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Determination of Initial Unconditional Solution of Heat Conductivity Equation for Evaluation of Temperature Variance in Finite Soil Layer

İmanverdi Ekberli
Department of Soil Science, Faculty of Agriculture,
Ondokuz Mayıs University, 55139 Samsun, Turkey

Abstract: Soil temperature has very significant effect on biological development of plant and physical, chemical, biological properties of soil. The variance of temperature depending on time and soil depth forms heat transportation and therefore it enables constitution of temperature regime. In this research, it has been determined with a help of experiments (held on 01-17 June, 2005) that daily average temperature on soil surface varies between 24.3-41.9°C and temperature amplitude varies between 7.1-12.5°C. In 10, 20, 30, 40, 50 and 60 cm of soil depth, average temperature, amplitude and diffusivity vary between intervals of 19.5-23.8°C; 0.6-3.5°C; 0.13×10^{-3}-2.51×10^{-3} ($cm^2\ sec^{-1}$), consecutively. The initial unconditional solution of heat conductivity equation in $T(0, \tau) = T_{or} + T_0 \cos\omega\tau$ in finite soil layer depending on condition of $T(l, \tau) \approx 0$ has been determined. The obtained solution, average temperature of soil layer, amplitude and diffusivity enable determination of temperature variance along soil layer depending on time. Based on the obtained solution, it has been determined that relative error between the calculated and observed temperature varies in interval of %2.41-5.66.

Key words: Temperature, harmonic variance, heat conductivity equation, limit conditions, model

INTRODUCTION

The mathematical modeling of soil processes, which is considered to be part of ecosystem, can be conducted by modeling physical, chemical, biological, etc. properties of soil together or separately. The modeling of soil processes is one of the important phases of increasing soil fertility, its preservation, therefore, management of agriculture on optimum level. Besides, the necessity of improvement of soil treatment methods and tools on higher standards enables the implication of mathematical models.

It is possible to express the relationships between parameters effecting soil processes with analytical mathematical models (Platonov and Chudnovskii, 1984). The usage of mathematical modeling method in research of soil properties makes management of soil processes easier. The complexity of soil structure necessitates implementation of simple analytical models to different soil processes. The fertility and energy-mass variation models are derived by assuming that parameters not included in the model are on optimum level (Nerpin and Chudnovskii, 1984; Poluektov, 1979; Sirotenko, 1981). The practicability of parameters in real applications has to be taken into account in determination of model parameters.

The solution of heat conductivity in solid substances equation is solved by taking different initial and limit conditions into account based on properties of substance (Luikov and Mikhailkov, 1965; Luikov, 1967; Isaçenko *et al*., 1981). The heat conductivity in soil occurs as a result of variance of soil warmth. And the soil warmth depends on daily and seasonal variation of sun radiation, weather temperature and especially, soil structure. The short interval of average daily temperature variance has a significant effect on biological development of plant and soil factors, which necessitates determination of possible errors in the process of modeling soil temperature and taking those errors into account during model calculations (Gorıshina *et al*., 1981; Timlin *et al*., 2002).

Many mass transportation processes are realized in soils that can be resembled to complex biochemical reactor (Richter, 1987). The model of heat conductivity in soil constitutes some of complex ecosystem models representing transportation of water, salt, nitrogen, etc. The regulation of soil temperature regime, which is very important factor in biological development of plant, necessitates usage of the solution of equation of heat conductivity in appropriate initial and limit conditions.

Kang *et al.* (1999), have derived model, which can be used in other ecosystem models considering soil temperature as a variable, for estimating daily variance of soil temperature by taking its topography, wastes on soil surface and its plant cover into account. Wang and Bras (1999), have stated practicability of model, which expresses practical relationship between soil temperature and heat transportation in soil.

Heat distribution in soil varies based on soil's general and specific properties in each soil layer and artificial source of heat. In that case, it is possible to derive model expressing moisture and heat distribution in non-saturated soil by using finite numerical method. (Santander and Bubnovich, 2002). The study of temperature loss resulted from vertical heat transfer from substances on soil surface throughout soil can be conducted by using heat conductivity equation, in case of presence of underground waters with stable temperature level in certain depth of soil (Choi and Krarti, 1998).

