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Homotopy Analysis of Slider Bearing Lubricated With Powell-Eyring Fluid

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Abstract: The analytical study of an infinite, lubricated slider bearing consisting of connected surfaces with Powell-Eyring fluid as lubricant is considered. Under the assumption of the order of magnitude of the variables, it is seen that only viscous and Non-Newtonian terms have effects, where as the inertia terms are negligible. The pressure distribution for inclined slider bearing is calculated approximately by using homotopy analysis method. The variation of pressure and from that the load carrying capacity of the bearing is presented for a range of fluid and bearing parameters.

Key words: Homotopy analysis method, slider bearing, Non-Newtonian fluid, Perturbation analysis, Lubrication theory, Powell-Eyring model

INTRODUCTION

The presence of fluid film greatly reduces the sliding friction between solid objects. The enormous practical importance of this effect has stimulated a great deal of research both theoretical and experimental. The problem of slider bearing with Non-Newtonian lubricants is difficult to analysis mathematically because of the nonlinear character of the governing equations of motion. Numerical methods remain available, but are some what more costly. Yürüsoy (2002) employed the perturbation method to study the problem by introducing a small parameter. In this study, we revisit the problem and solved it approximately by homotopy analysis method introduced by Liao (2004). The homotopy analysis is a powerful new analytic method that remain valid even with strong nonlinearity and with no small or large parameter. The method is successfully applied by (Ayub *et al.*, 2004; Hayat *et al.*, 2003) to discuss different problems of fluid flow. We see from our solution that homotopy analysis method is more general than the perturbation method. The analytical solution so obtained subsumes Yürüsoy's results.

Some relevant studies on Non-Newtonian lubrication in bearing have been published. Ng and Saibel (1962) used a third grade fluid and studied the flow occurring in the slider bearing. Buckholz (1986) used a power law model as a Non-Newtonian lubricant in a slider bearing. Yürüsoy and Pakdemirli (1999) studied the flow of a third grade fluid in a slider bearing and constructed a perturbation solution. Yürüsoy (2002) has investigated second and third grade fluid in a slider bearing by using perturbation technique.

Hansen and Na (1968) considered the similarity solution of the laminar boundary layer problem of the Powell-Eyring model.

FORMULATION OF THE PROBLEM AND FLOW EQUATIONS

In the two dimensional bearing (Fig. 1), the plane y = 0, moves with constant velocity U in the x-direction and the top of the bearing (the slider) is fixed. The variables are Non-dimensional with respect to U and the

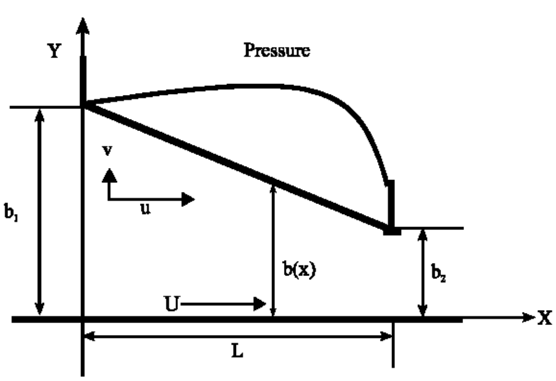

Fig. 1: Slider bearing

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length L of the bearing. We consider the steady flow of an incompressible, gravity free, Powell-Eyring fluid as lubricant in a slider bearing. The continuity and linear momentum equations governing the motion of the fluid are:

$$\mathrm{div}V = 0 \tag{1}$$

$$\mathrm{div}T = \rho\frac{dV}{dt} \tag{2}$$

where V (u, v) is the velocity, ρ is the density, d/dt is the material time derivative and T and is the Cauchy stress tensor.

The Powell-Eyring model can be written as (Hansen, and Na, 1968)

$$\tau_{xy} = \mu\frac{\partial u}{\partial y} + \frac{1}{B}\sinh^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right) \tag{3}$$

where τ_{xy} is a shear stress, μ is viscosity and B and C are constants of the Powell-Eyring model. Introducing the following Non-dimensional parameters

$$x' = x/L,\ y' = y/b_1,\ u' = u/U,\ v' = Lv/b_1U,\ b' = b/b_1,\ p' = pb_1/\rho LU^2 \tag{4}$$

