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The Optimal Method of Nonlinear Decoding for Nonlinear Encoding of QAM Signal

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Abstract: Nonlinear encoding and decoding are common techniques of digital signal processing that are widely used in digital communications. By using the principle of optimization, the paper proposes a simple method to decode for nonlinear encoding of QAM signal. This method can be used for both floating- point and fixed-point calculations. The calculating error can be easily controlled by using this method. This method has been successfully used for designing commercial communication products.

Key words: Nonlinear encoding, nonlinear decoding, QAM, optimization

INTRODUCTION

In modern digital communications, a nonlinear companding technique which is widely used in digital signal processing is also called nonlinear encoding and decoding (Gibent, 1992). For example, a technique of encoding and decoding of PCM signals (μ-law of A-law companding) (Proakis, 2001). This nonlinear companding technique can also be used in QAM signal communications. Before transmitting QAM signal, doing nonlinear companding encoding can make QAM signals with different energy can obtain new nonlinear scale relations, so as to improve accuracy of QAM signals discrimination (Xi and Zhao, 1983). Nonlinear encoding of QAM signal (Fig. 1).

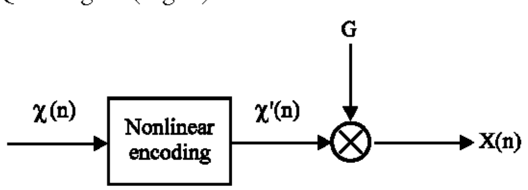

Fig. 1: Nonlinear encoding frame of QAM signal

As shown in Fig. 1, G is the energy adjusting factor. Its purpose is to make sure that passing through nonlinear encoding, signals transmission cannot change their original average energy. Represented by formula is:

$$\chi(n) = \chi_r(n) + j\chi_i(n) \tag{1}$$

$$X(n) = G\chi'(n) = G\chi(n)\Delta(n) \tag{2}$$

where, $\Delta(n)$ is the nonlinear function of $|x(n)|$.

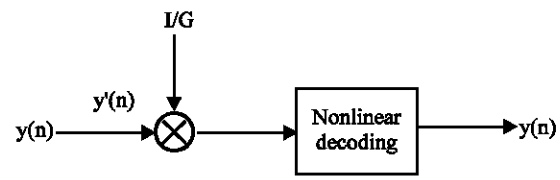

Fig. 2: Nonlinear decoding frame of QAM signal

For receiving signal Y(n) should do nonlinear decoding correspondingly, nonlinear decoding of QAM signal (Fig. 2). Expressed by formula is:

$$Y(n) = Y_r(n) + jY_i(n) \tag{3}$$

$$y'(n) = \frac{1}{G}Y(n) \tag{4}$$

$$y(n) = y'\Delta^{-1}(n) \tag{5}$$

where, $\Delta^{-1}(n)$ is the function of $|Y(n)|$.

The determination of $\Delta^{-1}(n)$ is the key of realizing nonlinear decoding. Generally, there are two difficulties of decoding for nonlinear encoding of QAM signal. Firstly, searching for the function of $\Delta(n)$directly. Owning to the nonlinear characteristic of $\Delta(n)$, it is usually difficult to find out the precise formulation of $\Delta^{-1}(n)$ directly in practical application. So in the processing of decoding, we introduce noise, losing the meaning of nonlinear encoding of QAM signal. Secondly, even through we can find out the precise formulation of inverse function of $\Delta(n)$, because $\Delta(n)$ and $\Delta^{-1}(n)$ are nonlinear function, it also has several difficulties to calculate precisely to the nonlinear function in practical program, especially to the fixed-point calculation. Next we will propose a simple method to solve the above two difficulties fundamentally.

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MATHEMATICAL MODEL

Given that

$$F(n) = \frac{1}{G}\Delta^{-1}(n) \tag{6}$$

Thus,

$$y(n) = F(n)Y(n) \tag{7}$$

This question is turned into finding out F(n). Suppose that

$$f(n) = \alpha_m |Y(n)|^m + \alpha_{m-1} |Y(n)|^{m-1} + ... + \alpha_0 \tag{8}$$

Where, f(n) is not only a polynomial approximant expression of F(n), but also a polynomial with m degree of |Y(n)|. So this question is turned into the problem of how to find out f(n), that is how to determine the coefficients with $\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0$ of f(n).