The temperature on soil surface practically demonstrates variation based on periodic law that the initial level doesn't have significant importance. Therefore, the initial temperature isn't taken into account in solution of equation of soil's heat conductivity (Gülser and Ekberli, 2002, 2004). The soil temperature decreases in lower soil layers and it becomes equal to zero in certain depth.

The objective of this study is determining initial unconditional solution of equation of heat conductivity in finite soil depth, in case of soil temperature's periodic variance on soil surface and zero temperature in certain soil depth.

MATERIALS AND METHODS

The experiment was carried out at the Agricultural Faculty Experimental Field in Ondokuz Mayýs University, Samsun (41°21.86' N; 36°11.41'E; 187 mH), Turkey in June 2005 in order to compare the theoric values obtained from the solution of heat conductivity equation with the values obtained from experiment. Some physical and chemical properties of the soil profile were determined as following: particle size distribution by hydrometer method (Day, 1965), soil reaction, pH, 1:1 (w:v) soil water suspension by pH meter, electrical conductivity (EC_{25}°C) in the same soil suspension by EC meter, Organic Matter (OM) contents by modified Walkley-Black method (Kacar, 1994), $CaCO_3$ by Scheibler calsimeter (Kacar, 1994) and total N by Kjeldahl method (Kacar, 1994). Exchangeable K, Ca, Na and Mg were determined with 1 N NH_4OAc (Sağlam, 1997). Micro elements (Mn, Zn and Cu) were estimated in the extraction solution of 0.005M DTPA+0.01 M $CaCl_2$ + 0.1 M TEA by using an Atomic Absorption Spectrometer (Lindsay and Norvell, 1978). Available phosphorus was measured by the spectrophotometer with molybdophosphoric blue colour method (Olsen *et al.*, 1954).

Measurements of soil temperature between 1st and 17th of June, 2005 were also done at 0, 10, 20, 30, 40, 50 and 60 cm depths by using mercury-in-glass thermometer (Sterling and Jackson, 1986). Temperature readings were taken six times in a day at 9 am, 11 am, 1 pm, 3 pm, 5 pm and 7 pm for 16 days. Heat diffusivity (b^2) at that depth (x) can be estimated using 16 days frequency ($\omega = 2\pi/1382400sn = 4.54\times10^{-6}$ sec^{-1}) by the following equations (Hillel, 1982; Nerpin and Chudnovskii, 1984; Gülser and Ekberli, 2004):

$$b^2 = \omega x^2 \left[2\ln^2 \frac{T_{0(x)}}{T_0} \right]^{-1}$$

where, T_0 is the amplitude of the surface temperature fluctuation, which is the range from maximum or minimum to average temperature and $T_{0(x)}$ is the amplitude of the surface temperature fluctuation at any given depth.

The solution of soil's heat conductivity equation has been determined by taking into consideration that initial unconditional limit problem of soil's heat conductivity equation for $0 \le x \le l$ ve $0 < \tau$, generally, is as following:

$$\begin{cases} T(0,\tau) = \mu_1(\tau) \\ T(l,\tau) = \mu_2(\tau) \end{cases}$$

(here, μ_1 (τ), μ_2 (τ) - are the functions expressing warmth values in τ time (sec.) on soil surface or any x = l layer (cm) of soil.

RESULTS AND DISCUSSION

Experimental results: The results showed that the textural class of soil is clay, low in organic matter (OM), neutral in pH, non saline according to EC value (Soil Survey Staff, 1993). In experimental soils, the amount of $CaCO_3$ diminishes along soil layer and demonstrates accumulation in soil depth of 40-50 cm. Total N and exchangeable K, Ca, Na, Mg vary in each layer downwards throughout the soil. Also, the amount of available phosphorus and micro elements (Mn, Zn and Cu) demonstrate decrease in each layer downwards throughout the soil (Table 1).