Using (3) and (4) in (1) and (2), the appropriate Non-dimensional equations governing the motion of the fluid (on dropping dashes) are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\delta}\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{1}{\delta^2}\frac{\partial^2 u}{\partial y^2} + \hat{\alpha}\frac{1}{\delta^2}\frac{\frac{\partial^2 u}{\partial y^2}}{\sqrt{\beta\left(\frac{\partial u}{\partial y}\right)^2 + 1}} \tag{6}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\delta^3}\frac{\partial p}{\partial y} + \hat{\alpha}\frac{1}{\delta^2}\frac{\frac{\partial^2 u}{\partial y \partial x}}{\sqrt{\beta\left(\frac{\partial u}{\partial y}\right)^2 + 1}} \tag{7}$$

where, the non dimensional parameters are

$$\mathrm{Re} = \frac{\rho UL}{\mu},\quad \hat{\alpha} = \frac{1}{\rho LBCU},\quad \beta = \frac{U^2}{b_1C^2},\quad \frac{1}{\delta} = \frac{L}{b_1} \tag{8}$$

By assuming that $\delta << 1$, $\frac{1}{\mathrm{Re}}$ is of order δ and $\hat{\alpha}$ is also of order δ($\hat{\alpha} = \delta\hat{\gamma}$). Under these assumptions, the largest terms in Eq. (6) and (7) are:

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} + \hat{\gamma}\frac{\frac{\partial^2 u}{\partial y^2}}{\sqrt{\beta\left(\frac{\partial u}{\partial y}\right)^2 + 1}} \tag{9}$$

$$\frac{\partial p}{\partial x} = 0 \tag{10}$$

making use of (10) in (6), we get

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \hat{\gamma}\frac{\frac{\partial^2 u}{\partial y^2}}{\sqrt{\beta\left(\frac{\partial u}{\partial y}\right)^2 + 1}} \tag{11}$$

Equation (11) is highly Non-linear differential equation and cannot be solved easily. On expanding the term $\sqrt{\beta\left(\frac{\partial u}{\partial y}\right)^2 + 1}$ as a power series up to $O(\beta^2)$, we have

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \hat{\gamma}\frac{\partial^2 u}{\partial y^2}\left(1 - \frac{1}{2}\beta\left(\frac{\partial u}{\partial y}\right)^2 + O(\beta^2)\right) \tag{12}$$

The continuity and momentum equations governing the motion of the fluid (after truncating the series) are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + \hat{\gamma}\frac{\partial^2 u}{\partial y^2}\left(1 - \frac{1}{2}\beta\left(\frac{\partial u}{\partial y}\right)^2\right). \tag{13}$$

After some re-arrangement, Eq. (13) becomes

$$\frac{dp}{dx} = (1+\hat{\gamma})\frac{\partial^2 u}{\partial y^2} - \frac{1}{2}\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \tag{14}$$

where $\hat{\gamma}$ and β are dimensionless material constants.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{d\hat{p}}{dx} = \frac{\partial^2 u}{\partial y^2} + \hat{R}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \tag{15}$$

subject to the dimensionless boundary conditions

$$u(0)=1,\quad u(b)=0,\quad v(0)=0,\quad v(b)=0 \tag{16}$$

where $\hat{p}$ and $\hat{R}$ are defined as:

$$\hat{p}=\frac{1}{1+\tilde{\gamma}}p \quad\text{and}\quad \hat{R}=-\frac{1}{2}\frac{\beta\tilde{\gamma}}{(1+\tilde{\gamma})} \tag{17}$$

In the sequel, we use the homotopy analysis technique to give analytic and uniformly valid solution of the problem.

HOMOTOPY ANALYSIS METHOD

We construct the zeroth order deformation equation, as in reference (Ayub *et al*., 2004)

$$(1-q)\ell\left[\tilde{u}(y,q)-u_0(y)\right]= qh\left[\frac{\partial^2\tilde{u}(y,q)}{\partial y^2}+\hat{R}\left(\frac{\partial\tilde{u}(y,q)}{\partial y}\right)^2\frac{\partial^2\tilde{u}(y,q)}{\partial y^2}-\frac{d\hat{p}}{dx}\right] \tag{18}$$

subject to the boundary conditions

$$\left.\begin{aligned}\tilde{u}(y,q)&=1 \ \text{at}\ y=0\\ \tilde{u}(y,q)&=0 \ \text{at}\ y=b\end{aligned}\right\} \tag{19}$$

where h is an auxiliary parameter, $u_0(y)$ is an initial guess approximation and q is an embedding parameter such that $q \in [0, 1]$.

We choose the auxiliary linear operator

$$\ell=\frac{d^2}{dy^2} \tag{20}$$

and an initial guess approximation

$$u_0(y)=\frac{1}{2}\frac{d\hat{p}}{dx}(y^2-yb)+\left(1-\frac{y}{b}\right) \tag{21}$$

which can be obtained by solving Eq. (15) with $\hat{R}=0$ subject to the boundary conditions.

