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Laminar Natural Convection in Saltbox Roofs for Both Summerlike and Winterlike Boundary Conditions

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Abstract: Saltbox roofs have trapezoidal geometry whose one wall is vertical and they have wide applications in buildings. In the present study, laminar natural convection heat transfer and fluid flow are performed in the saltbox roofs for summerlike (bottom is cold, inclined ceiling is hot) and winterlike (bottom is hot, inclined ceiling is cold) boundary conditions. In both cases, vertical wall is adiabatic. Governing equations for natural convection in streamline-vorticity form are solved using the finite difference method with successive under relaxation (SUR) technique. Numerical computations were examined for Ra = 10^3, 10^4, 10^5, 10^6, 5×10^6 and 10^7. Prandtl number is chosen as 0.71 which corresponds to air and inclination angle (18°) is chosen according to climate of Elazig, Turkey (38.7°N). Results are compared with the gable roof (triangle geometry) with the same bottom distance and observed that lower heat transfer is obtained when saltbox roof were used.

Key words: Heat transfer, natural convection, saltbox roof, finite difference method

INTRODUCTION

Natural convection heat transfer forms in buildings due to temperature difference and buoyancy including floor heating, heat transfer from radiators and roofs. Roofs are the main part of the buildings that protects the building from rain, storm, and snow. The geometry of roofs can be different. For instance, it can be Gable roof (triangular), Gambrel (pentagon) or saltbox (trapezoidal). Heat transfer will be reduced with the well designed roof on behalf of energy saving. Different natural convection heat transfer mechanisms depend on boundary conditions such as winterlike and summerlike. In the winterlike boundary conditions, due to heating of rooms, the ceiling (bottom of the roof) is hot while inclined surface of the roof is cold. Similarly, in the summerlike conditions, the ceiling has cold temperature whereas inclined boundary has hot temperature because of the radiation heat transfer between inclined boundary and sun.

Though there are many studies on natural convection in square and rectangle cavities (Ostrach, 1988; Gebhart *et al*., 1988; Vahl Davis and Jones, 1983; Küblbeck *et al*., 1980; Oztop and Bilgen, 2006) or inclined enclosure with differentially heated side walls (Baytas and Pop, 1999; Bilgen and Oztop, 2005) or bottom walls (Sahoo and Sastri, 1997 and Yilbas *et al*., 1998), the number of studies on natural convection heat transfer in roofs are very limited. Moukalled and Acharya (2001) conducted a numerical study for buoyancy induced flow problem inside a trapezoidal shaped geometry with baffles for building roofs. They solved the laminar conjugate heat transfer problem for different parameters of baffles in the summerlike and winterlike conditions. They observed that in winterlike conditions, convection starts to dominate at a Ra number much lower than that in summerlike conditions. Recently, Tzeng *et al*. (2005) proposed the Numerical Simulation Aided Parametric Analysis method to solve natural convection equations in streamline-vorticity form. They applied this method to laminar natural convection in triangle shaped roof. They developed correlations for parameters which are effective on flow and heat transfer. In addition to Tzeng's study, the laminar natural convection in triangular cross-sectional roofs, namely, Gabel Roofs with different inclination angle and Ra number in summer day conditions is investigated numerically using finite volume method by Asan and Namli (2000). In their study, both aspect ratio and Ra number affect the temperature and flow field. Heat transfer decreases with the increasing of aspect ratio. In their other study, the same geometry was studied for winter day conditions. It is found that in this situation, the Nusselt number strongly depends on both height-base ratio and Ra number where as for summer day boundary conditions (Asan and Namli, 2001).

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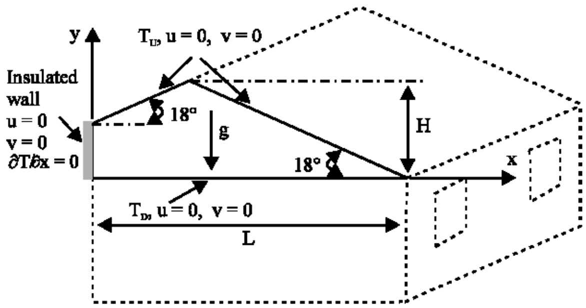

Fig. 1: Schematic of Saltbox roof with boundary conditions and coordinates

To the best of the authors' knowledge, the natural convection heat transfer and fluid flow in saltbox roofs has not yet been investigated; therefore, the present study is the first attempt to analyze natural convection for that physical model. The aim of the present study is to examine natural convection heat transfer in saltbox roof which is widely used in residential and commercial building both in urban and rural areas. With this aim, streamlines, isotherms and heat transfer is obtained for different Ra number values and different boundary conditions in winterlike and summerlike conditions.