Assume that $\overrightarrow{x(n)}$ n=0,1,2,...,N are a set of QAM signals without nonlinear encoding, $\overrightarrow{Y(n)}$ n=0,1,2...,N are corresponding receiving signals passing through nonlinear decoding. So, nonlinear decoding successfully should be represented as decoding error shown in Eq. 9.

$$E = \left|\overrightarrow{y(n)} - \overrightarrow{x(n)}\right| \tag{9}$$

reaching a minimum.

According to Eq. 7 and 8, structuring function $H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0)$,

$$H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0) = \sum_{n=0}^{N} |(Y(n)f(n) - x(n))|^2 \tag{10}$$

Then according to Eq. 1 and 3, we can obtain

$$H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0) = \sum_{n=0}^{N} \{Y_r(n)f(n) - x_r(n))^2 + (Y_i(n)f(n) - x_i(n))^2\} \tag{11}$$

Therefore, the question of decoding error minimum is turned into determining the value about $\alpha_m, \alpha_{m-1}, \Lambda, \alpha_1, \alpha_0$, to make sure function $H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0)$ reaching a minimum

$$H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0) = \min \sum_{n=0}^{N} \{Y_r(n)f(n) - x_r(n))^2 + (Y_i(n)f(n) - x_i(n))^2\} \tag{12}$$

That is, the optimization question of Eq. 11. According to the condition of function $H(\alpha_m, \alpha_{m-1}, ..., \alpha_1, \alpha_0)$ reaching a minimum, suppose, $\frac{\partial H}{\partial \alpha_i} = 0$ (i = m,m-1,....,0),

$$\frac{\partial H}{\partial \alpha_i} = \sum_{n=0}^{N} \left\{2(Y_r(n)f(n) - x_r(n))Y_r |Y(n)|^i + 2(Y_i(n)f(n) - x_i(n))Y_i(n)|Y(n)|^i\right\} \quad i = 0,1,....,m \tag{13}$$

$$\frac{\partial H}{\partial \alpha_i} = \sum_{n=0}^{N} 2\left\{Y_r^2(n)f(n) - x_r(n)Y_r(n) + Y_i^2(n)f(n) - x_i(n)Y_i(n)\right\}|Y(n)|^i \tag{14}$$

$$\frac{\partial H}{\partial \alpha_i} = \sum_{n=0}^{N} 2\left\{Y_r^2(n)f(n) - x_r(n)Y_r(n) + Y_i^2(n)f(n) - x_i(n)Y_i(n)\right\}|Y(n)|^i \tag{15}$$

$$\frac{\partial H}{\partial \alpha_i} = \sum_{n=0}^{N} 2\left\{|Y(n)|^i f(n) - (x_r(n)Y_r(n) + x_i(n)Y_i(n))\right\}|Y(n)|^i \quad i = 0,1,....,m \tag{16}$$

Because of $\frac{\partial H}{\partial \alpha_i} = 0$ (i = m, m-1,....,0),

$$\sum_{n=0}^{N} f(n)\left|Y(n)\right|^{i+2} = \sum_{n=0}^{N}(x_r(n)Y_r(n) + x_i(n)Y_i(n))\left|Y(n)\right|^{i} \quad i = 0,1,...,m \tag{17}$$

Substituting (8) into (16), we can obtain

$$\sum_{n=0}^{N}\left|Y(n)\right|^{i+2}(\alpha_m\left|Y(n)\right|^{m} + \alpha_{m-1}\left|Y(n)\right|^{m-1} + ... + \alpha_0) = \sum_{n=0}^{N}(x_r(n)Y_r(n) + x_i(n)Y_i(n))\left|Y(n)\right|^{i} \quad i = 0,1,...,m \tag{18}$$

Suppose that $a_{ij} = \sum_{n=0}^{N}\left|Y(n)\right|^{m+i+2-j}$ i,j=0,1,2,....,m (19)

$$b_i = \sum_{n=0}^{N}(x_r(n)Y_r(n) + x_i(n)Y_i(n))\left|Y(n)\right|^{i} \tag{20}$$

Then we can obtain linear system of equations of α_m, α_{m-1}, ..., α_0,

$$\begin{pmatrix} a_{00} & a_{01} & \cdots & a_{0m} \\ a_{10} & a_{11} & \cdots & a_{1m} \\ \cdots & & & \\ a_{m0} & a_{m1} & \cdots & a_{mm} \end{pmatrix} \begin{pmatrix} \alpha_m \\ \alpha_{m-1} \\ \cdots \\ \alpha_0 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ \cdots \\ b_m \end{pmatrix} \tag{21}$$

Solving about (21) so as to get f(n).