As a result of temperature measurements conducted during the research period (01-17 June, 2005), average temperature, amplitude and diffusivity values in soil surface and its lower layers enabling the control of initial unconditional solution of equation of heat conductivity in finite soil layer have been obtained (Table 2).

Table 1: Some physical and chemical properties of the soil used in the study

Soil depth (cm)	(%) Sand	Silt	Clay	OM	$CaCO_3$	Total N	pH (1:1)	$EC_{25.4}$°C (dS m^{-1})	(me 100 g^{-1}) K	Ca	Na	Mg	(mg kg^{-1}) P	Mn	Zn	Cu
0-10	25.6	21.8	52.6	3.55	0.34	0.46	7.3	0.348	2.03	36.0	2.40	10.5	31.77	36.71	5.12	0.37
10-20	27.2	19.0	53.8	2.39	0.19	0.36	6.9	1.070	1.66	39.0	2.48	12.2	26.44	32.96	3.30	0.30
20-30	43.5	14.9	41.6	0.77	0.16	0.16	7.3	0.397	0.94	38.6	2.94	14.7	16.40	20.23	1.05	0.17
30-40	61.7	18.7	19.6	0.33	0.18	0.07	7.0	0.198	0.52	36.8	3.60	15.2	9.84	11.20	0.95	0.05
40-50	65.5	16.0	18.4	0.16	2.96	0.08	7.7	0.248	0.55	34.7	3.74	12.8	6.15	11.91	0.91	0.07
50-60	62.1	19.2	18.7	0.18	0.18	0.26	7.5	0.248	1.47	33.0	1.83	11.5	7.99	14.42	0.96	0.04

Table 2: Measured average values of soil temperature (T, °C), amplitude (T_o, °C) and diffusivity (b^2, cm^2 sec^{-1}) at the different soil depths (01-17 June, 2005)

Soil depth (cm)	Day time (h) 9:00 T	T_o	b^2	11:00 T	T_o	b^2	13:00 T	T_o	b^2	15:00 T	T_o	b^2	17:00 T	T_o	b^2	19:00 T	T_o	b^2
0	31.1	7.1	0.00	37.3	11.3	0.00	41.9	7.3	0.00	38.6	7.8	0.00	32.0	8.3	0.00	24.3	12.5	0.00
10	22.2	2.6	0.23×10^{-3}	23.2	3.0	0.13×10^{-3}	23.7	3.5	0.42×10^{-3}	23.8	2.5	0.18×10^{-3}	23.8	2.7	0.18×10^{-3}	23.8	3.0	0.11×10^{-3}
20	22.1	2.1	0.61×10^{-3}	22.5	1.5	0.22×10^{-3}	22.8	2.2	0.63×10^{-3}	22.5	2.0	0.49×10^{-3}	22.6	2.2	0.52×10^{-3}	22.8	2.6	0.37×10^{-3}
30	22.0	1.6	0.92×10^{-3}	22.4	1.3	0.44×10^{-3}	22.1	1.1	0.57×10^{-3}	22.2	1.4	0.69×10^{-3}	22.0	2.0	1.01×10^{-3}	21.6	1.9	0.58×10^{-3}
40	21.4	1.5	1.50×10^{-3}	21.6	1.2	0.72×10^{-3}	21.7	0.6	0.58×10^{-3}	21.7	1.1	0.95×10^{-3}	21.5	1.2	0.97×10^{-3}	21.0	1.5	1.24×10^{-3}
50	20.8	1.5	2.35×10^{-3}	21.0	1.4	1.30×10^{-3}	21.1	0.7	1.03×10^{-3}	21.0	1.4	1.93×10^{-3}	20.8	1.2	1.52×10^{-3}	20.5	1.5	1.94×10^{-3}
60	19.4	0.6	1.34×10^{-3}	19.6	0.8	1.17×10^{-3}	19.6	1.2	2.51×10^{-3}	19.7	0.8	1.58×10^{-3}	19.5	1.0	1.83×10^{-3}	19.5	1.2	2.19×10^{-3}