PHYSICAL MODEL

Geometric configuration of the model (Saltbox roof) is shown in Fig. 1 with the Boundary Conditions (BC) and coordinates. The figure is curvilinear enclosure which is filled with air (Pr = 0.71) and it is heated from the inclined boundary and cooled from the entire bottom wall uniformly in summer day conditions. However, inclined surface has uniform cold temperature while the bottom is uniform hot in winter day conditions. Left vertical boundary is adiabatic for both conditions. The length of bottom wall is depicted by L and maximum height of the roof is shown with H. Inclination angles are fixed at 18° for the inclined top walls which is suitable most geographic area for Turkey according to climate.

GOVERNING EQUATIONS AND NUMERICAL METHOD

The system was considered to be two-dimensional, incompressible, steady-state. The Boussinesq approximation (Gray and Giorgini, 1976) was assumed for fluid with constant physical properties. Dimensionless governing equations in streamline-vorticity form can be obtained via introducing dimensionless variable as follows:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ \Psi = \frac{\psi \Pr}{\upsilon},\quad \Omega = \frac{\omega (L)^2 \Pr}{\upsilon},\ \theta = \frac{T - T_c}{T_h - T_c} \tag{1}$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ \omega = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right),\quad Ra = \frac{\beta g (T_h - T_c) L^3 \Pr}{\upsilon^2},\ \Pr = \frac{\upsilon}{\alpha} \tag{2}$$

Based on the dimensionless variables above governing equations (stream function, vorticity and energy equations) can be written as:

$$-\Omega = \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} \tag{3}$$

$$\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = \frac{1}{\Pr}\left(\frac{\partial \Psi}{\partial Y}\frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \Omega}{\partial Y}\right) - Ra\left(\frac{\partial \theta}{\partial X}\right) \tag{4}$$

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{\partial \Psi}{\partial Y}\frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} \tag{5}$$

The numerical method used in the present study is based on finite difference method to discretize the governing equations (Eq. 3-5) and the set of algebraic equations are solved using Successive Under Relaxation (SUR) technique. The solution technique is well described in the literature (Ozisik, 1994; Oosthuizen and Naylor, 1998) and has been widely used to solve natural convection equations. The convergence criterion 10^{-4} is chosen for all depended variables and 0.1 is taken for under-relaxation parameter. The present solutions are compared with the known results for triangular shaped roofs from the open literature. As indicated in the Fig. 2,

