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Effect of Scale Parameters in the Performance of Shewhart Control Chart with Interpretation Rules

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Abstract: It is common in practice that the Shewhart $\overline{X}$ chart usually supplemented with interpretation rules in order to detect the small shifts in the process mean. In this study the effect of scale parameters in the performance of the Shewhart $\overline{X}$ control chart of the most used seven interpretation rules were obtained through by developing a spread sheet for normal and gamma distributions when process remains in control. The results show that the in-control performance can be fairly high for particular rules when the underlying data from a gamma distribution rather than normal distribution considering scale parameters. It was also observed that the in-control performance of gamma distribution grows larger when the scale parameter increases.

Key words: Interpretation rules, Shewhart control chart, scale parameter, normal distribution, gamma distribution

INTRODUCTION

The Shewhart $\overline{X}$ chart for monitoring the mean of the distribution (usually normal distribution) of a quality characteristic of the items produced by the certain process. The standard control chart utilizes three-sigma control limits and indicates an out-of-control signal of a single point falls beyond the control limits. It is known that the Shewhart control charts are efficient in detecting quickly medium to large shifts in the process mean but are insensitive to small shifts. To increase the sensitivity of the Shewhart control charts to small shifts in the process mean additional interpretation rules has been suggested. These rules and their purpose were first summarized in the West Electric Handbook in 1956. The Shewhart control chart with interpretation rules were divided in to zone boundaries. The zone boundaries are placed ±1σ, ±2σ and the center line and the control limits (±3σ), divided the Shewhart control chart into six zones. These zones are represented in Fig. 1.

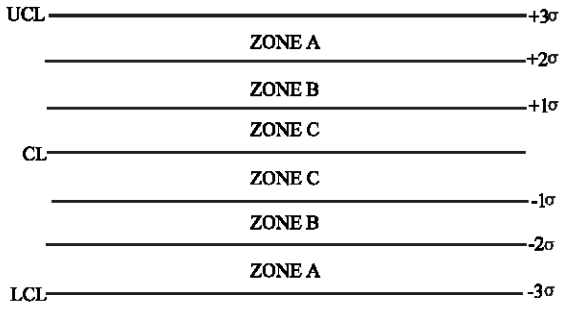

Fig. 1: $\overline{X}$ Chart with different zones

There are number of sources including Bell Laboratories and Juran have compiled a list of rules for interpreting Shewhart control charts. The most common seven interpretation rules for Shewhart charts are as under

Rule 1: Any point falls outside the control limits (μ ± 3σ)
Rule 2: 2/3 consecutive points fall between μ-2σ and μ-3σ or between μ+2σ and μ+3σ
Rule 3: 4/5 consecutive points fall between μ-1σ and μ-3σ or between μ+1σ and μ+3σ
Rule 4: 8 consecutive points fall between μ and μ-3σ or between μ and μ+3σ
Rule 5: 15 consecutive points fall between μ-1σ and μ+1σ
Rule 6: 8 consecutive points fall on both sides of center line with none of the points fall between μ ±1σ
Rule 7: 7 consecutive points without a change in direction (trend)

Several studies have been done to investigate the performance of the Shewhart $\overline{X}$ chart with interpretation rules for example Champ and Woodall (1987, 1990), Do Sun Bai (2000) and Khoo M.B.C (2004). The common things for all these authors were incorporated in their research for investing the performance of only first four rules and some new rules through by simulation studies. Therefore, the object of this research is to further explore the effect of scale parameter in the performance of Shewhart $\overline{X}$ control chart with interpretation rules when process mean is in control. In order to accomplish this object, normal and gamma distribution were considered

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for analysis. These seven rules were considered because of mostly used by the practitioners. The results were obtained analytically by providing very simple methodology through developing of EXCEL Spread Sheet using statistical functions or from the Table of normal curve.