Due to the fact that soil surface gets warmer and cools downs comparatively faster, average temperature varies in larger interval (24.3-41.9°C). In lower soil layers temperature variance occurs in narrow interval. In 60 cm soil depth, average temperature varies between 19.4-19.7°C. The values of average temperature amplitude on soil surface vary between 7.1-12.5°C and this interval gets narrower in lower soil layers. The average diffusivity values vary between 0.13×10^{-3}-2.51×10^{-3} ($cm^2 sec^{-1}$). This interval is getting more stretched in lower soil layers, therefore, results in diminishing temperature level. This increase occurs more in >30 cm soil depth. Along with other factors (climate, plant cover, grass cover, etc.), the variance of physical and chemical properties of soil in >30 cm soil depth also have significant effect on diffusivity.

Mathematical solution: Heat conductivity problem in finite soil depth, in case of harmonic variance of temperature on soil surface and in case of zero temperature level in any l soil depth, will be as following:

$$\frac{\partial T(x,\tau)}{\partial \tau} = b^2 \frac{\partial^2 T(x,\tau)}{\partial x^2} \qquad (1)$$

$$T(0,\tau) = T_{or} + T\cos\omega\tau, \quad T(l,\tau) \approx 0 \qquad (2)$$

where, T (x, τ) is the soil temperature when x = 0, x is the vertical coordinate, τ is the time computed from a given initial moment (°C), b^2 is the diffusivity ($cm^2 sec^{-1}$), T_{or} is the average temperature of the soil surface, T_0 is the amplitude of the surface temperature fluctuation, which is the range from maximum or minimum to average temperature and $\omega = 2\pi/P$ - is radian variance (1/sec) of temperature, where P- is period (sec).

Let's solve Eq. 1 in:

$$\tilde{T}(x,\tau) = T_{or} + X(x)\, e^{i\omega\tau} \qquad (3)$$

form by expressing limit conditions (2) as:

$$\tilde{T}(0,\tau) = T_{or} + T_0\, e^{-i\omega\tau}, \quad \tilde{T}(l,\tau) = T_{or} \qquad (4)$$

If

$$\frac{\partial \tilde{T}(x,\tau)}{\partial \tau} = -i\omega X(x)\, e^{-i\omega\tau} \text{and} \frac{\partial^2 \tilde{T}(x,\tau)}{\partial x^2} = X''(x) e^{-i\omega\tau}$$

expressions that are obtained from solution (3) are taken into account,

$$X''(x) + \frac{i\omega}{b^2} X(x) = 0 \;\text{ or }\; X''(x) + \gamma^2 X(x) = 0 \qquad (5)$$

will derive. Here,

$$\gamma = \sqrt{i\omega/b^2} = \sqrt{\omega/2b^2}\,(1+i) = k\,(1+i),\ k = \sqrt{\omega/2b^2}\,.$$

The additional conditions derived from the expressions (3) and (4) will be as following:

$$X(0) = T_0,\ X(l) = 0 \qquad (6)$$

From $\lambda^2 + \gamma^2 = 0$ characteristic equation of equation (5), $\lambda_1 = \gamma i$ ve $\lambda_2 = -\gamma i$, therefore the solution will be in the following form: $X(x) = C_1 e^{\gamma i x} + C_2 e^{-\gamma i x}$ (C_1, C_2 - are constants). The general solution can be written as following, by using Eyler's formula:

$$X(x) = C_1 (\cos \gamma x + i \sin \gamma x) + C_2 (\cos \gamma x - i \sin \gamma x) = (C_1 + C_2) \cos \gamma x + i (C_1 - C_2) \sin \gamma x \quad (7)$$

$$C_1 = \frac{T_0}{2} - \frac{T_0 \cos \gamma l}{2 i \sin \gamma l} \text{ and } C_2 = \frac{T_0}{2} + \frac{T_0 \cos \gamma l}{2 i \sin \gamma l}$$

are obtained from the solution of (7) by taking conditions (6) into account. Then, the solution (7) of equations (5)-(6) will be expressed in the following form:

$$X(x) = T_0 \left[\frac{\sin \gamma (l - x)}{\sin \gamma l} \right] = X_1(x) + i X_2(x) \quad (8)$$

Here, $X_1(x)$, $X_2(x)$ functions are real and imaginary parts of $X(x)$ solution, consecutively.

