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Solving Information Fusion Problems on Unreliable Evidential Sources with Generalized DSmT

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Abstract: How to deal with information fusion problems on unreliable evidential sources? Because the kind of fusion problem is everywhere in information society, DSmT recently developed from DST and probability theory is much more powerful than the other method (DST, etc.). However, it needs to be improved for its limitation that its author didn't mention of solving information fusion problem on unreliable evidence source. Therefore in this study we ourselves extend DSmT and solve the fusion problem on not only equal-reliable sources of evidence, as well as unreliable evidence source. At last, An example is illustrated to verify the benefit of this kind of method and a conclusion is reached that generalized DSmT method can fuse any class information and knowledge effectively whether handing static fusion problems or dynamic fusion problems in real time directly.

Key words: Information fusion, uncertain, DST, DSmT, plausible and paradoxical reasoning

INTRODUCTION

Information fusion technology was a new subject that originated from martial application C^4I system and IW system involving multi-target detection, recognizance, tracking and warfare field surveillance, evaluation of situation and menace, etc. With the transferring of information fusion technology from the martial application to civil one, presently the multi-sensors fusion technology won a chance to develop rapidly. In fact, since 1950, the development of new original theories dealing with uncertain and imprecise information has become very prolific. There are three major theories available now as alternative to the theory of probabilities for automatic plausible reasoning in expert systems as follows: Firstly, the fuzzy set theory developed by Zadeh (1965); Secondly, the Shafer's theory of evidence in 1976 developed from his teacher (Dempster, 1967), known as DST (Shafer, 1976) and the theory of possibilities by Dubois and Prade (1988). Thirdly, recently, the unifying avant-gardise neutrosophy theory proposed by Smarandache (2000) especially, Dezert and Smarandache (2003a) proposed a new theory known as DSmT on which a panel discussion and a special session were also carried out at The 7th International Conference on Information Fusion 2004 (Dezert *et al*., 2004). Though obviously DSmT is a new and creative theory, which can deal with a wide range of evidence sources, it don't distinctly propose how to deal with the unreliable sources.

THE D-S THEORY OF EVIDENCE

In this section we simply recall DST as follows:

The idea of using belief functions for representing someone's subjective feeling of uncertainty was first proposed by Shafer (1976), following the seminal work of Dempster (1967) about upper and lower probabilities induced by a multi-valued mappings. The use of belief functions as an alternative to subjective probabilities for representing uncertainty was later justified axiomatically by Smets and Kennes (1994), who introduced the Transferable Belief Model (TBM), providing a clear and coherent interpretation of the various concepts underlying the theory.

Let θ_i (i = 1, 2, 3...n) be some exhaustive and exclusive elements (hypotheses) of interest taking on values in a finite discrete set Ω, called the frame of discernment. Let us assume that an agent entertains beliefs concerning the value of θ_i, given a certain evidential corpus. We postulate that these beliefs may be represented by a belief structure (or belief assignment), i.e., a function from 2^Ω to [0,1] verifying $\Sigma_{A\subseteq\Omega}$ m (A) = 1 and m (ϕ) = 0, here ϕ is empty set. For all $A\subseteq\Omega$, the quantity m (A) represents the mass of belief allocated to proposition "$\theta_i \subseteq A$" and that cannot be allocated to any strict sub-proposition because of lack of evidence. The subsets A of Ω such that m (A) are called the focal elements of m. The information contained in the belief structure may be equivalently represented as a belief function bel, or as a plausibility function pl, defined respectively as bel (A) = $\Sigma_{B\subseteq A}$ m (B)

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and pl (A) = $\Sigma_{B \cap A \neq \phi}$ m (B). The quantity bel (A), called the credibility of A, is interpreted as the total degree of belief in A (i.e., in the proposition $\theta_i \subseteq A$), whereas pl (A) denotes the amount of belief that could potentiality be transferred to A, taking into account the evidence that does not contradict that hypothesis.

