2S}, associated with the measured unsteady supercritical flow depth, y_{1U}, was also calculated based on the well known steady state Bélanger equation and included in the tabulation. In this regard the difference between y_{2S} and y_{2U} was considered as a factor that influences the moving jump characteristics. Deep inspection of the data indicated that the flow depth form the gate outlet to the position of the moving jump remained almost unchanged during each run. Therefore, the spatial variation of discharge, $\partial Q/\partial x$, along the mentioned reach was considered nil and ignored.

In addition to the variables mentioned earlier it seems appropriate to define all the variables used herein in order to present an overall view and eliminate any ambiguity. Hence, they are listed as follows: flow depth, y; average flow velocity, V; discharge at a given section and instant, Q = Q(t); Froude number at a given section, Fr; total energy of unit weight of flow at a specified section, H; pressure force at a specified section, F_P; and unit weight of water, γ. The subscripts 1 and 2 associated the pertinent variable with super and subcritical flow directly upstream and downstream of the moving jump front, respectively.

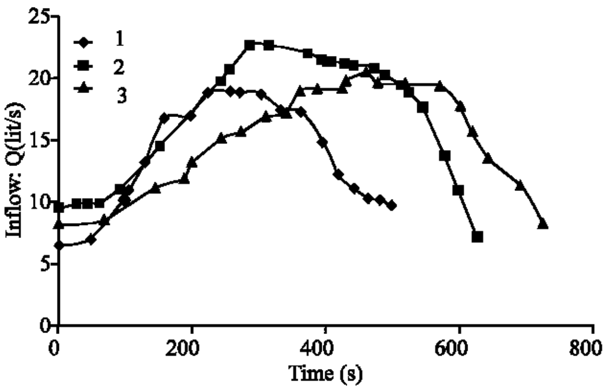

Fig. 3: Recorded hydrographs

Also, the subscripts S and U indicated steady and unsteady state conditions, respectively.

As mentioned earlier Bélanger equation was used to evaluate y_{2S}:

$$y_{2S} = \frac{1}{2} y_1 \left(\sqrt{1+8Fr_1^2} - 1 \right) \quad (2)$$

If the pressure distribution at section 2 is considered hydrostatic and the discharge variation along the relatively short distance between sections 1 and 2 is ignored, the total energy at section 2 could be determined based on measured flow parameters and steady state flow parameters by Eq. (3) and (4) respectively:

$$H_{2U} = Z_2 + \frac{Q_g^{\ 2}}{2gy_{2U}^2} + \frac{P_{2U}}{\gamma} = S_o\left(L - x_{y2}\right) + \frac{Q_g^{\ 2}}{2gy_{2U}^2} + y_{2U} \quad (3)$$

$$H_{2S} = Z_2 + \frac{Q_g^{\ 2}}{2gy_{2S}^2} + \frac{P_{2S}}{\gamma} = S_o\left(L - x_{y2}\right) + \frac{Q_g^{\ 2}}{2gy_{2S}^2} + y_{2S} \quad (4)$$

In which H_{2U} = total energy computed based on measured unsteady flow depth at the downstream side of the jump front; H_{2S} = total energy computed based on expected steady conjugate depth; Z = invert elevation; S_o = longitudinal slope; L = total flume length; and x_{y2} = the distance form the sluice gate to the point where y_{2U} (or the computed y_{2S}) take place.

In this study the data pertinent to three specific hydrographs were presented and analyzed. The hydrographs, which were recorded at the upstream end of the experimental setup-sluice gate, were depicted in Fig. 3. The markers shown on each hydrograph indicate the points whose data were tabulated and analyzed. As was justified earlier the discharge along the supercritical reach at a given moment was considered constant and equals Q_g.-the discharge at the sluice gate. However, the discharge at the downstream side of the moving jump, Q_U, differs form that of the upstream side, Q_g. In fact the influences of unknown momentum equation terms, the backwater effects, and the unsteadiness of flow could have an impact on the discharge value.