MATERIALS AND METHODS

The EXCEL spread sheet was developed for the calculations of the effect of the scale parameters in the performance of Shewhart control charts with interpretation rules. Firstly, the EXCEl spread sheet was developed for the normal distribution to find out the performance of Shewhart control chart with interpretation rules when process remains in control. After the verification and validation of the results obtained through by EXCEL model was extended for gamma distribution. The beauty of these models is that it can be extended to any distribution by little modifications.

When the process is in control, the performance of the Shewhart interpretation rule 1 (i.e., P(x>3σ) and P(x<3σ)) were obtained, when the area under curve less than μ-3σ is 0.00135 or the area under the normal curve greater than μ+3σ is 0.00135. As the normal curve is symmetric around mean. Therefore the total area is 0.0027 of the total area under the normal curve. While the in control performance for the rule 2 to rule 6 were obtained using the binomial distribution with SUCCESS being a sample that falls in the range being examined. The probabilities for SUCCESS used in the binomial distribution were obtained from the probability density function of normal distribution.

Further more the performance for rule 3 having following 4 additional probabilities when process is in control:

- The probability of 4/4 points falling in the range μ-1σ to μ-3σ
- The probability of 4/5 points falling in the range μ-1σ to μ-3σ with one of the first four points not falling in the range.
- The probability of 4/4 points falling in the range μ+1σ to μ+3σ.
- The probability of 4/5 points falling in the range μ+1σ to μ+3σ with one of the first four points not falling in the range

As in the 7th rule, the points do not lie in range being examined but continuously increasing or decreasing. So the performance of this rule can not calculated as previous mention rules but rather calculated using the number of SUCCESS of the rule occurring dividing by the number of possible outcomes. Therefore, the performance of this rule can be calculated by taking the number of successes (2→1 increasing and 1 decreasing) by the number of possible outcomes (7!=5040)

$$P(\text{All Ascending}) = 1/5040 = 0.000198$$
$$P(\text{All Descending}) = 1/5040 = 0.000198$$
$$\alpha = P(\text{All Ascending}) + P(\text{All Descending})$$
$$\alpha = 0.000198 + 0.000198 = 0.000397$$

The values of all rules for in-control performance of Shewhart $\overline{X}$ control chart with interpretation rules are shown in Table 1 for normal distribution.

The PDF of Gamma Distribution is

$$f(x \mid a,b) = \frac{1}{b^{a}\Gamma(a)} * x^{a-1} * e^{-\frac{x}{b}} \qquad x,a,b > 0$$

Where $\Gamma(\bullet)$ = gamma function, a = Shape parameter and b = Scale parameter.

Table 1: The in-control performance of Shewhart $\overline{X}$ control charts with interpretation rules considering normal distribution

Normal distribution					
Mean (μ) = 0					
SD (σ) = 1					
Rule	test	p(s)	Binomial	Probability	In-control performance
1	P(x>3σ)	0.001350	1 of 1	0.001350	0.002700
	P(x<-3σ)	0.001350	1 of 1	0.001350	
2	P(-3σ < x < -2σ)	0.021400	2 of 2	0.000458	0.002704
			2 of 3	0.000894	
	P(2σ < x < 3σ)	0.021400	2 of 2	0.000458	
			2 of 3	0.000894	
3	P(-3σ < x < -1σ)	0.157305	4 of 4	0.000612	
			4 of 5	0.002057	0.005339
	P(1σ < x < 3σ)	0.157305	4 of 4	0.000612	
			4 of 5	0.002057	
4	P(x < μ)	0.500000	8 of 8	0.003906	0.007813
	P(x > μ)	0.500000	8 of 8	0.003906	
5	P(-1σ < x < 1σ)	0.682689	15 of 15	0.003261	0.003261
6	P(-1σ < x < 1σ)	0.6826895		0.000103	
	1-P(-1σ < x < 1σ)	0.3173105	8 of 8	0.000103	0.000103
			All Ascending	0.000198	
7	Combination of 7 items = 5040		All Descending	0.000198	0.000397

