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Technical Note-An Integrated Single-vendor Single-buyer Inventory System with Shortage Derived Algebraically

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Abstract: Earlier studies derived the integrated vendor-buyer system with shortages using algebraic method to determine the optimal replenishment policy. In this note, we will offer a simple algebraic approach to replace their sophisticated algebraic skill.

Key words: Integrated inventory system, single-vendor single-buyer, shortage

INTRODUCTION

The EOQ (Economic Order Quantity) model is widely used by practitioners as a decision-making tool for the control of inventory. However, for minimizing the total relevant costs, in most previous all published papers that have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. But Ronald *et al*. (2004) thought that their algebraic procedure is too sophisticated to be absorbed by ordinary readers. Hence, Ronald *et al*. (2004) derived a procedure to transform a two-variable problem into two steps and then, in each step, they solve a one-variable problem using only the algebraic method without referring to calculus. Recently, Chang *et al*. (2005) rewrote the objective function of Ronald *et al*. (2004) such that the usual skill of completing the square can handle the problem without using their sophisticated method.

Recently, we study the paper of Wu and Ouyang (2003) that extended Yang and Wee's model (2002) to investigated the integrated single-vendor single-buyer inventory system with shortage using algebraic method. But Wu and Ouyang's (2003) method had the same problem as Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) described as Ronald *et al*. (2004). Therefore, in this note, we will offer a simple algebraic approach same as Chang *et al*. (2005) to replace Wu and Ouyang's (2003) sophisticated algebraic skill. This method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus as Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) stated.

ALGEBRAIC IMPROVEMENT IN THE WU AND OUYANG'S MODEL

We adopt the same notation and assumptions as Wu and Ouyang (2003) in this note. From Eq. 4 in Wu and Ouyang (2003), we know the integrated vendor-buyer total cost per year, TC(Q, B), can be expressed as

$$TC(Q,B)=\frac{dC_b}{Q}+\frac{H_b(Q-B)^2}{2Q}+\frac{S_bB^2}{2Q}+\frac{dC_v}{nQ}+\frac{QH_v}{2}\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right] \quad (1)$$

Our goal is to find the minimum solution of TC(Q, B) by algebraic approach. Then we rewrite Eq. 1 as

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$$TC(Q,B)=\frac{S_b+H_b}{2Q}\left[B-\frac{H_b}{S_b+H_b}Q\right]^2-\frac{H_b^{\ 2}}{2(S_b+H_b)}Q+\frac{H_b}{2}Q+\frac{d\left(C_b+\frac{C_v}{n}\right)}{Q}+\frac{QH_v}{2}\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right] \quad (2)$$

It implies that when Q is given, we can set B as $B=B(Q)=\frac{H_b}{S_b+H_b}Q$ to get the minimum value of TC(Q, B) as follows:

$$TC(Q,B(Q))=-\frac{H_b^{\ 2}}{2(S_b+H_b)}Q+\frac{H_b}{2}Q+\frac{d\left(C_b+\frac{C_v}{n}\right)}{Q}+\frac{QH_v}{2}\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right] \quad (3)$$

Then we rewrite Eq. 3 as

$$TC(Q,B(Q))=\left[\sqrt{\left[\frac{H_b+H_v\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right]}{2}-\frac{H_b^{\ 2}}{2(S_b+H_b)}\right]Q}-\sqrt{\frac{d\left(C_b+\frac{C_v}{n}\right)}{Q}}\right]^2 \quad (4)$$
$$+\sqrt{2d\left(C_b+\frac{C_v}{n}\right)\left(\frac{H_bS_b}{S_b+H_b}+H_v\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right]\right)}$$

Then we can obtain the optimal buyer's lot size

$$Q^*=\sqrt{\frac{2d(S_b+H_b)\left(C_b+\frac{C_v}{n}\right)}{H_bS_b+H_v(S_b+H_b)\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right]}} \quad (5)$$

and the optimal buyer's maximum shortage level

$$B^*=\frac{H_b}{S_b+H_b}Q^* \quad (6)$$

Therefore, the minimum value of the integrated vendor-buyer total cost per year TC(Q*, B*) is

$$TC(Q^*,B^*)=\sqrt{2d\left(C_b+\frac{C_v}{n}\right)\left(\frac{H_bS_b}{S_b+H_b}+H_v\left[n\left(1-\frac{d}{p}\right)+\frac{2d}{p}-1\right]\right)} \quad (7)$$

Equation 5-7, in this note, are the same as equations in Wu and Ouyang (2003), respectively. Our procedure avoids the difficult decomposition, as in equations in Wu and Ouyang (2003). We think this method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

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REFERENCES

Cárdenas-Barrón, L.E., 2001. The economic production quantity (EPQ) with shortage derived algebraically. Intl. J. Prod. Econ., 70: 289-292.

Chang, S.K. J., J.P.C. Chuang and H.J. Chen, 2005. Short comments on technical note-The EOQ and EPQ models with shortages derived without derivatives. Intl. J. Prod. Econ., 97: 241-243.

Grubbström, R.W. and A. Erdem, 1999. The EOQ with backlogging derived without derivaties. Intl. J. Prod. Econ., 59: 529-530.

Ronald, R.J., G.K. Yang and P. Chu, 2004. Technical note. The EOQ and EPQ models with shortages derived without derivatives. Intl. J. Prod. Econ., 92: 181-184.

Wu, K.S. and L.Y. Ouyang, 2003. An integrated single-vendor single-buyer inventory system with shortage derived algebraically. Prod. Plan. Contl., 14: 555-561.

Yang, P.C. and H.M. Wee, 2002. The economic lot size of the integrated vendor-buyer inventory system derived without derivatives. Opt. Contl. Applied Meth., 23: 163-169.