Now we assume the simplest situation that two distinct pieces of evidence induce two belief structures m_1 and m_2. The orthogonal sum of m_1 and m_2, denoted as $m = m_1 \oplus m_2$ is defined as:

$$m(A) = K^{-1} \sum_{B \cap C = A} m_1(B) m_2(C) \quad (1)$$

Here $K = 1 - \sum_{B \cap C = \phi} m_1(B) m_2(C)$, for $A \neq \phi$ and m (ϕ) = 0

The orthogonal sum (also called Dempster's rule of combination) is commutative and associative. It plays a fundamental operation for combining different evidential sources in evidential sources in evidence theory. Decision-making is an important issue in any theory of uncertainty. In the TBM, a distinction is made between two levels of uncertainty representation: a credal level at which beliefs are entertained and represented using the formalism of belief functions and a decision level at which belief functions are converted to probability distributions to allow coherent betting behaviors (Smets and Kennes, 1994). Given a belief structure m, the Generalized insufficient reason principle leads to the definition of the pignistic probability distribution BetP as

$$\text{BetP}(\theta) = \Sigma_{\theta \in A} \frac{m(A)}{|A|}$$

where |A| denotes the cardinality of A. Of course, Jean Dezert and Florentin Smarandache have extended it and introduce a Generalized Pignistic Transformation (GPT) as a tool for decision-making (Dezert *et al.*, 2004).

DEZERT-SMARANDACHE THEORY

Here we will simply introduce DSmT to the reader. If the reader want to know it in detail, please refer to the work by Smarandache and Dezert (2004). The practical limitations of the Dempster-Shafer Theory (DST) come essentially from its inherent following constraints, which are closely related with the acceptance of the third exclude principle.

(C1) The DST only considers a discrete and finite frame of discernment Ω based on a set of exhaustive and exclusive elementary elements θ_i (i = 1, 2, 3 ... n).

(C2) The evidential sources are assumed independent and provide their own belief function on the power-set 2^Ω but for same interpretation for Ω.

In most of practical fusion system, conflicts are unavoidable which can lead to the failure of making decision by using the DST. To solve it, some ad-hoc or heuristic techniques must always be added to the fusion process to manage or reduce the possibility of high degree of conflict between sources so that the complexity of reckoning increases. To overcome these major limitations and drawbacks relative to the Dempster's rule of combination, The Dezert- Smarandache Theory (DSmT) of plausible and paradoxical reasoning emerges as the times require.

The foundations of the DSmT is to refute the principle of the third middle excluded and to allow the possibility for paradoxes (partial overlapping) between elements of the frame of discernment. The relaxation of the constraint C1 can be justified since the elements of Ω correspond generally only to imprecise/vague notions or concepts so that no refinement for satisfying C1 is actually possible (specially if natural language is used to describe elements of Ω). The DSmT refutes also the excessive requirement imposed by C2 since it seems clear that the frame is usually interpreted differently by the distinct sources of evidence (experts). Some subjectivity on the information provided by a source of information is almost unavoidable, otherwise, this would assume, as within the DST, that all corpora of evidence have an objective/universal (possibly uncertain) interpretation or measure of the phenomena under consideration, which unfortunately rarely (never) occurs in reality. Actually in most of cases, the sources of evidence provide their beliefs about some hypotheses only with respect to their own worlds of knowledge and experience without reference to the (inaccessible) absolute truth of the space of possibilities. The DSmT includes the possibility to deal with evidences arising from different sources of information, which don't have access to absolute interpretation of the elements θ_i (i = 1,2,3...n) under consideration and can be interpreted as a general and direct extension of probability theory and the DST in the following sense. Let $\Omega = \{\theta_i, \theta_2\}$ be the simplest frame of discernment involving only two elementary hypotheses (with no more additional assumptions on θ_1 and θ_2), then

- The probability theory solve basic probability assignments m (.) $\in$ [0, 1] such that m (θ_1) + m (θ_2) = 1
- The DST deals with basic belief assignments (bba) m (.) $\in$ [0, 1] such that m (θ_1) + m (θ_2) + m ($\theta_1 \cup \theta 2$) = 1
- The DSmT theory deals with new bba m (.) $\in$ [0, 1] such that m (θ_1) + m (θ_2) + m ($\theta_1 \cup \theta_2$) + m ($\theta_1 \cap \theta_2$) = 1