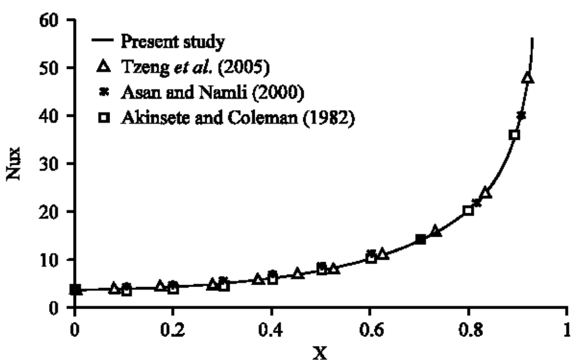

Fig. 2: Comparisons of local Nusselt numbers with literature for Ra = 2772

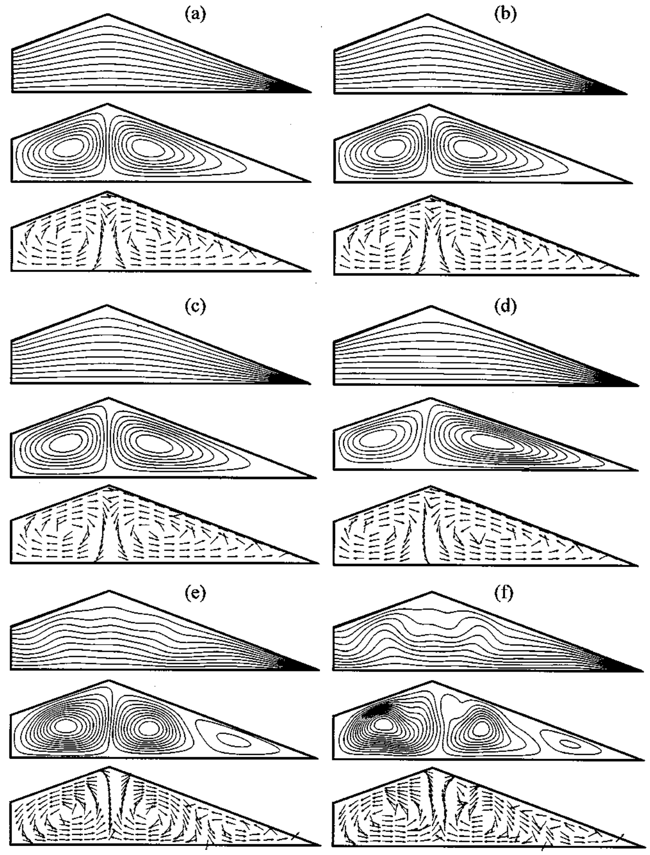

Fig. 3: Isotherms, streamlines and velocity vectors (from top to bottom in each groups) in summerlike BC a) Ra = 10^3, b) Ra = 10^4, c) Ra = 10^5, d) Ra = 10^6, e) Ra = 5×10^6, f) Ra = 10^7

it was found that these results of the present code are in very good agreement with the literature (Asan and Namli, 2000).

As indicated earlier, the problem is defined for two different boundary conditions as summerlike and winterlike. The physical boundary conditions are illustrated in the physical model (Fig. 1) and they can be defined as follows:

On the inclined surfaces, $u = v = 0$, $T_U = T_{hot}$ for summerlike BC, $T_U = T_c$ for winterlike BC (6)

On the bottom wall, $u=v=0$, $T_D=T_{cold}$ for summerlike BC, $T_D=T_h$ for winterlike BC (7)

On the vertical wall, $u = 0$, $\partial T/\partial x = 0$ for both summerlike BC and winterlike BC (8)

In the study, a regular grid is used and increasing the mesh size is from 34×23 to 145×97. Optimum grid dimension is chosen according to variation of Nu number for both boundary conditions. Thus, it is identified as 76×51 for summerlike BC and 97×65 for winterlike BC. These mesh sizes are sufficient to solve temperature and flow field. Detailed study for this grid study can be found Koca (2005). Calculation of the local Nusselt number was performed by

$$Nu_x = -\left.\frac{\partial\theta}{\partial Y}\right|_{y=0} \tag{9}$$

RESULTS AND DISCUSSION

Laminar natural convection heat transfer and fluid flow is analyzed numerically in saltbox roofs for both winterlike and summerlike boundary conditions at different Ra number. Even though the chosen angle of the roof is suitable for climate of Elazig, Turkey (38.7°N), it is applicable for some part of Turkey.

Fig. 3 (a-f) shows the streamlines, isotherms and velocity vectors in the range of Ra number 10^3-10^7 for the case of summerlike boundary conditions where temperature of bottom wall higher than that of inclined

