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Characteristics of Nonlinear Tm Surface Waves in an Interface of Antiferromagnet and Left-handed Metamaterial Structure

¹M.S. Hamada, ²H.S. Ashour, ¹A.I. Ass'ad and ³M.M. Shabat

¹Department of Physics, Al-Aqsa University, Gaza Strip, Palestinian Authority

²Department of Physics, Al-Azhar University, Gaza Strip, Palestinian Authority

³Department of Physics, Islamic University of Gaza,
 Gaza, P.O. Box 108, Gaza Strip, Palestinian Authority

Abstract: In the present study, nonlinear TM surface wave characteristics have been studied theoretically along a single interface of antiferromagnet and Left-Handed Metamaterial (LHM). The complex wave number of TM surface wave is computed by solving the dispersion equation in order to find out reduced phase and attenuation constants. The effects of operating angular frequency (ω) and non-linearity of antiferromagnet on reduced phase and attenuation constants have been examined. The power flow has also been studied as a function of reduced phase and attenuation constants.

Key words: Antiferromagnet, left-handed material, dispersion relation, power flow

INTRODUCTION

Since the discovery of LHM (Shelby *et al.*, 2001a, b; Veselago, 1968; Shadrivov *et al.*, 2004; Hamada, 2003), there have been attracted attention due to their recent experimental realization and a number of unusual properties observed in experiment. A LHM can also be used as a perfect lens for focusing both propagating and evanescent waves (Shadrivov *et al.*, 2004). A LHM is one that has a frequency band with simultaneously negative $\epsilon_{\text{eff}}(\omega)$ and $\mu_{\text{eff}}(\omega)$ and also have imaginary part, thereby the refractive index of such metamaterial exhibits a negative value in this frequency range as predicted by Veselago (1968). Also the study of nonlinear surface waves in an antiferromagnet have been attracted a significant degree of attention (Wang and Awai, 1998; Wang *et al.*, 2000; Hamada *et al.*, 2002a, b; Beletskii *et al.*, 2002). These studies are considered a key problem of the simulation of a number of opto-microwave-electronic devices. However these studies have been done on structures of an antiferromagnet and right-handed material (RH). Therefore, in the present study we present a nonlinear TM surface waves at microwave frequencies in a layered structure of nonlinear antiferromagnet and linear left-handed metamaterial. The dispersion relation have been solved to calculate both the reduced phase and attenuation constants. The effect of LHM on the power flow has also been examined.

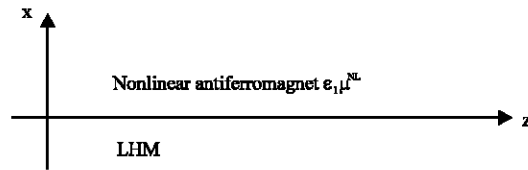


Fig. 1: Coordinate system for the single interface structures

THEORY

The waveguide geometry consists of the LHM which occupies the semi-infinite region $x < 0$ and the two-sublattice uniaxial antiferromagnet medium occupies the region $x > 0$, as shown in Fig. 1.

The TM electromagnetic waves propagate along the z-axis in the xz-plane with a complex wave vector k and an angular frequency ω . The electric and magnetic vectors of the electromagnetic field take the form:

$$\vec{E} = (E_x, 0, E_z) \exp[ik_0(\beta x - ct)], \quad (1-a)$$

$$\vec{H} = (0, H_y, 0) \exp[ik_0(\beta x - ct)], \quad (1-b)$$

where $\beta = k/k_0$ is the complex effective wave index, k is the complex wave number propagation constant, k_0 is the wave number of the free space and c is the velocity of light in vacuum. The complex effective wave index can be written as:

$$\beta = \beta' + i\beta'' \tag{2}$$

where β' is the reduced phase constant and β'' is the reduced attenuation constant.

The permeability tensor, for the absence of an applied Zeeman field, describing the nonlinear response of the crystal to the intense radio frequency (rf) field is a diagonal one (Wang and Awai, 1998; Wang *et al.*, 2000):

$$\mu_{yy}(\omega) = \mu^{NL}(\omega) = \mu^L(\omega) + \chi_{NL}(\omega)|H|^2 \tag{3}$$

where $\mu^L(\omega)$ is the linear permeability and has the form:

$$\mu^L(\omega) = 1 + \frac{2\omega_M\omega_A}{\omega_c^2 - \omega^2} \tag{4}$$

where, $\omega_M = \gamma\mu_0M_s$, $\omega_A = \gamma\mu_0H_A$, $\omega_E = \gamma\mu_0H_E$ and $\omega_c = \sqrt{\omega_A^2 + 2\omega_A\omega_E}$ is the resonance frequency of the system. M_s is the saturation magnetization field, H_A is the anisotropy field, H_E is the exchange field of the crystal and γ is the gyromagnetic ratio. The nonlinear part $\chi_{NL}(\omega)|H|^2$ is always positive for a linearly polarized electromagnetic wave and this means that the crystal is self-focus crystal. The nonlinear susceptibility χ_{NL} can be regarded as a constant in off-resonance frequency

The negative effective dielectric permittivity and magnetic permeability constant, respectively (Shelby *et al.*, 2001) for LHM can be read as:

$$\epsilon_{eff} = 1 - \frac{\omega_p^2 - \omega_{eo}^2}{\omega^2 - \omega_{eo}^2 + i\alpha\omega} \tag{5}$$

where ω_p is the electric plasma frequency and ω_{eo} is the low-frequency edge of the electrical forbidden band and α represents the losses.

$$\mu_{eff} = 1 - \frac{\omega_{mp}^2 - \omega_{mo}^2}{\omega^2 - \omega_{mo}^2 + i\alpha\omega} \tag{6}$$

where ω_{mp} is the magnetic plasma frequency and ω_{mo} is the low-frequency edge of the magnetic forbidden band.