Next we continue introduce the combinational rule about free model and hybrid model of DsmT. Let $\Omega = \{\theta_1, \theta_2, \theta_3 \ldots \theta_n\}$ be a set of elements which can't be precisely defined and separated so that no refinement of Ω in a new larger set Ω_{ref} of disjoint elementary hypotheses is possible. The hyper-power set D^{Ω} is defined as the set of all compositions built from elements of Ω with $\cup$ and $\cap$ (Ω generates D^{Ω} under operators $\cup$ and $\cap$) operators such that

- $\Phi, \theta_1, \theta_2, \theta_3 \ldots \theta_n \in D^{\Omega}$
- If $A, B \in D^{\Omega}$, then $A \cap B \in D^{\Omega}$ and $A \cup B \in D^{\Omega}$
- No other elements belong to D^{Ω}, except those obtained by using rules 1 or 2

The cardinality of D^{Ω} is majored by 2^{2^n} when Card (Ω) = $|\Omega|$ = n. The generation of hyper-power set D^{Ω} is closely related with the famous Dedekind's problem on enumerating the set of monotone Boolean functions. An algorithm for generating D^{Ω} based on monotone Boolean functions can be found by Dezert and Smarandache (2003a). Here we at first give the combinational rule of free DSmT model. Let us define a map m (.): $D^{\Omega} \in [0, 1]$ associated to a given source of evidence(abandoning Shafer's model) by assuming here that the fuzzy/vague/relative nature of elements θ_i (i = 1, 2, 3 ... n) can be non-exclusive, as well as no refinement of Ω into a new finer exclusive frame of discernment Ω_{ref} is possible. m (ϕ) = 0 and $\sum_{A \in D^{\Omega}} m(A) = 1$, here m (A) is called A's generalized basic belief (gbba). The belief function is defined in almost the same manner as within the DST, i.e., $bel(A) = \sum_{B \in D^{\Omega}, B \subseteq A} m(B)$. We has the classical DSm rule for $k \geq 2$ sources for free DSmT model (Smarandache and Dezert, 2004):

$$\forall A \neq \Phi \in D^{\Omega}$$

$$m_{M^{f(\Omega)}}(A) \cong [m_1 \oplus \cdots m_k](A) = \sum_{\substack{X_1 \cdots X_i \in D^{\Omega} \\ (X_1 \cap \cdots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i) \quad (2)$$

The DSm rule of combination is still commutative and associative.

While we also may give the presentation of DSm rule of combination for hybrid DSm models proposed by Smarandache and Dezert (2004), due to the limitation of length, we will write them directly here. (Smarandache and Dezert, 2004).

$$\forall A \in D^{\Omega}$$

$$m_{M(\theta)}(A) = \phi(A)\left[S_1(A) + S_2(A) + S_3(A)\right] \quad (3)$$

where

$$S_1(A) = m_{M^{f(\Omega)}}(A) \cong \sum_{\substack{X_1 \cdots X_i \in D^{\Omega} \\ (X_1 \cap \cdots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i)$$

$$S_3(A) \cong \sum_{\substack{X_1 \cdots X_i \in D^{\Omega} \\ (X_1 \cap \cdots \cap X_k) = \phi \\ (X_1 \cup \cdots \cup X_k) = A}} \prod_{i=1}^{k} m_i(X_i)$$

$$S_2(A) \cong \sum_{\substack{X_1 \cdots X_i \in \phi \\ \{(u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k)) = A\} \vee \\ \{[(u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k)) \in \phi] \wedge (A = I_t)\}}} \prod_{i=1}^{k} m_i(X_i)$$

In fact, the hybrid DSm model is a genuine model, which has been extended to take into account all possible integrity constraints (if any) of the problem under consideration due to the true nature of elements/concepts involved into it. But whether the free DSmT or the hybrid DSmT just proposes the combinational role aimed at the equal-reliable evidential source. How to solve the unequal-reliable evidential sources with DSmT? We will extend DSmT in next section for this.