Setting q = 0 in (18), we get

$$\tilde{u}(y,0)=u_0(y),\quad y>0 \tag{22}$$

and setting q = 1 in (18), we have

$$\tilde{u}(y,1)=u(y)\,. \tag{23}$$

Therefore, according to (22) and (23) the variation of q from 0 to 1 is just the continuous variation $\tilde{u}(y,q)$ from the initial guess approximation $u_0(y)$ to the unknown solution u(y) of (15) and (16).

Assume that the deformation u(y,q) governed by (18) and (19) is smooth enough so that

$$u_0^{(k)}(y)=\left.\frac{\partial^k\tilde{u}(y,q)}{\partial q^k}\right|_{q=0}\quad k\geq 1 \tag{24}$$

namely the k-th order deformation derivative exists.

Then, in view of Eq. (24) and Taylor's formula, we expand $\tilde{u}(y,q)$ in the power series

$$\tilde{u}(y,q)=u_0(y)+\sum_{k=1}^{\infty}\left(\frac{u_0^{(k)}(y)}{k!}\right)q^k \tag{25}$$

We note that the convergence region of the above infinite series is dependent upon h ($\neq 0$).

We define

$$u_k(y)=\frac{u_0^{(k)}(y)}{k!},\quad k\geq 1 \tag{26}$$

Using (21), (23) and (24), we get at q = 1, the important relationship

$$u(y)=\sum_{k=0}^{\infty}u_k(y) \tag{27}$$

between the initial guess approximation $u_0(y)$ and the unknown solution u(y).

Now, differentiating the zeroth order deformation (18) and (19) k-times with respect to q and then setting q = 0 we obtain for $k\geq 1$ the kth-order deformation equation

$$\ell\left[u_k(y)-\chi_k u_{k-1}(y)\right]=hR_k(y), \tag{28}$$

with the following boundary conditions

$$u_k(0)=u_k(b)=0 \tag{29}$$

in which

$$R_k=u''_{k-1}+\hat{R}\sum_{j=0}^{k-1}u''_{k-1-j}\sum u'_i u'_{j-1}-(1-\chi_k)\frac{d\hat{p}}{dx} \tag{30}$$

and

$$\chi_k=\begin{cases}0, & k\leq 1\\ 1, & k\geq 2\end{cases} \tag{31}$$

where prime denotes derivative with respect to y.

First order homotopy solution: By putting k = 1 in Eq. (26) and (28), we obtained first order solution. In particular differentiating (18) with respect to q, making use of (24) and setting q = 0, we have

$$\ell\left\{u_0^{(1)}\right\} = h\left\{\frac{\partial^2 u_0}{\partial y^2} + \hat{R}\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial y^2} - \frac{d\hat{p}}{dx}\right\} \tag{32}$$

Using (20) and (21) in (32), we obtained a second order differential equation of the form

$$\frac{d^2 u_0^{(1)}}{dy^2} = h\left[\frac{d\hat{p}}{dx} + \hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(y^2 + \frac{b^2}{4} - by\right) + \frac{1}{b^2}\left(\frac{d\hat{p}}{dx}\right) - \left(\frac{2y}{b} - 1\right)\left(\frac{d\hat{p}}{dx}\right)^2 - \frac{d\hat{p}}{dx}\right\}\right] \tag{33}$$

which after simplification is

$$\frac{d^2 u_0^{(1)}}{dy^2} = h\hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(y^2 + \frac{b^2}{4} - by\right) + \frac{1}{b^2}\left(\frac{d\hat{p}}{dx}\right) - \left(\frac{2y}{b} - 1\right)\left(\frac{d\hat{p}}{dx}\right)^2\right\} \tag{34}$$

Now integrating (34) twice with respect to y, we get

$$u_0^{(1)} = h\hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12} + \frac{b^2y^2}{8} - \frac{by^3}{6}\right) + \left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^3}{3b} - \frac{y^2}{2}\right) + \left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2}\right) + \alpha_1 y + \alpha_2\right\} \tag{35}$$

where α_1 and α_2 are integration constants. Using boundary conditions (16) in (35), we get

$$u_0^{(1)} = h\hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12} + \frac{b^2y^2}{8} - \frac{by^3}{6} - \frac{b^3y}{24}\right) + \left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2} - \frac{y^3}{3b} - \frac{by}{6}\right) + \left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2} - \frac{y}{2b}\right)\right\} \tag{36}$$

by putting value of $\hat{R}$, we have

$$u_0^{(1)} = -\frac{1}{2}\frac{\beta\tilde{\gamma}h}{(1+\tilde{\gamma})}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12} + \frac{b^2y^2}{8} - \frac{by^3}{6} - \frac{b^3y}{24}\right) + \left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2} - \frac{y^3}{3b} - \frac{by}{6}\right) + \left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2} - \frac{y}{2b}\right)\right\} \tag{37}$$