The difference between H_{2U} and H_{2S} could be attributed to the unbalance forces acting across the jump front. The mentioned difference in total energy is plotted against difference in related depths in both dimensional and dimensionless forms in Fig. 4. Figure 4a reveals satisfactory linear relationship as follows:

$$H_{2U} - H_{2S} = 0.684\left(y_{2U} - y_{2S}\right) \quad (5)$$

Therefore, based on the available data and the linear relationship obtained in Fig. 4 the discharge of the downstream side of the transcritical front could be estimated. That is substituting the terms of total energy in Eq. (5) yields:

$$\left(y_{2U} - y_{2S}\right) + \left(\frac{V_{2U}^2 - V_{2S}^2}{2g}\right) = 0.684\left(y_{2U} - y_{2S}\right) \quad (6)$$

Substituting the discharges and cross section areas for velocities in Eq. (6) yields:

$$2g\,0.316\left(y_{2U} - y_{2S}\right) = \left(\frac{Q_U y_{2S} - Q_g y_{2U}}{b\left(y_{2U} \cdot y_{2S}\right)}\right)^2 \quad (7)$$

Solving Eq. (7) for Q_U yields:

$$Q_U = \sqrt{\frac{2g\,0.316.b^2 y_{2U}^2 y_{2S}^2\left(y_{2U} - y_{2S}\right) + \left(Q_g y_{2U}\right)^2}{y_{2S}^2}} \quad (8)$$

The values of Q_U were determined for the selected moments' data and included in the tabulation; therefore, the ratio Q_g/Q_U could be evaluated as well. Plotting the values of $(H_{2U}\text{-}H_{2S})/H_{2U}$ or $(y_{2U}\text{-}y_{2S})/y_{2U}$ against Q_U/Q_g reveals reliable relationships between the actual discharge directly at the downstream side of the jump front and the discharge released form the gate opening (Fig. 5).

Having Q_U, the steady state momentum equation for the control volume that includes both sides of the jump front takes the form:

$$\sum F_x = F_{P1} - F_{P2U} + W\sin\alpha - F_F = \rho Q\left(V_{2U} - V_1\right) \quad (9)$$

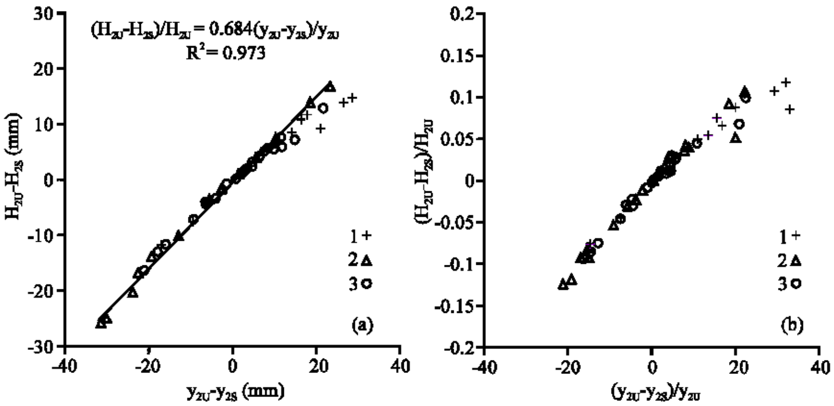

Fig. 4: Relation between (a) depth differences and energy differences (b) relative depth differences and relative energy differences

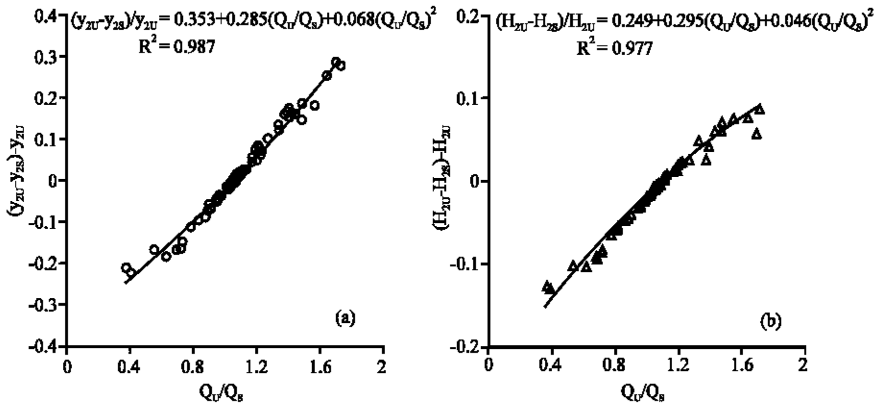

Fig. 5: Relationship between discharge ratio and (a) relative subcritical flow depth variation (b) relative energy variation

In which F_{P1} and F_{P2U} are the pressure forces acting on the relevant sections; W sinα and F_F are the weight component in the flow direction and friction force, respectively. The later forces are nil and could be ignored without spoiling the results; because the length of the control volume (the distance between two sides of the jump front) was short and the longitudinal slope of the channel was small. Therefore, the following equation is obtained:

$$F_{P1}-F_{P2u} = \rho\left(Q_U V_{2U} - Q_g V_1\right) = \rho\left(\frac{Q_U^2}{b.y_{2U}} - \frac{Q_g^2}{b.y_1}\right) \qquad (10)$$