Table 2: The in-control performance of Shewhart $\overline{X}$ control charts with interpretation rules (Gamma Distribution) with different values of scale parameters

Gamma distribution Shape parameter (a) = 0.50 Scale parameter (b) = 1.00(1.00)10.00 Mean 0.50 when b = 1.00 SD 0.7071 when b = 1.00							
Scale parameter	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7
1.0	0.0143	0.0015	0.0007	0.0473	0.0000	0.2478	0.0004
2.0	0.0833	0.0090	0.0049	0.0082	0.0000	0.2478	0.0004
3.0	0.1573	0.0135	0.0070	0.0115	0.0000	0.2738	0.0004
4.0	0.2207	0.0149	0.0070	0.0215	0.0000	0.3060	0.0004
5.0	0.2733	0.0150	0.0063	0.0340	0.0000	0.3354	0.0004
6.0	0.3173	0.0144	0.0054	0.0475	0.0000	0.3616	0.0004
7.0	0.3545	0.0137	0.0047	0.0614	0.0000	0.3850	0.0004
8.0	0.3865	0.0129	0.0040	0.0753	0.0000	0.4059	0.0004
9.0	0.4142	0.0121	0.0035	0.0889	0.0000	0.4246	0.0004
10.0	0.4386	0.0114	0.0030	0.1021	0.0000	0.4417	0.0004

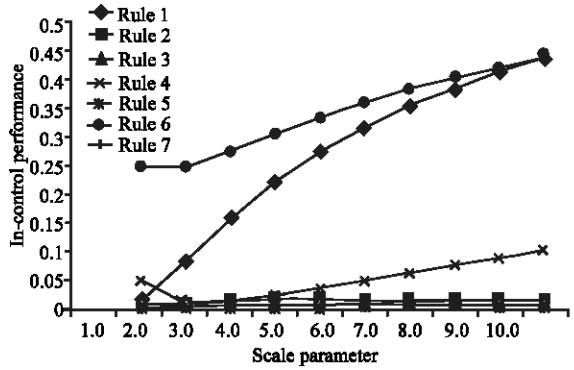

Fig. 2: The in-control performance of Shewhart $\overline{X}$ control charts with interpretation rules (Gamma distribution) with different values of scale parameters

$$E(x) = \mu = a * b = \text{Scale} * \text{Shape}$$

$$\text{var}(x) = a * b^2 = \text{Scale}^2 * \text{Shape}$$

In order to see the effect of scale parameter in the performance of Shewart $\overline{X}$ control chart with interpretation rules by fixing the shape parameter (a) = 0.50 and varying the different value of scale parameter ranging from 1.00(1.00)10.00. From these parameters a mean and standard deviation were calculated. After the calculation of mean and standard deviation then we have adopted above same methodology as the results shown in below Table 2 and graphically in Fig. 2.

CONCLUSIONS

Interpretation rules were designed to increase the effectiveness of Shewhart $\overline{X}$ chart to detect the presence of assignable causes. It was found in this study, that the performance for individual Shewhart control chart interpretation rules when process remains in control for the statistical distributions has significant differences. It was found from Table 1 and 2 that the performance for the rules 1 4, 6 are much larger for the gamma distribution than the normal distribution. While the performance for rule 5 of Gamma distribution is zero. After the analysis of distributions that the interpretation rules 6 and 7 for normal distribution and rules 2, 3 and 7 for gamma distribution have a very low performance and very low power to detect the changes in the process mean in terms of standard deviation. After the analysis of Table 2 and Fig. 2 that the scale parameter has an effect in the performance of Shewhart control charts with interpretation rules considering only gamma distribution. By keeping the shape parameter constant and varying the scale parameters from the Table 2 and Fig. 2 show that the values of Rule 1, Rule 6 increases as the scale parameter increases and the values of rule 2 and rule 3 decreases as the scale parameter increases. While Rule 5 has no effect of scale parameters and Rule 7 has the same effect in all cases of the scale parameters.

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