Second order homotopy solution: Now Differentiating (18) twice with respect to q, making use of (24) and setting q = 0, we have

$$\ell\left\{u_0^{(2)}\right\} = 2\ell\left\{u_0^{(1)}\right\} + 2h\left(\frac{\partial^2 u_0^{(1)}}{\partial y^2}\right) + 2h\hat{R}\left\{\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0^{(1)}}{\partial y^2} + 2\left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial^2 u_0}{\partial y^2}\right)\left(\frac{\partial u_0^{(1)}}{\partial y}\right)\right\} \tag{38}$$

Using (20) and (21) in (38), we obtained a second order differential equation of the form

$$\frac{d^2 u_0^{(2)}}{dy^2} = 2(1+h)h\hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(y^2 + \frac{b^2y}{4} - \frac{by^2}{2}\right) + \left(\frac{d\hat{p}}{dx}\right)^2\left(y - \frac{y^2}{b}\right) + \left(\frac{d\hat{p}}{dx}\right)\left(\frac{y}{b^2}\right) + O(\beta^2)\right\} \tag{39}$$

Integrating (39) twice with respect to y and using boundary condition (19), gives

$$u_0^{(2)} = (2+2h)h\hat{R}\left\{\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12}+\frac{b^2y^2}{8}-\frac{by^3}{6}-\frac{b^3y}{24}\right)+\left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2}-\frac{y^3}{3b}-\frac{by}{6}\right)+\left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2}-\frac{y}{2b}\right)+O(\beta^2)\right\} \tag{40}$$

Summing up the result, we write

$$\begin{aligned} u &= u_0^{(1)} + \frac{u_0^{(1)}}{1!} + \frac{u_0^{(2)}}{2!} + \ldots = u_0 + u_1 + u_2 + \ldots \\ &= \frac{1}{2}\frac{d\hat{p}}{dx}(y^2 - yb) + \left(1-\frac{y}{b}\right) - \frac{1}{2}\frac{\beta\tilde{\gamma}h}{(1+\tilde{\gamma})}\left\{\begin{aligned}&\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12}+\frac{b^2y^2}{8}-\frac{by^3}{6}-\frac{b^3y}{24}\right)\\ &+\left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2}-\frac{y^3}{3b}-\frac{by}{6}\right)+\left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2}-\frac{y}{2b}\right)\end{aligned}\right\} \\ &\quad - \frac{(1+h)h\beta\tilde{\gamma}}{2(1+\tilde{\gamma})}\left\{\begin{aligned}&\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12}+\frac{b^2y^2}{8}-\frac{by^3}{6}-\frac{b^3y}{24}\right)\\ &+\left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2}-\frac{y^3}{3b}-\frac{by}{6}\right)+\left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2}-\frac{y}{2b}\right)\end{aligned}\right\} + O(\beta^2) \end{aligned} \tag{41}$$

v-COMPONENT OF THE VELOCITY

Substituting (41) in (5), we get

$$\frac{\partial v}{\partial y} = -\frac{d}{dx}\left[\begin{aligned}&\frac{1}{2}\frac{d\hat{p}}{dx}(y^2 - yb) + \left(1-\frac{y}{b}\right)\\ &-\frac{1}{2}\frac{\beta\tilde{\gamma}h}{(1+\tilde{\gamma})}\left\{\begin{aligned}&\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^4}{12}+\frac{b^2y^2}{8}-\frac{by^3}{6}-\frac{b^3y}{24}\right)\\ &+\left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^2}{2}-\frac{y^3}{3b}-\frac{by}{6}\right)+\left(\frac{d\hat{p}}{dx}\right)\left(\frac{y^2}{2b^2}-\frac{y}{2b}\right)\end{aligned}\right\}\end{aligned}\right] \tag{42}$$

Integrating (42) with respect to y and using the condition v(0) = 0 gives

$$v = -\frac{d}{dx}\left[\begin{aligned}&\frac{1}{2}\frac{d\hat{p}}{dx}\left(\frac{y^3}{3} - \frac{by^2}{2}\right) + \left(y-\frac{y^2}{2b}\right)\\ &-\frac{1}{2}\frac{\beta\tilde{\gamma}h}{(1+\tilde{\gamma})}\left\{\begin{aligned}&\left(\frac{d\hat{p}}{dx}\right)^3\left(\frac{y^5}{60}+\frac{b^2y^3}{24}-\frac{by^4}{24}-\frac{b^3y}{48}\right)\\ &+\left(\frac{d\hat{p}}{dx}\right)^2\left(\frac{y^3}{6}-\frac{y^4}{12b}-\frac{by^2}{12}\right)+\left(\frac{d\hat{p}}{dx}\right)\left(\frac{y3}{6b^2}-\frac{y^2}{4b}\right)\end{aligned}\right\}\end{aligned}\right] \tag{43}$$

and using v(b) = 0, in (43) gives

$$\frac{d}{dx}\left[b^3\frac{d\hat{p}}{dx} - \frac{\beta\tilde{\gamma}h}{1+\tilde{\gamma}}\left\{\frac{b^5}{40}\left(\frac{d\hat{p}}{dx}\right)^3 + \frac{b}{2}\frac{d\hat{p}}{dx}\right\}\right] = 6\frac{db}{dx} \tag{44}$$