As for number of terms N of summation Σ, 6 120 we can determine it in accordance with the number of constellation points of QAM signals.

APPLICATION

Now, we will give an illustration of application. This is a modem adopting this optimal method of nonlinear decoding for nonlinear encoding of QAM signals. Next, we will introduce this modem roughly.
In this modem, Δ(n) is defined as follows:

$$\Delta(n) = 1 + \frac{\xi(n)}{6} + \frac{\xi^2(n)}{120} \tag{22}$$

Where, $\phi = 0.3125$, ρ is average energy of signal χ(n) (Anonymous, 1996).

$$\xi(n) = \frac{\phi(\chi_r^2(n) + \chi_i^2(n))}{\rho} \tag{23}$$

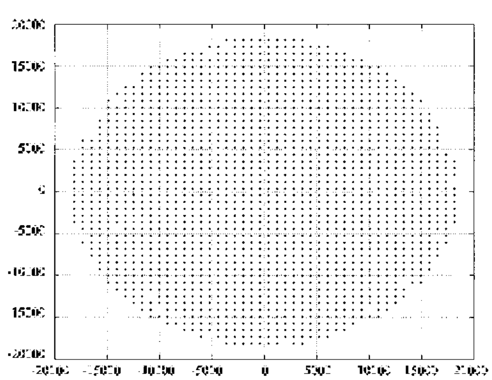

Fig. 3: Original signals without nonlinear encoding

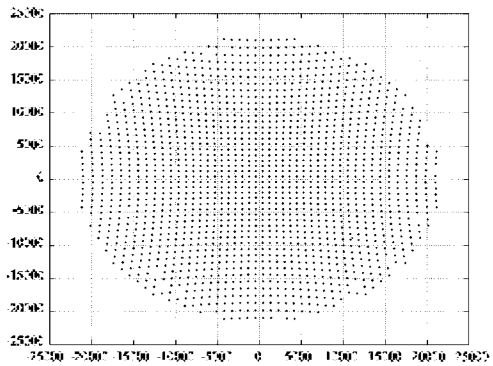

Fig. 4: Signals passing through nonlinear encoding

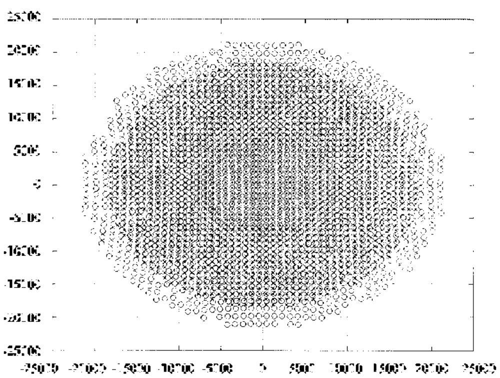

Fig. 5: Comparison of signals with and without nonlinear encoding

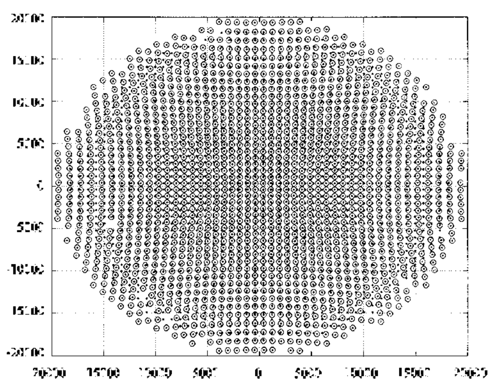

Fig. 6: Comparison of signals after nonlinear encoding multiplied by energy adjusting factor and original signals

Figure 3 shows the 15760-point QAM signal constellation for this modem (Tang and Song, 2001), which achieves a data rate of 33.6 kbs^{-1}.