FURTHER EXTENSION FOR DSmT

Necessity of further extension for DSmT: With the rapid development of science and technology, the precision, correctness and real-time for acquiring and processing information and knowledge from the nature to serve for human are more and more required necessarily. Technique of information fusion experiences from the single source to multi-source and multi-sensor involving homogeneous and heterogeneous information. It is well known that the homogeneous sensors can't give the same reliability sometimes for the sake of the difference of designing or operating, so that they can't give the coherent result, let alone the heterogeneous sensors. To make the reader see it clearly, let us introduce three simple examples as follows:

Firstly, let us assume two homogeneous sensors (A and B), one (A) has a high occurrence of malfunction, however, the other (B) have a low one. If we use DST or DSmT (either the free model or the hybrid model) and then the equal quantity of m (•) is allocated respectively to each one (i.e., m_A (•) = m_B (•)). We all know, it is obvious to be unreasonable.

Secondly, let us assume someone goes to hospital, he sees two doctors (A, B), who have the same diploma from the same university, the same certification of qualification as a doctor, as well as the same age and length of service. But before giving a decision, one (A) answers for his diagnosis on the patient and asks some relative question of the symptom and examines carefully. While the other (B) don't examine the patient throughout and then give a mistaken diagnosis result at once. Under this condition, can you assign the equal quantity of m (•) to two doctors?

Thirdly, supposed two experts (one (A) is younger, while the other is older) have a discussion about a plan to develop the company together. At first sight, the two experts all own abundant professional knowledge. We know that although the older is affluent in working experience, he has conservative thought, which might bring on blundering away. On the contrary, the younger has an open idea and powerful ability in assimilating exoteric knowledge and information, however, he is short of abundant working experience, which leads to rash advance (i.e., more haste, less speed). Regarding to this condition, when we allocate the quantity of m (•) to them, how will we solve this fusion problem?

Defining a generalized DSm theory: Let n evidential sources (S_1, S_1 ... S_k), here we work out a uniform way in dealing with the homogeneous and heterogeneous information sources. So we get the discernment frame $\Omega = \{\theta_1, \theta_2 \dots \theta_n\}$, m (•) is the basic belief assignment, let m_i (•) (i = 1, 2, 3 ... k) be the evidential source S_i's observation and let p_i represent its corresponding estimation correctness rate in history, considering $\sum_{A\in D^\Omega} m_i(A) = 1$, let $m_i(\theta_1 \cup \theta_2 \cup \dots \cup \theta_n) = 1 - p_i + p_i q_i$, here q_i represents the original quantity allocated to the total ignorance before Generalization and then this is because of existing occurrence of malfunction, that is, $\sum_{A\in D^\Omega} m_i(A) = p_i$, we assign the quantity 1 - p_i to the total ignorance again. Here we give a very simple instance ($\Omega = \{\theta_1, \theta_2\}$ and two evidential source m_1 (•), m_2 (•)) as shown in Table 1.

So we may generalize the combinational rule of DSmT as follows:

The classical DSm rule for $k \geq 2$ sources for free DSmT model is expressed as:

$$\begin{gathered}\forall A \neq \Phi \in D^{\Omega} \\ m_{M^{f(\Theta)}}(A) \cong [m_1 \oplus \cdots m_k](A) = \\ \sum_{\substack{X_1 \cdots X_i \in D^{\Theta} \\ (X_1 \cap \cdots \cap X_k) = A}} \prod_{i=1}^{k} p_i * m_i(X_i)\end{gathered} \quad (4)$$

The hybrid DSm rule of combination for $k \leq 2$ sources is expressed as:

$$\begin{gathered}\forall A \in D^{\Omega} \\ m_{M(\theta)}(A) = \phi(A)\left[S_1(A) + S_2(A) + S_3(A)\right]\end{gathered} \quad (5)$$

Table 1: Different in the BBA between DsmT and G-DSmT

	DSmT		G-DSmT	
	m_1 (A)	m_1 (A)	m_1 (A)	m_1 (A)
ϕ	0	0	0	0
$\theta_1 \cap \theta_2$	0	0	0	0
θ_1	0.9	0.7	0.81	0.56
θ_2	0.1	0.3	0.09	0.24
$\theta_1 \cup \theta_2$	0	0	0.1	0.2
Remark	$p_1 = 0.9$, $p_2 = 0.8$; $q_1 = 0$, q_2=0			

where

$$S_1(A) = m_{M^{f(\Theta)}}(A) \cong \sum_{\substack{X_1 \cdots X_i \in D^{\Theta} \\ (X_1 \cap \cdots \cap X_k) = A}} \prod_{i=1}^{k} p_i * m_i(X_i)$$