Equation (44) is a second order nonlinear differential equation in $\hat{p}$ with variable coefficient b(x) subject to the boundary conditions

$$\hat{p}(0) = \hat{p}(1) = 0$$

PRESSURE DISTRIBUTION

We rewrite (44) with h = -1 as:

$$\frac{d}{dx}\left[b^3\frac{d\hat{p}}{dx}-\left(-\frac{1}{2}\frac{\beta\tilde{\gamma}}{(1+\tilde{\gamma})}\right)\left\{\frac{b^5}{20}\left(\frac{d\hat{p}}{dx}\right)^3+b\frac{d\hat{p}}{dx}\right\}\right]=6\frac{db}{dx}, \tag{45}$$

or

$$\frac{d}{dx}\left[b^3\frac{d\hat{p}}{dx}-\hat{R}\left\{\frac{b^5}{20}\left(\frac{d\hat{p}}{dx}\right)^3+b\frac{d\hat{p}}{dx}\right\}\right]=6\frac{db}{dx} \tag{46}$$

Integration w.r.t x, yields

$$\frac{d\hat{p}}{dx}-\frac{\hat{R}}{b^2}\frac{d\hat{p}}{dx}-\frac{\hat{R}b^2}{20}\left(\frac{d\hat{p}}{dx}\right)^3=\frac{6}{b^2}+\frac{A_1}{b^3}, \tag{47}$$

where A_1 is constant of integration.

Again we wish to solve (47) for $\hat{p}$ by using homotopy analysis technique. We construct the zeroth order deformation equation as in (18),

$$(1-q)\ell_1\left[\tilde{\hat{p}}(x,q)-\hat{p}_0(x)\right]=qh\left[\frac{d\hat{p}_0}{dx}-\frac{\hat{R}}{b^2}\frac{d\hat{p}_0}{dx}-\frac{\hat{R}b^2}{20}\left(\frac{d\hat{p}_0}{dx}\right)^3-\frac{6}{b^2}-\frac{A_1}{b^3}\right] \tag{48}$$

subject to the boundary conditions

$$\left.\begin{aligned}\tilde{\hat{p}}(x,q)=0 \quad \text{at} \quad x=0\\ \tilde{\hat{p}}(x,q)=0 \quad \text{at} \quad x=1\end{aligned}\right\} \tag{49}$$

taking the initial gauss approximation as:

$$\hat{p}_0(x)=\frac{6x(b-r)}{b^2(1+r)}=\frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)},$$

where

$$b(x)=(1-x+rx), \qquad r=\frac{b_2}{b_1} \tag{50}$$

in which b_1 is the maximum value of b and b_2 is the minimum value of b.
Defining the linear operator as:

$$\ell_1=\frac{d}{dx} \tag{51}$$

and an embedding parameter q such that $q \in [0, 1]$.
Setting q = 0 in (48), we get

$$\tilde{\hat{p}}(x,0)=\hat{p}_0(x), \qquad x>0, \tag{52}$$

setting q = 1 in (48), we get

$$\tilde{\hat{p}}(x,1)=\hat{p}(x) \tag{53}$$

Therefore, according to (52) and (53), the variation of q from 0 to 1 is just the continuous variation $\tilde{\hat{p}}(x,q)$ from the initial guess approximation $\hat{p}_0(x)$ to the unknown solution $\hat{p}(x)$ of (48).

Assume that the deformation $\tilde{\hat{p}}(x,q)$ governed by (48), (49), (52) and (53) is smooth enough so that

$$\hat{p}_0^{(k)}=\left.\frac{\partial^k\tilde{\hat{p}}(x,q)}{\partial q^k}\right|_{q=0}\qquad k\geq 1 \tag{54}$$

namely the k-th order deformation derivative exists.
Then, according to (54) and Taylor's formula, we have

$$\tilde{\hat{p}}(x,q)=\hat{p}_0(x)+\sum_{k=1}^{\infty}\left[\frac{\hat{p}_0^{(k)}(x)}{k!}\right]q^k \tag{55}$$