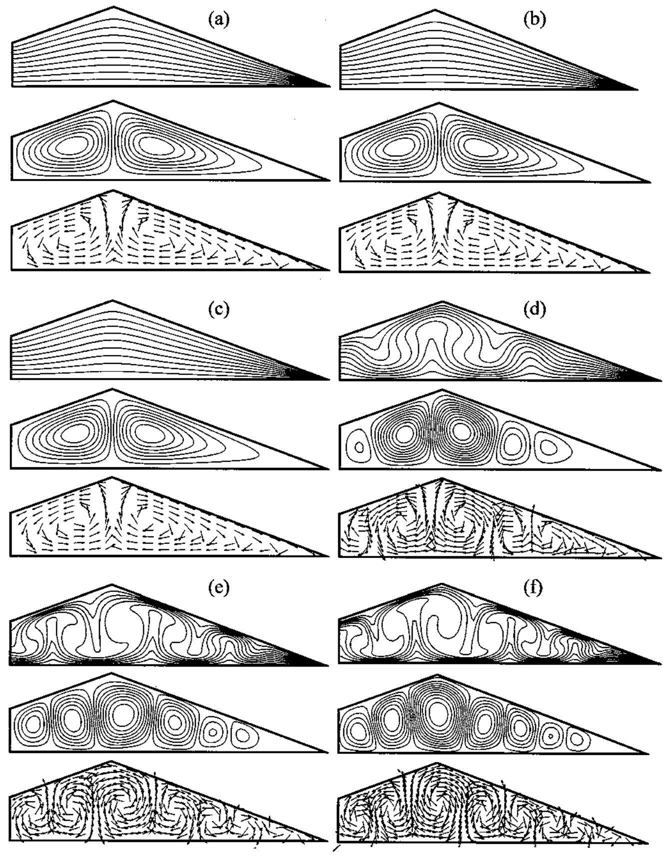

Fig. 4: Isotherms, streamlines and velocity vectors (from top to bottom in each groups) in winterlike BC a) Ra = 10^3, b) Ra = 10^4, c) Ra = 10^5, d) Ra = 10^6, e) Ra = 5×10^6, f) Ra = 10^7

walls. In this case, multiple cells with different dimensions are formed at all Ra number due to two inclined wall. Due to heated inclined walls, the flow moves insides the enclosure. Streamlines elongates with inclined walls and two vortexes are formed under in each inclined wall. The left one moves in clockwise and the right one counter clock wise (Fig. 3 a-c). These directions can be seen from the velocity vectors clearly. Their dimensions are almost equal due to small Ra number. As Ra number increases the right vortex becomes dominant to left one due to long distance of heater zone (Fig. 3d). Up to Ra=10^5 conduction heat transfer becomes dominant to convection. Thus, isotherms show the same behavior for Fig 3 (a-d). However, as Ra number increases due to distraction of boundary layer, heat transfer is increased and three vortex centers are obtained (Fig. 3e-f). Interestingly, the results show that direction of the left cell was changed from clockwise to counterclockwise on the contrary of lower Ra number as indicated in Fig. 3 (e-f). Plumelike temperature distributions were obtained in higher Ra number values. Figure 4 presents the streamline, isotherms and velocity vectors for the same parameters as in Fig. 3 for winterlike BC. In this case, heat transfer between hot and cold wall becomes stronger because this type of boundary condition is more suitable to mechanism of natural convection. In Fig. 4 (a-c), conduction mechanism is stronger and two different streamline centers are obtained. These two cells push each other and a stagnation point is obtained. However, isotherms are fit with the geometry of the enclosure. As Ra number increases (Ra>10^5) heat transfer becomes stronger and plumelike thermal distribution is obtained. They represent the lines with equal intervals between zero and unity. Any further increase in the Ra number will enhance the stratification in the core, and heat transfer by convection near the walls will increase. With the increase of Ra number, the effect of convection moves to upward and wavy motion is obtained in the streamlines. The cell under the long inclined boundary becomes thinner as Ra number increases due to high velocity and flowrate of the fluid. Impinged air to inclined wall turns in different directions and multiple cells are formed in different directions (Fig. 4 d-f). Note that directions of flow can be seen from the velocity vectors.