The solution (4) is written as following by using the expression (8):

$$\tilde{T}(x, \tau) = T_{or} + T_0 \left[\frac{\sin \gamma (l - x)}{\sin \gamma l} \right] e^{-i\omega\tau} \quad (9)$$

The solution of initial unconditional Eq. 1 will be obtained as following by spliting real and imaginary parts of solution (9):

$$T(x, \tau) = T_{or} + T_0 [X_1(x) \cos \omega t + X_2(x) \sin \omega t] \quad (10)$$

The functions of $X_1(x) = \mathrm{Re}X(x)$ and $X_2(x) = \mathrm{Im}X(x)$ are calculated from the expression (8) by using equations of $\sin z = (e^{iz} - e^{-iz})/2i$, $\cos z = (e^{iz} + e^{-iz})/2$, $e^z = e^{x+iy} = e^x (\cos y + i \sin y)$, for $\forall z \in C$.

$$\frac{\sin \gamma (l - x)}{\sin \gamma l} = \frac{\sin[k(1+i)(l-x)]}{\sin[k(1+i)l]} = \frac{e^{ik(1+i)(l-x)} - e^{-ik(1+i)(l-x)}}{e^{ik(1+i)l} - e^{-ik(1+i)l}} = \frac{e^{-k(l-x)+ik(l-x)} - e^{k(l-x)-ik(l-x)}}{e^{-kl+ikl} - e^{kl-ikl}} =$$

$$= \frac{e^{-k(l-x)}[\cos k(l-x) + i \sin k(l-x)] - e^{k(l-x)}[\cos k(l-x) - i \sin k(l-x)]}{e^{-kl}(\cos kl + i \sin kl) - e^{kl}(\cos kl - i \sin kl)} =$$

$$= \frac{\left[e^{-k(l-x)} \cos k(l-x) - e^{k(l-x)} \cos k(l-x)\right] + i\left[e^{-k(l-x)} \sin k(l-x) + e^{k(l-x)} \sin k(l-x)\right]}{(e^{-kl} \cos kl - e^{kl} \cos kl) + i(e^{-kl} \sin kl + e^{kl} \sin kl)} =$$

$$= \left[\frac{(e^{-2kl+kx} + e^{2kl-kx}) \cos kx - (e^{kx} + e^{-kx}) \cos(2kl - kx)}{e^{-2kl} + e^{2kl} - 2\cos 2kl} \right] +$$

$$+ i \left[\frac{\left(e^{-kx} - e^{kx}\right) \sin(2kl - kx) + (e^{2kl-kx} - e^{-2kl+kx}) \sin kx}{e^{-2kl} + e^{2kl} - 2\cos 2kl} \right].$$ From this,

$$X_1(x) = \frac{\left(e^{-2kl+kx} + e^{2kl-kx}\right) \cos kx - \left(e^{kx} + e^{-kx}\right) \cos(2kl - kx)}{e^{-2kl} + e^{2kl} - 2\cos 2kl}$$

$$X_2(x) = \frac{\left(e^{-kx} - e^{kx}\right) \sin(2kl - kx) + (e^{2kl-kx} - e^{-2kl+kx}) \sin kx}{e^{-2kl} + e^{2kl} - 2\cos 2kl}$$ or

$$X_1(x) = \frac{\mathrm{ch}[(2l - x)k] \cos(kx) - \mathrm{ch}(kx) \cos[(2l - x)k]}{\mathrm{ch}(2kl) - \cos(2kl)} \quad (11)$$

$$X_2(x) = \frac{\mathrm{sh}[(2l - x)k] \sin(kx) - \mathrm{sh}(kx) \sin[(2l - x)k]}{\mathrm{ch}(2kl) - \cos(2kl)} \quad (12)$$

are obtained.