Defining

$$\hat{p}_k(x)=\frac{\hat{p}_0^{(k)}(x)}{k!} \tag{56}$$

Using (53), (55) and (56), we get at q = 1, the important relationship

$$\hat{p}(x)=\sum_{k=0}^{\infty}\hat{p}_k(x) \tag{57}$$

between the initial guess approximation $\hat{p}_0(x)$ and the unknown solution.
Setting q = 0 in (48), gives

$$\tilde{\hat{p}}(x,0)=\hat{p}_0(x)\cdot \tag{58}$$

In particular, differentiating (48) with respect to q, making use of (54) and setting q = 0 we have

$$\ell_1\left\{\hat{p}_0^{(1)}\right\}=h\left[\frac{d\hat{p}_0}{dx}-\frac{\hat{R}}{b^2}\frac{d\hat{p}_0}{dx}-\frac{\hat{R}b^2}{20}\left(\frac{d\hat{p}_0}{dx}\right)^3-\frac{6}{b^2}-\frac{A_1}{b^3}\right] \tag{59}$$

$$\ell_1\left\{\hat{p}_0^{(1)}\right\}=h\left[\begin{array}{l}\dfrac{6(x+rx-1)(r-1)}{(1-x+rx)^3(1+r)}-\dfrac{\hat{R}}{(1-x+rx)^2}\left(\dfrac{6(x+rx-1)(r-1)}{(1-x+rx)^3(1+r)}\right)\\ -\dfrac{\hat{R}(rx-x+1)^2}{20}\left(\dfrac{6(x+rx-1)(r-1)}{(1-x+rx)^3(1+r)}\right)^3-\dfrac{6}{(rx-x+1)^2}-\dfrac{A_1}{(rx-x+1)^3}\end{array}\right] \tag{60}$$

Using equation (51) we obtain

$$\frac{d\hat{p}_0^{(1)}}{dx}=h\left[\begin{array}{l}\dfrac{6(r-1)}{(r+1)}\times\dfrac{(x+rx-1)}{(rx-x+1)^3}-\dfrac{6\hat{R}(r-1)}{(r+1)}\times\dfrac{(x+rx-1)}{(rx-x+1)^5}\\ -\dfrac{216\hat{R}(r-1)^3}{20(r+1)^3}\times\dfrac{(x+rx-1)}{(rx-x+1)^7}-\dfrac{6}{(rx-x+1)^2}-\dfrac{A_1}{(rx-x+1)^3}\end{array}\right] \tag{61}$$

Integrating (61) with respect to x, we have

$$\hat{p}_0^{(1)} = h\left[\begin{array}{l} -\frac{6}{(r-1)(r+1)}\times\frac{r^2x-x+1}{(rx-x+1)^2}+\frac{\hat{R}}{(r-1)(r+1)}\times\frac{2r^2x-2x-r+2}{(rx-x+1)^4} \\ +\frac{9\hat{R}}{25(r+1)^3(r-1)(rx-x+1)^6} \\ \times\left(\begin{array}{l} 30rx-30x-15r+12r^2-3r^3+30x^2-10x^3-15rx^2+18r^2x-30r^3x \\ +12r^4x-60r^2x^2+30r^2x^3+30r^3x^2+30r^4x^2-30r^4x^3-15r^5x^2 \\ +10r^6x^3+10 \end{array}\right) \\ +\frac{6}{(r-1)(rx-x+1)}+\frac{A_1}{2(r-1)(rx-x+1)^2}+A_2 \end{array}\right] \tag{62}$$

A_2 is an integration constant.
Using the boundary conditions $\hat{p}(0)=\hat{p}(1)=0$, we have

$$\hat{p}_0^{(1)} = -\frac{4}{25}\times\frac{h\hat{R}(r-1)(x-1)x}{r(r+1)^3(rx-x+1)^6}\times\left(\begin{array}{l} 77r-91x-100rx-32r^2+79r^3+117x^2 \\ -65x^3+13x^4-75rx^2+136r^2x+142rx^3 \\ -8r^3x-44rx^4+63r^4x-54r^2x^2-109r^2x^3 \\ -48r^3x^2+59r^2x^4+112r^3x^3-27r^4x^2 \\ -56r^3x^4-127r^4x^3+87r^5x^2 \\ +59r^4x^4+34r^5x^3-44r^5x^4+13r^6x^3 \\ +13r^6x^4+26 \end{array}\right) \tag{63}$$