Figure 4 shows the 15760-point QAM signal constellation passing through nonlinear encoding for this modem, which achieves a data rate of 33.6 kbs^{-1}.

Figure 5 shows the 15760-point QAM signal constellation with and without nonlinear encoding for this modem, which achieves a data rate of 33.6 kbs^{-1}. In Fig. 5, point is represented as signals before nonlinear encoding, ring is represented as signals after nonlinear encoding.

Figure 6 shows the comparison of signals with nonlinear encoding multiplied by energy adjusting factor and original signals for this modem, which achieves a data rate of 33.6 kbs^{-1}. Figure 6 indicates the nonlinear feature of encoding. In Fig. 6, point is represented as signals before nonlinear encoding, ring is represented as signals after nonlinear encoding multiplied by energy adjusting factor.

Suppose that

$$f(n) = 1 + \alpha_1 |Y(n)|^2 + \alpha_2 |Y(n)|^4 + \alpha_3 |Y(n)|^6 \qquad (24)$$

When N, 15760 we can obtain the linear system of equations corresponding to (21).

$$\begin{pmatrix} 109.107130 & 33.413446 & 11.090156 \\ 33.413446 & 11.090156 & 3.880389 \\ 11.090156 & 3.880389 & 1.408472 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -40.1250226 \\ -11.980031 \\ -3.914613 \end{pmatrix} \qquad (25)$$

Solving about (25) so as to get

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -0.492102 \\ 0.530889 \\ -0.367193 \end{pmatrix}$$

So with regard to 16-fixed point calculation,

$$f(n) = 1 - 16125\,|Y(n)|^2 + 17396\,|Y(n)|^4 - 12032\,|Y(n)|^6 \qquad (26)$$

Figure 7 shows the 15760-point QAM signal constellation passing through nonlinear decoding

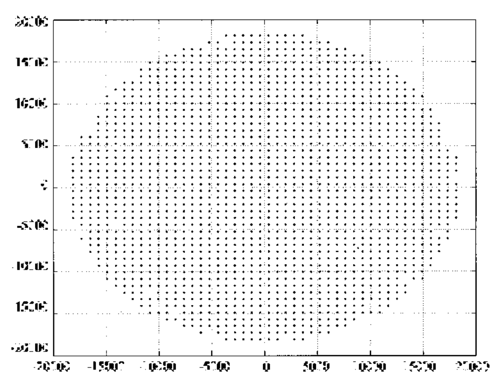

Fig. 7: Restoring signal with nonlinear encoding, coinciding with Fig. 3

Table 1: Error distribution of 15760-point QAM signal with nonlinear decoding of 16-fixed point calculation

Error δ	0≤δ<1	1≤δ<2	2≤δ<3	3≤δ<4	4≤δ<5	5≤δ<6	6≤δ<7
Point	3057	11559	996	28	26	23	21

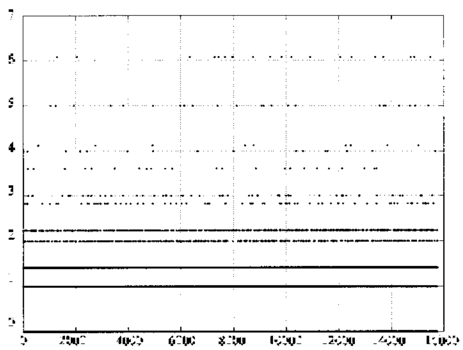

Fig. 8: Error distribution of signals with nonlinear decoding and original signals

for this modem, which achieves a data rate of 33.6 kbs^{-1}.

Figure 8 shows error distribution of 15760-point QAM signal passing through nonlinear decoding and original signals on the condition of 16-fixed point calculation (Chen *et al*., 2003). Table 1 lists the data of error distribution.

CONCLUSIONS

Above application use a optimal method solving the problem of decoding for nonlinear encoding of QAM signal. It is simple to control range of error. No matter whether floating-point or fixed-point calculation, it is also effective to adoption. This method has been successfully used for designing commercial communication products.

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