$$S_3(A) \cong \sum_{\substack{X_1 \cdots X_i \in D^{\Theta} \\ (X_1 \cap \cdots \cap X_k) = \phi \\ (X_1 \cup \cdots \cup X_k) = A}} \prod_{i=1}^{k} p_i * m_i(X_i)$$

$$S_2(A) \cong \sum_{\substack{X_1 \cdots X_i \in \phi \\ \{(u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k)) = A\} \vee \\ \{[(u(X_1) \cup u(X_2) \cup \cdots \cup u(X_k)) \in \phi] \wedge (A = I_t)\}}} \prod_{i=1}^{k} p_i * m_i(X_i)$$

Supposed let p_i = (i = 1, 2, 3, ... k), in fact it becomes the free and hybrid DSmT model at once. So the combinational rules for free and hybrid DSmT model are a kind of special situation of G-DSmT. The G-DSmT pushes further the application of plausible and paradoxical reasoning. It not only may apply to any field where the DST or DSmT works, but also can deal with unreliable and independent evidential sources and even can solve the coupling ones (we will versify it in detail in next paper). We have recognized that the DSmT originated from the Probability theory and DST can settle a wider class of fusion problem directly and efficiently and owns a greater superiority than DST and the others, which can be found by Dezert and Smarandache (2003c) etc, let alone the G-DSmT.

ANALYSIS ON AN NUMBER EXAMPLE

In this section we give an example of the mobile robot, supposed where our new G-DSm theory is applied. To make the mobile robot located precisely, two different kinds of sensors (ultrasonic and laser) are used to work together. For the convenience of calculation, here we only assume one laser and one sonar rangefinder and the precision of location is scaled as precise and not precise, which is represented respectively as $\Omega = \{\theta_1, \theta_2\}$ in the discernment frame Ω. Let m_u (•), m_l (•) represent the basic belief assignment to ultrasonic and laser sensor and assume correctness rate p_u of ultrasonic telemeter in history is 0.9, the laser one p_l = 0.8. We know the hyper-power set $D^\Omega = \{\phi, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$ and then may calculate it with DSmT and G-DSmT only with regards as the free model respectively in the following Table 2:

Table 2: Calculating result from DSmT and G-DSmt shown

	DSmT			G-DSmT		
	m_u (A)	m_l (A)	m^f (A)	m_u (A)	m_l (A)	m^f (A)
ϕ	0	0	0	0	0	0
$\theta_1 \cap \theta_2$	0	0	0.34	0	0	0.2448
θ_1	0.9	0.7	0.63	0.81	0.56	0.6716
θ_2	0.1	0.3	0.03	0.09	0.24	0.0636
$\theta_1 \cup \theta_2$	0	0	0	0.1	0.2	0.02
Σ	1	1	1	1	1	1
Remark	$p_1 = 0.9$, $p_2 = 0.8$; $q_1 = 0$, $q_2 = 0$					

Results from the comparison in Table 2:

- For existing unreliability, the basic belief assignment of each source will decrease, for example, m_u (θ_1) transfers from 0.9 to 0.81, m_u (θ_2) transfers from 0.1 to 0.09; at the same time, the laser rangefinder also has the similar situation. However, the belief vacuous belief assignment VBA) will increase. It means that with the incensement of unreliability, the supporting measure to elements over the hyper-power set except the unknown hypotheses will decrease, which answers for the physical system and intuition of human.
- The fusion result of two unreliable evidence sources is fused according to the Eq. 4. Though here $m_g^f(\theta_i) > m^f(\theta_i)(i \in 1,2)$, we don't give a conclusion at once that anytime the basic belief assignments of all focal elements will satisfy the condition. That is, the real state must be reflected through the computation.
- It is shown clearly from the above Table 2 that the G-DSmT deal with the fusion problem more rational than DSmT, because it at least assigns the sensors' occurrence of malfunction to the total ignorance of system and even increases the ability in fusing information.

CONCLUSIONS

In this study, considering the limitation of DSm evidence theory in unreliable evidence sources, we generalize it to G-DSmT, in order to extend the application range of information fusion theory (DSmT) and improve the precision of fusion. We believe that G-DSmT will have more powerful vitality in society of information fusion presently or in the future.

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