The solution of Maxwell's equations, one can get the following nonlinear differential equation in the crystal:

$$\partial^2 h_y / \partial x^2 - (k_1^2 - k_0^2 \epsilon_1 \chi_{NL} |h_y|^2) h_y = 0, \tag{7}$$

where, $k_1^2 = k_0^2(\beta^2 - \epsilon_1 \mu^L)$, ϵ_1 is the relative dielectric constant.

For the surface wave h_y and $\partial h_y / \partial x$ approach zero as $x \rightarrow \infty$, the solution of Eq. (7) has now a well-known form for TE nonlinear (Mihalache *et al.*, 1989):

$$h_y = \frac{k_1}{k_0} \sqrt{\frac{2}{\chi_{NL} \epsilon_1}} \operatorname{sech}[k_1(x - x_0)], \tag{8}$$

where x_0 indicates the position of the maximum electric field component in the nonlinear cover.

For the superconductor region, the solution of Maxwell's equations for TM wave is given by:

$$h_y = h_2 e^{k_2 x} \tag{9}$$

where h_2 is the amplitude of h_y at $x = 0$ and $k_2 = k_0 \sqrt{\beta^2 - \epsilon_{eff} \mu_{eff}}$. The application of the appropriate boundary conditions for electric and magnetic fields at $x = 0$, leads to the dispersion equation

$$\tanh(k_1 x_0) = \frac{\epsilon_1 k_2}{\epsilon_{eff} k_1} \tag{10}$$

The complex effective wave index and the nonlinearity are then related by:

$$\beta^2 = \frac{\epsilon_1 \epsilon_{eff}}{\epsilon_1^2 - \epsilon_{eff}^2} \left(\mu_1 \mu_{eff} - \epsilon_{eff} \mu^L(\omega) - \frac{1}{2} \epsilon_{eff} \chi_{NL} h_2^2 \right), \tag{11}$$

which is known as another form of the dispersion equation.

The total Power flux (P) of the wave propagation in the z-direction can be written as]:

$$P_{LHM} = \frac{k k_1^2}{2\epsilon_0 \epsilon \epsilon_{eff} \omega \chi_{NL} k_2 k_0} \left[1 - \left(\frac{k_2 \epsilon}{k_1 \epsilon_{eff}} \right)^2 \right], \tag{12-a}$$

$$P_{NL} = \frac{k k_1}{\epsilon_0 \epsilon^2 \omega \chi_{NL} k_0^2} \left[1 + \frac{k_2 \epsilon}{k_1 \epsilon_{eff}} \right], \tag{12-b}$$

where P_{LHM} and P_{NL} is the power flux in LHM and nonlinear media.

COMPUTER SIMULATION AND DISCUSSIONS

In present study, we have used the data for the antiferromagnetic material Ferrous Florid (FeF₂) (Hamada, 2002a,b), where the anisotropy field H_A ,

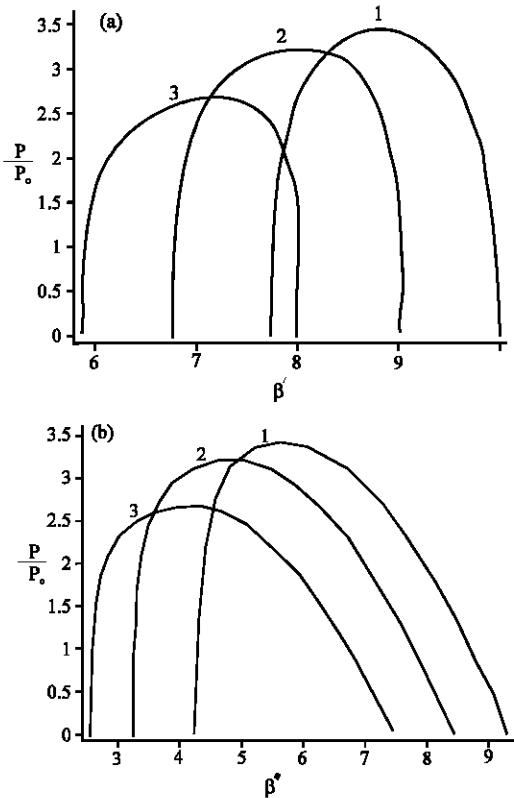


Fig. 2a-b: Normalized total power versus reduced phase constant β' and reduced attenuation constant β'' , respectively, for different values of frequency (f): (1) f = 10.6; (2) f = 10.63 and (3) f = 10.66 (GHz)

exchange field H_E , saturation magnetization M_s , the gyromagnetic ratio γ and the relative dielectric constant ϵ_s of FeF_2 crystal are 1.59×10^4 A/m, 4.3×10^4 A/m, 4.46×10^4 A/m, 1.7×10^{11} (Ts) $^{-1}$ and 4, respectively. The data parameters for the LHM as in (Shelby *et al.*, 2001a, b): $f_{mp} = 10.95$ GHz, $f_{mo} = 10.05$ GHz, $f_{cp} = 12.8$ GHz, $f_{co} = 10.3$ GHz and $\alpha = 10$ MHz.