The general solution of equation (1) will be as following, if we place the expressions (11) and (12) in (10) to their appropriate spots:

$$T(x,\tau) = T_{or} + T_0 \left\{ \frac{ch\left[(2l-x)k\right]\cos(kx) - ch(kx)\cos\left[(2l-x)k\right]}{ch(2kl)-\cos(2kl)} \right\} \cos(\omega\,\tau) +$$

$$+T_0 \left\{ \frac{sh\left[(2l-x)k\right]\sin(kx) - sh(kx)\sin\left[(2l-x)k\right]}{ch(2kl)-\cos(2kl)} \right\} \sin(\omega\,\tau) \qquad (13)$$

The solution (13) can be written without any units in the following form:

$$\theta(x,\tau) = \frac{1}{\Delta}\left[\Phi_1(x,l,k)\cos(\omega\tau) + \Phi_2(x,l,k)\sin(\omega\tau)\right] \qquad (14)$$

Here,

$$\Phi_1(x,l,k) = ch\left[(2l-x)k\right]\cos(kx) - ch(kx)\cos\left[(2l-x)k\right]$$
$$\Phi_2(x,l,k) = sh\left[(2l-x)k\right]\sin(kx) - sh(kx)\sin\left[(2l-x)k\right]$$
$$\Delta = ch(2kl) - \cos(2kl),\ \theta(x,\tau) = \frac{T(x,\tau) - T_{or}}{T_0}$$

Theoretical testing of the solution: The limit conditions (2) are justified, when the solutions of (13) or (14) are x = 0 and x = l. Based on expression (10):

$$\frac{\partial T}{\partial \tau} = \omega T_0\left[-X_1(x)\sin\omega\tau + X_2(x)\cos\omega\tau\right]$$
$$\frac{\partial T(x,\tau)}{\partial \tau} = \omega\,T_0 \left\{ \frac{ch(kx)\cos\left[(2l-x)k\right] - ch\left[(2l-x)k\right]\cos(kx)}{ch(2kl)-\cos(2kl)} \right\} \sin(\omega\tau) + \qquad (15)$$
$$+\omega\,T_0 \left\{ \frac{sh\left[(2l-x)k\right]\sin(kx) - sh(kx)\sin\left[(2l-x)k\right]}{ch(2kl)-\cos(2kl)} \right\} \cos(\omega\tau)$$

From expression (11) [ch (2kl) - cos (2kl)] X_1 (x) = ch [(2l - x) k] cos (kx) - ch (kx) cos [2l - x)k] is derived and used in expression (15) as following:

$$\frac{\partial^2 X_1(x)}{\partial x^2} = \frac{2k^2}{ch(2kl)-\cos(2kl)}\left\{sh\left[(2l-x)k\right]\sin(kx) - sh(kx)\sin\left[(2l-x)k\right]\right\} \qquad (16)$$

And from expression (12)
[ch (2kl) - cos (2kl)] X_2 (x) = sh [(2l - x) k] sin (kx) - sh (kx) sin [2l - x) k] is derived and used in expression (15) as following:

$$\frac{\partial^2 X_2(x)}{\partial x^2} = \frac{2\,k^2}{ch(2kl)-\cos(2kl)}\left\{ ch(kx)\cos\left[(2l-x)k\right] - ch\left[(2l-x)k\right]\cos(kx)\right\}. \qquad (17)$$

If the expressions (16) and (17) are placed in expression

$$\frac{\partial^2 T}{\partial x^2} = T_0\left[\frac{\partial^2 X_1(x)}{\partial x^2}\cos\omega\tau + \frac{\partial^2 X_2(x)}{\partial x^2}\sin\omega\tau\right] \text{ obtained from (10),}$$

Table 3: Comparison of values (T_m, °C) obtained from experiment and calculations