Rearranging Eq. (10) yields:

$$\frac{F_{P2U}}{\gamma} = f(y_{2U}) = \frac{y_1^2 b}{2} - \frac{1}{gb}\left(\frac{Q_U^2}{y_{2U}} - \frac{Q_g^2}{y_1}\right) \qquad (11)$$

Evaluating the right hand side of Eq. (11) based on the measured values provides the values of F_{P2U}/γ for any given moment. F_{P2U} should be considered as the actual pressure force (which deviate form the hydrostatic one) at the downstream side of the moving jump front and denoted by F_{P2U}^{nh} . The hydrostatic values of these forces are denoted by F_{P2U}^{h} and F_{P2S}, respectively which could be determined by applying flow depths y_{2U} and y_{2S} in the traditional hydrostatic force term of rectangular cross sections, i.e. $\gamma y_{2S}^2 b/2$ and $\gamma y_{2U}^2 b/2$, respectively. Based on the three mentioned definitions the pressure force at the subcritical section of the jump front was plotted in Fig. (6). The figure shows the variations of this parameter according to the flow discharge of the first hydrograph; other hydrographs exhibited same behavior.

The flow condition of each run exhibited exclusive behavior different from that observed in other runs. However, all these hydrographs' data follow the trend indicated by Eq. (5). Although, the longitudinal slope for all runs was the same, it seems that factors such as longitudinal slope and boundary roughness do not violate the drawn conclusion and manifest their influence only in the

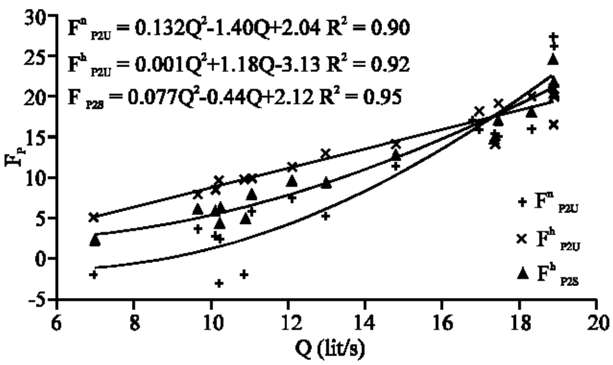

Fig. 6: Pressure force directly downstream of the transcritical flow front

magnitude of the slope of the line in Eq. (5). Nevertheless drawing general conclusion requires more investigation and data.

Two basic assumptions were considered in determining the total energy by Eq. (3) and (4) that is, (I) the pressure distributions across the jump fronts were hydrostatic and (ii) same discharge values present at both sides of the jump front. These assumptions manifest themselves explicitly in the pressure and velocity head terms; therefore, the simplifying assumptions influence the velocity head term (proportional to y_{2U}^{-2}) and the pressure head term (proportional to y_{2U}^{+1}) of Eq. (4) which represents the total energy computed based on y_{2U}. It is clear that the discharge at section 2 at any moment deviates form Q_g-value and the pressure distribution seems to vary from the hydrostatic one. The pressure deviation could be attributed to the presence of turbulence and the influence of the downstream boundary conditions. Therefore, it could be concluded that the slope of the line presented by Eq. (5) bear special physical meaning. That is, it provides a value independent of various boundary conditions which represents the influence of many factors such as y_{2U} exponents, the discharge difference of two flow regimes, and the non-hydrostatic pressure distribution in section 2. In fact this equation facilitates next step computation and provides a means to predict flow parameters.

According to Fig. 6, the pressure distribution in subcritical section is lower than the hydrostatic value for small discharges and higher than that for high discharges. The method presented for computing pressure force in the subcritical section by means of Eq. (11) shows that the pressure force is always less than the hydrostatic pressure force computed based on y_{2S}. The difference however becomes smaller as the discharge increase.

CONCLUSIONS

An experimental program was conducted to study the characteristics of unsteady transcritical flow. Three hydrographs with different shapes were generated at the upstream end of the flume. The unsteady flow parameters observed for each hydrograph were recorded continuously. Recorded data of some parameters related to specific discharges of each hydrograph were correlated to steady parameters associated with the mentioned discharges and their suprecritical flow characteristics.

The correlation reveals the presence of reliable relations between the steady and unsteady parameters. Although some simplifying assumption had been applied for driving such relations, estimation of unsteady mixed flow condition based on steady state relationships seems to be reliable and practical. However, further studies and data were required for drawing general conclusion, especially for different boundary roughness and longitudinal slopes.

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