Once the propagation characteristics are determined from the dispersion Eq. (10-11), the obtained values of the complex propagation constants can be fed to the power expression mentioned in Eq. 12 as shown in Fig. 2. The normalized power P/P_0 has been plotted against β' as shown in Fig. 2a and β'' as shown in Fig. 2b for different values of frequency. The normalized power with $P_0 = \frac{1}{2\chi_{NL}\omega\epsilon_0}$. Fig. 2a shows the strong dependence of the dimensionless normalized power P/P_0 frequency. It is found that P/P_0 increases up to some critical value and then falls to zero. This is related to the fact that in a medium characterized by negative dielectric constant, the energy flux and wave vector have opposite directions

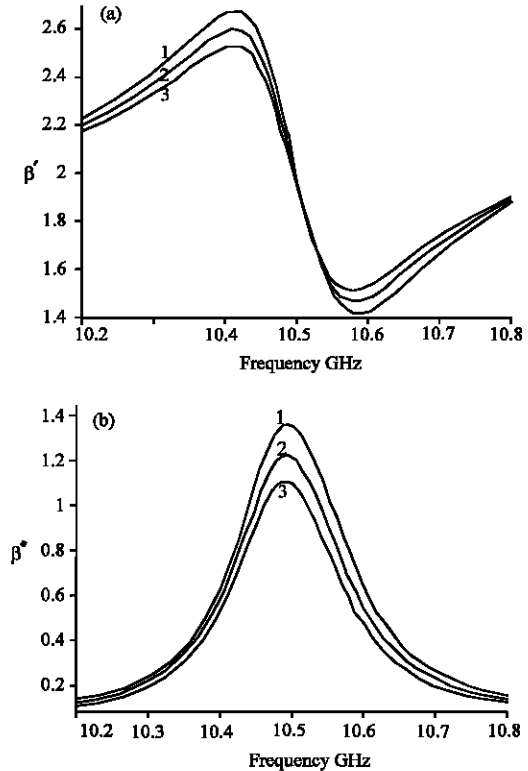


Fig. 3a-b: Computed reduced phase constant β' and reduced phase constant β'' vs. Frequency for different values of $\tan h(k_s x_0)$: (1) 0.8; (2) 0.85; (3) 0.9.

(Mihalache *et al.*, 1989) and for a certain range of β' the normalized power decreases with increasing β'' . It means that the power propagates in LHM substrate more than in the antiferromagnet cladding, where the power takes an opposite direction in LHM to the direction of antiferromagnet. These results, is in contrast with the results for the case of single interface of nonlinear antiferromagnet and Right-handed material (RH) (Wang and Awai, 1998). It has also been noticed that, for a single power level corresponds to two waves with different speeds, that is, one power can excites tow spatial TM waves. This behavior shows some kind of bistability for short range of reduced phase constant, which could be used in the design and construction some opto-microwave devices. Figure (2a-b) show a cut-off in power, which exhibits a different type of optical limiter action under appropriate conditions, that is, the TM surface wave cuts off at a finite guided wave velocity. It has been also seen that the maximum value of power can be reduced by tuning the operating frequency. Figure 2b confirms the bistability which have been noticed in Fig. 2a. The present study also indicates that the attenuation is also changed by tuning the frequency.

The nonlinear dispersion Eq. 11 and 12 have also been solved to compute the complex effective wave index (β) within the frequency interval 10.2-10.8 GHz, where the LHM has a negative refractive index. The nonlinear dispersion equation versus frequency has been calculated for different values of nonlinearity, (Fig. 3). In Fig. 3a, one can notice a region of β' increases from low frequency up to a upper band edge, followed by a sharp decreasing and then the lower passband is found. The region of sharp decreasing can be confirmed in Fig. 3b, where the attenuation constant β'' has a maximum value. The sharp decrease or attenuation can be explained by the fact that the surface waves propagate in the LHM substrate, where the waves take an opposite direction. This phenomena, disappear when the substrate is RH (Hamada *et al.*, 2002a, b). The present study also shows that the attenuation can be controlled both by frequency and nonlinearity.

CONCLUSIONS

Nonlinear TM surface waves in an antiferromagnet - LHM structure has been studied. It has been found that the variation of both β' and β'' with power at different values of frequency, is often regarded as implying the possibility of optical switching. It has also been found some kinds of bistability for short range of reduced phase constant. We have also found that attenuation of TM surface waves in single interface containing LHM is a highly dependent on frequency and nonlinearity.

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