There fore, the final pressure distribution would then be

$$\begin{array}{l} \hat{p}(x)=\hat{p}_0+\hat{p}_0^{(1)}=\frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)}+\frac{2\beta\tilde{\gamma}h}{25(1+\tilde{\gamma})}\times\frac{(r-1)(x-1)x}{r(r+1)^3(rx-x+1)^6} \\ \times\left(\begin{array}{l} 77r-91x-100rx-32r^2+79r^3+117x^2-65x^3+13x^4 \\ -75rx^2+136r^2x+142rx^3-8r^3x-44rx^4+63r^4x-54r^2x^2 \\ -109r^2x^3-48r^3x^2+59r^2x^4+112r^3x^3-27r^4x^2-56r^3x^4 \\ -127r^4x^3+87r^5x^2+59r^4x^4+34r^5x^3-44r^5x^4+13r^6x^3 \\ +13r^6x^4+26 \end{array}\right) \end{array} \tag{64}$$

for h = -1, we get the pressure equation as:

$$\begin{array}{l} \hat{p}(x)=\frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)}-\frac{2}{25(1+\tilde{\gamma})}\times\frac{\beta\tilde{\gamma}(x-1)(r-1)x}{r(r+1)^3(rx-x+1)^6} \\ \times\left(\begin{array}{l} 77r-91x-100rx-32r^2+79r^3+117x^2-65x^3+13x^4 \\ -75rx^2+136r^2x+142rx^3-8r^3x-44rx^4+63r^4x \\ -54r^2x^2-109r^2x^3-48r^3x^2+59r^2x^4+112r^3x^3 \\ -27r^4x^2-56r^3x^4-127r^4x^3+87r^5x^2+59r^4x^4 \\ +34r^5x^3-44r^5x^4+13r^6x^3+13r^6x^4+26 \end{array}\right) \end{array} \tag{65}$$

Using Eq. (17) in Eq. (65), we obtain the lubricated pressure distribution in slider bearing as:

$$p(x) = (1+\tilde{\gamma}) \times \frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)} - \frac{2}{25} \times \frac{\beta\tilde{\gamma}(x-1)(r-1)x}{r(r+1)^3(rx-x+1)^6} \times \begin{pmatrix} 77r - 91x - 100rx - 32r^2 + 79r^3 + 117x^2 - 65x^3 \\ +13x^4 - 75rx^2 + 136r^2x + 142rx^3 - 8r^3x - 44rx^4 \\ +63r^4x - 54r^2x^2 - 109r^2x^3 - 48r^3x^2 + 59r^2x^4 \\ +112r^3x^3 - 27r^4x^2 - 56r^3x^4 - 127r^4x^3 + 87r^5x^2 \\ +59r^4x^4 + 34r^5x^3 - 44r^5x^4 + 13r^6x^3 + 13r^6x^4 + 26 \end{pmatrix} \quad (66)$$

NUMERICAL RESULTS

In this section, the pressure distribution in the bearing is determined for various values of the parameters $\tilde{\gamma}$, β and clearance ratio r.

Figure 2 indicates variation of the pressure with respect x when $\beta = 0$ and $\tilde{\gamma}$ is varied. For $\tilde{\gamma} = 0$, the unbroken line shows the pressure distribution in the Newtonian fluid. For $\tilde{\gamma} = 0.3$ and $\tilde{\gamma} = 0.5$, the circles and dots, respectively show the effect of Powell-Eyring model. It is seen that the pressure increases with increasing $\tilde{\gamma}$ which means higher loading capacity for the bearing. Lubricant possessing higher $\tilde{\gamma}$ values of the Powell-Eyring model bears higher load capacities.

Figure 3, shows the manner in which pressure varies with $\tilde{\gamma}$, when β is held fixed at some nonzero value. As before, increasing $\tilde{\gamma}$, (circles and dots show the graphs for $\tilde{\gamma} = 0.3$ and $\tilde{\gamma} = 0.5$, respectively) increases pressure, also can see the reverse effect of β which is clear from Fig. 4.

In Fig. 4, for different β, $\tilde{\gamma}$ is fixed (line circles and dots show the graphs for $\beta = 0.2$, $\beta = 0.4$ and $\beta = 0.6$, respectively). It is seen that when $\tilde{\gamma} > 0$ the pressure decreases with increasing β, which means lower loading capacity for the bearing. Lubricant possessing higher β values of the Powell-Eyring model bears lower load capacity.

In Fig. 5, for $\beta = \tilde{\gamma} = 0.01$, the dimensionless length versus dimensionless pressure is plotted for different clearance ratios (line circles and dots show the graphs for

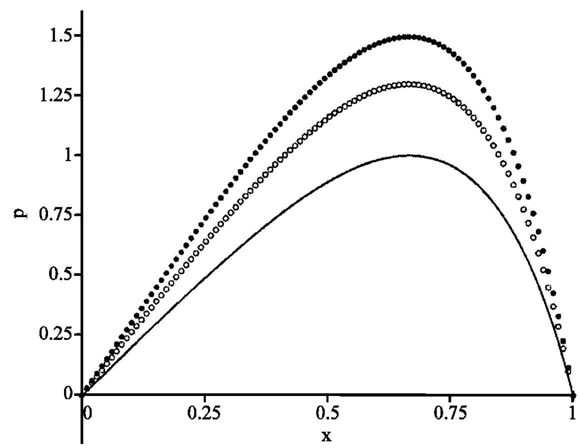

Fig. 3: r = 0.5, [$\beta = 0$, $\tilde{\gamma} = 0$(Newtonian); $\beta = 0.01$, $\tilde{\gamma} = 0.3$; $\beta = 0.01$, $\tilde{\gamma} = 0.5$]