Soil depth (cm)	Day time (h) 9:00			11:00			13:00			15:00			17:00			19:00		
	T_m	Δ	δ, (%)	T_m	Δ	δ, (%)	T_m	Δ	δ, (%)	T_m	Δ	δ, (%)	T_m	Δ	δ, (%)	T_m	Δ	δ, (%)
0	29.7	1.4	4.71	40.3	3.0	7.44	38.7	3.2	8.27	36.6	2.0	5.47	31.4	0.6	1.91	25.5	1.2	4.71
10	21.7	0.5	2.30	25.4	2.2	8.66	24.8	1.1	4.44	23.4	0.4	1.71	22.0	1.8	8.18	21.1	1.9	8.68
20	21.5	0.6	2.79	24.4	1.9	7.79	23.7	0.9	3.78	22.6	0.1	0.44	21.6	1.0	4.63	20.8	2.0	9.62
30	21.2	0.8	3.77	23.4	1.0	4.27	22.9	0.8	3.49	22.1	0.1	0.45	21.3	0.7	3.29	20.8	0.8	3.85
40	20.9	0.5	2.39	22.5	0.9	4.00	22.1	0.4	1.81	21.5	0.2	0.93	20.9	0.6	2.87	20.5	0.5	2.44
50	20.4	0.4	1.96	22.0	1.0	4.55	21.6	0.5	2.32	20.9	0.1	0.48	20.3	0.5	2.46	19.8	0.7	3.54
60	19.0	0.4	2.11	20.4	0.8	3.92	20.1	0.5	2.49	19.6	0.1	0.51	19.1	0.4	2.09	18.8	0.7	3.72

$$\frac{\partial^2 T}{\partial x^2} = \frac{2k^2 T_0}{ch(2kl) - \cos(2kl)} \left\langle \begin{array}{l} \{ sh[(2l-x)k]\sin(kx) - sh(kx)\sin[(2l-x)k] \}\cos(\omega\tau) + \\ + \{ ch(kx)\cos[(2l-x)k] - ch[(2l-x)k]\cos(kx) \}\sin(\omega\tau) \end{array} \right\rangle =$$
$$= 2k^2 T_0 \left\{ \frac{ch(kx)\cos[(2l-x)k] - ch[(2l-x)k]\cos(kx)}{ch(2kl) - \cos(2kl)} \right\} \sin(\omega\tau) + \tag{18}$$
$$+ 2k^2 T_0 \left\{ \frac{sh[(2l-x)k]\sin(kx) - sh(kx)\sin[(2l-x)k]}{ch(2kl) - \cos(2kl)} \right\} \cos(\omega\tau), \quad 2k^2 = \omega / b^2$$

will be derived.

As seen from the expression (15) and (18), the solution (13) or (14) justifies Eq. 1.

Comparison of values obtained from calculation and experiment: Based on solution (13) of heat conductivity equation, the average absolute ($\Delta = |T - T_m|$) and relative ($\delta = (\Delta/|T_m|)\times 100$, %) errors between calculated temperature values by soil layers and temperature values obtained from experiment vary between 0.5-1.9 and %2.41-5.66, consecutively (Table 3). And limit values for agricultural researches are in (< %10) interval (Dospexov, 1973).

The temperature variance in soil effects biological development of plant and soil properties in significant way. Therefore, the estimation of temperature along soil layer and regulation of temperature regime is one of the main problems in Agriculture. According to heat conductivity equation's solution (13) or (14), the estimation of temperature depending on time in any soil layer is possible, when average temperature by soil layer, amplitude and diffusivity values are known.

REFERENCES

Choi, S. and M. Krarti, 1998. Heat transfer for slab-on-grade floor with stepped ground. Energy Convers, Manag., 39: 691-701.

Day, P.R., 1965. Particle Fractionation and Particle Size Analysis. In: C.A.Black (Ed.) Methods of soil analysis. Agron. No. 9, Part 1, Am. Soc. Agron. Madison, WI., pp: 545-567.

Dospexov, B.A., 1973. Methodolgy of Experimental Experience (With Essentials of Statistical Processing). Moscow, Kolos, pp: 336.