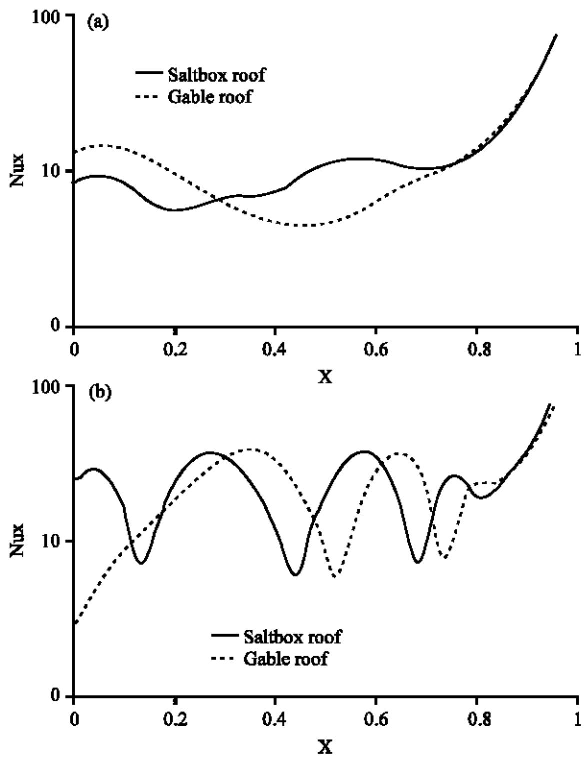

Fig. 5: Comparisons of local Nusselt numbers between Gable and Saltbox roofs for a) summerlike BC and b) winterlike BC at Ra = 5×10^6

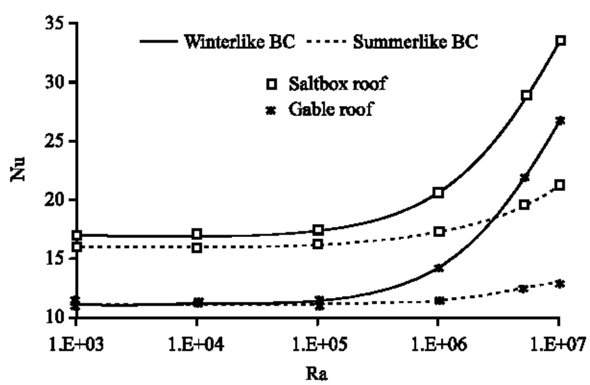

Fig. 6: Comparisons of mean Nusselt numbers with different Rayleigh numbers for both summerlike and winterlike BCs.

Figure 5a and b show the variation of local Nu number along the hot wall for summerlike BC and winterlike BC, respectively, for the value of Ra number 5×10^6. Also, values of local Nu number are compared with the Gable roof, which is the most common roof type and which is benchmark (triangular enclosure) in the open literature. Nu number is higher at the middle of the bottom wall for the case of Saltbox roof than that of Gable roof. Gable type and Saltbox roofs are compared from the heat transfer point of view. Wavy variations can be seen in winterlike BC for both geometries. Values of peak points are almost the same. In the summerlike BC, heat transfer increases at the middle of the roof in case of Saltbox roof. A comparison was performed for variations of the mean Nu number with the Ra number for different cases in Fig. 6. As plotted in the figure, there is a big difference, especially at higher Ra number values on mean Nu number between summerlike and winterlike BCs. Higher heat transfer is obtained in the case of winterlike BC. The values are almost constant up to value of Ra number 10^5 but for the higher Ra numbers mean Nu number value is increased because of the increasing flow velocity inside the roof. This trend is clear for winterlike BC. Similar results are indicated in the literature by Tzeng *et al.* (2005).

CONCLUSIONS

The present numerical study investigates the laminar natural convection heat transfer and fluid flow in both summerlike and winterlike boundary conditions for different Ra numbers. Two different circulation cells are formed for both conditions in different rotational direction for all Ra numbers. Heat transfer is increased with the increasing of Ra number, as expected. When the mean and local Nu numbers are compared with the Gable roof, more heat transfer is obtained for winterlike BC. However, heat transfer is almost the same for summerlike BC. It is found that heat transfer becomes stronger for winterlike BC than that of summerlike BC, which is the most significant observation in the present study. For further research, this study can be extended for turbulent regime. It is possible to find an optimum inclination angle for heat transfer in specific applications.

Nomenclature

g	Gravitational acceleration (m sec^{-2})
Gr	Grash of number
L	Length of bottom wall (m)
H	Maximum height of roof (m)
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
T	Temperature (°C)
u, v	Velocities (m^2 sec^{-1})
X, Y	Non-dimensional coordinates

Greek letters

υ	Kinematic viscosity ($m^2\ sec^{-1}$)
Ω	Vorticity
θ	Non-dimensional temperature
β	Thermal expansion coefficient (K^{-1})
α	Thermal diffusivity ($m^2\ sec^{-1}$)
ψ	Stream function

Subscript

U	Upper
D	Down
c	Cold
h	Hot

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