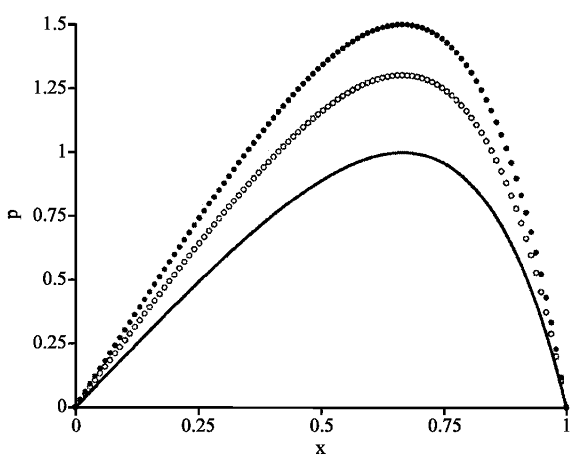

Fig. 2: r = 0.5, $\beta = 0$, [$\tilde{\gamma} = 0$(Newtonian), $\tilde{\gamma} = 0.3$, $\tilde{\gamma} = 0.5$]

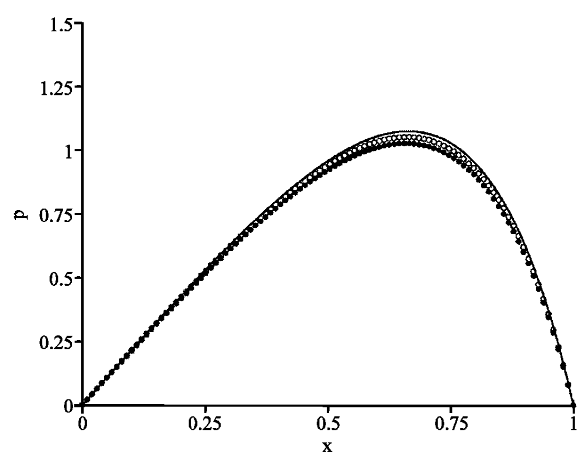

Fig. 4: r = 0.5, $\tilde{\gamma} = 0.1$, [$\beta = 0.2, 0.4, 0.6$]

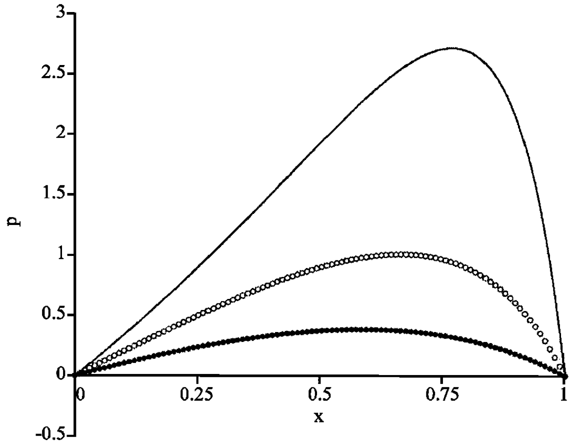

Fig. 5: $\beta = \tilde{\gamma} = 0.01$, [r = 0.3, 0.5, 0.7]

r = 0.3, r = 0.5 and r = 0.7, respectively). Similar to Newtonian behavior, in the Non-Newtonian case pressure build up in the bearing for lower clearance ratios.

CONCLUSIONS

In this study, the homotopy analysis technique, proposed by Liao (2004), is used to give an approximate analytical solution of Non-linear differential equation arising in slider bearing. As a result, we obtain a family of solution expression (41). In this study we don't need the so called small parameter assumption at all, which is necessary in perturbation method. That is the homotopy analysis method is independent of any small or large quantities. Also the homotopy analysis method provides us with great freedom to choose initial approximation, the auxiliary linear operator and the auxiliary parameter h. Thus we need to focus on choosing proper initial approximation, auxiliary linear operator and proper values of h to ensure that solution series converge. Therefore, it is this kind of freedom that establishes a cornerstone of the validity and flexibility of the homotopy analysis method. Thus from the above discussion we conclude that the analytical method used in this study is to be useful for the analysis of lubrication theory and also for solving nonlinear problems with strong nonlinearity and with no small or large parameter. (Incidentally there appears to be an error in reference (Yürüsoy, 2003). The term $yb^3/3$ of Eq. (21) in his study should be read as $yb^3/48$).

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