Gorışina, N.G., N.P. Nikiforova and V.A. Platonov, 1981. The role of micro climate in application of technological solutions in Agriculture. Trudı GGO, Issue No. 460, pp: 82-88.

Gülser, C. and İ. Ekberli, 2002. Change in soil temperature along profile. J. Fac. Agric. OMU., 17: 43-47.

Gülser, C. and İ. Ekberli, 2004. A comparison of estimated and measured diurnal soil temperature through a clay soil depth. J. Applied Sci., 4: 418-423.

Hillel, D., 1982. Introduction to soil physics. Academic Press, Inc. San Dieoga, California.

Isaçenko, V.P., V.A. Osipova and A.S. Sukomel, 1981. Heat Conductivity. Moscow, Energoizdat, pp: 417.

Kacar, B., 1994. Plant and soil chemical analyses III. Soil analyses. Ankara Univ. Agric. Fac. Education Research and Ext. Publications, No. 3, Ankara, Turkey.

Kang, S., S. Kim, S. Oh and D. Lee, 1999. Predicting spatial and temporal patterns of soil temperature based on topography, surface cover and air temperature. For. Ecol. Manag., 136: 173-184.

Lindsay, W.L. and W.A. Norwell, 1978. Development of a DTPA soil test for zinc, iron, manganese and copper. Soil Sci. Soc. Am. J., 42: 421-428.

Luikov, A.V. and Y.A. Mikhailkov, 1965. Theory of energy and mass transfers. Pergamon Press. Oxford, UK.

Luikov, A.V., 1967. Theory of Heat Conductivity. Vısşaya Shkola. Moscow, pp: 599.

Nerpin, S.V. and A.F. Chudnovskii, 1984. Heat and Mass Transfer in the Plant-Soil-Air System. Translated from Russian. Published for Usda and National Sci. Found., Washington, DC, by Amerind Publishing Co. Pvt. Ltd. New Delhi, India.

Olsen, S.R., U. Cole, F.S. Watanable and L.A. Dean, 1954. Estimating of Available Phosphorus in Soils by Extraction with Sodium Bicarbonate. US Dept. of Agr. Cir. 939, Washington, DC.

Platonov, V.A. and A.F. Chudnovsky, 1984. Modeling Agro-meteorological Conditions and Optimaztion of Agriculture Technology. Leningrad, Gidrometeoizdat, pp: 280.

Poluektov, R.A., 1979. Imitated models of productivity of agro-ecosystem. İmitaçionnıye modeli produktivnosti agroekosistem. Theoretic and quantitative methods of modeling crop. Leningrad, pp: 14-23.

Richter, J., 1987. The Soil as a Reaktor. Modelling Processes in the Soil. Gremlingen: Catena Verlag, W. Germany, pp: 182.

Sağlam, T., 1997. Chemical Analysis of soil and water. Trakya Univ. Tekirdağ Agriculture Faculty, Issue No. 189, pp: 164.

Santander, R.E. and V. Bubnovich, 2002. Assesssment of mass and heat transfer mechanisms in unsaturated soil. Intl. Comm. Heat Transfer, 19: 531-545.

Sirotenko, O.D., 1981. Mathematical Modeling of Water-heat Regime and Productivity of Agro-ecosystem. Leningrad, Gidrometeoizdat, pp: 167.

Soil Survey Staff, 1993. Soil Survey Manuel. USDA Handbook No. 18 Washington.

Sterling, A.T. and R.D. Jackson, 1986. Temperature. In: Klute, A. (Ed.), Methods of Soil Analysis Part 1. Physical and Mineralogical Methods. Agronomy Monograph No. 9, ASA, SSSA, Madison WI.

Timlin, D.J., Y.A. Pachepsky, B.A. Acock, J. Simunek, G. Flerchinger and F. Whisler, 2002. Error analysis of soil temperature simulations using measured and estimated hourly weather data with 2DSOIL. Agric. Sys., 72: 215-239.

Wang, J. and R.L. Bras, 1999. Ground heat flux estimated from surface soil temperature. J. Hydrol., 216: 214-226.