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Efficient Load Flow Method for Radial Distribution Feeders

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Abstract: This study presents a simple and efficient solution of load flow in distribution systems characterised by their radial configuration and laterals. This iterative method, based on Kirchoff laws, have the merit to evaluate for each node both the voltage root mean square (rms) value and phase-angles. The phase-angles although of weak values become necessary in the reactive energy optimisation problem. To solve the lines with laterals, a simple technique of determining nodes belonging branches is given. The method, requires a few computational time and have solved successfully several line examples. The results obtained for the voltage magnitudes and deviation-angles are compared to those of authors having worked on the subject.

Key words: Radial lines, load flow, distribution feeders, voltage

INTRODUCTION

The increasing of the electric energy needs has forced the power suppliers to pay more attention to the analysis of the distribution networks. Generally, distribution feeders have a high R/X ratio and their configuration is radial. These reasons make that distribution systems are ill-conditioned and thus conventional method as Newton Raphson (Tinney and Hart, 1967), fast decoupled load flow (Stott and Alsac, 1974) and their modifications (Amerongen, 1989) to (Haque, 1993) are unsuitable for solving load flow for most cases and fail to converge.

Literature survey shows that several non-Newton efficient algorithms based on backward and forward sweeps are reported (Haque, 1996) to (Ranjan and Das, 2003). Haque (1996) has developed a method for radial and mesh networks. In the mesh networks the loops are opened and in the loop break point a dummy bus is added. The power flow through the branch that makes the loop is simulated by injection of the same power in the dummy bus. The method uses the backward and forward sweeps with initial voltage of all the nodes assumed to be equal to that of the source bus which is took as reference. No algorithm for determining automatically the nodes after each branch is given. Ghosh and Das (1999) also uses backward and forward sweeps with an initial voltage of all the nodes put to be equal to 1 per-unit (p.u). In the solution methodology Ghosh and Das (1999) gives an algorithm for identifying the nodes beyond the line branches. The method involve the evaluation of algebraic expressions. It only permit the calculation of the nodes voltage rms values. Nanda *et al.* (2000) solves the load flow problem by going up and down the line and but it assumes a voltage of 1 p.u only at the end buses of the line (main feeder and laterals end buses). The convergence criterion is this case based on the voltage at the supply node. If the difference between the source voltage calculated and specified is within a certain tolerance, the solution is reached. Aravindhababu *et al.* (2001) proposes an iterative method in which the nodes voltage are assumed to be the voltage of the source. It first form the branch to node matrix for than calculating the loads and branches current; the branches voltage drop and the nodes voltage. The convergence criterion is based upon the voltage difference of two consecutive iterations. Mekhamer *et al.* (2002) uses the equations developed by Baran and Wu for each node of the feeder but with a different procedures. In this method, the load flow problem is solved by considering the larerals as a concentrated load of the main feeder. Once the voltage of the main feeder calculated, the first node voltage of each lateral is put equal to the voltage of the same node of the main feeder. The nodes voltage of the laterals are than calculated using Baran and Wu equations. The convergence criterion is made upon the active and reactive power fed through the terminal nodes of laterals and main feeder. Afsari *et al.* (2002) also uses the Baran and Wu equations. In their method Afsari *et al.* (2002), initially estimate the voltage of the terminal nodes which are used as an initial values in the backward sweep instead of a flat start value of 1 p.u. Any lateral is assumed to be replaced by the total lateral load on the main feeder. The authors method gives both voltage rms

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values and voltage phase-angles. Ranjan *et al*. (2003) presents a method based on the load flow algorithm developed by Das which is modified to incorporate a composite load models. This method also apply the backward and forward sweeps with an initial values of the nodes voltage assumed to be of 1 p.u. Ranjan and Das (2003) method uses the basic principal of circuit theory but first, the authors have developed an algorithm for determining the nodes after each branch of the network. However the method gives only the voltage magnitude of each node on the basis of algebraic equations. As convergence criterion of the algorithm, author have proposed the difference of the active and reactive power at the sub-station end of two successive iterations. If this difference is less than 0.1 kW and 0.1 kvar; the solution is reached.

In the present study, our main aim is the development of an efficient method for solving radial distribution feeders with laterals. A fast and easy to understand algorithm for determining nodes after branches based on the study presented by Augugliaro *et al*. (2001) is given. Other benefits of the method lies in the evaluation of the voltage phase-angles which becomes necessary if the load flow solution is used in the capacitors sizing problem. The tests carried out on several feeders with laterals shows that our method takes few time to reach the solution. The results obtained by our method are compared to those of some authors cited above.

Node and branch numbering: The numbering scheme is not required for the proposed load flow solution of radial distribution networks. However and for convenience, to perform the numbering scheme we consider the example line of Fig. 1.

First, we number the nodes of the main feeder. The source node is numbered as bus number 0. The node just ahead the source node is labelled node 1 and so on until the end-node 4 of the main feeder. Thereafter, the nodes of the main feeder are explored for laterals. The lateral that branches out from the bus nearest to the source bus is chosen and their buses are numbered from 5 to 6 as shown in Fig. 1. Similarly, the bus numbers of the next lateral (lateral out from node 3 in Fig. 1) are numbered following the end-node of the previous lateral (7 to 8) and so on until all the laterals nodes are numbered. For the branch numbering, we give each branch the same number of its receiving end-node. The feeder connectivity of Fig. 1 is presented in Table 1.

Node after branch determination: To determine the nodes after each branch of the feeder, we first construct the branch-to-node incidence matrix IM. In IM the rows indicate the identification numbers of branches and the columns the identification numbers of nodes. The generic elements IM(i, j) are assumed to have the values signification of which are given below.

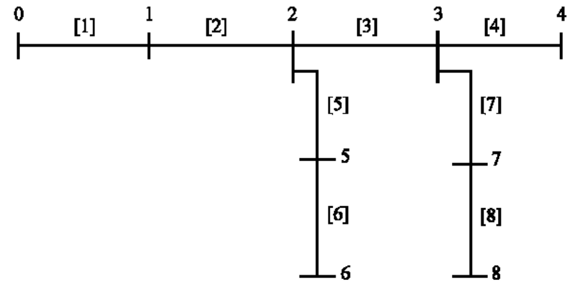

Fig. 1: Nodes and branches numbering scheme

Table 1: Feeder connectivity

Branch	Sending-end [SE(i)]	Recieving-end [RE(i)]
1	0	1
2	1	2
3	2	3
4	3	4
5	2	5
6	5	6
7	3	7
8	7	8

$$\begin{cases} +1 & \text{if } j \text{ is the receiving end of branch } i \\ -1 & \text{if } j \text{ is the sending end of branch } i \\ 0 & \text{otherwise} \end{cases}$$

For the feeder example of Fig. 1, the branch-to-node incidence matrix is:

$$IM = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

From the branch-to-node incidence matrix, we deduce the node-to-branch incidence matrix G by simple inversion of IM. In the node-to-branch incidence matrix, the rows numbers are the nodes identifiers and the column numbers identify the branches numbers. For the feeder of Fig. 1, the node-to-branch incidence matrix G is:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

For the branch 1(column 1), the nodes which belong it are all the rows of a non-zero values i.e., 1, 2,…,8. For the third branch (column 3), the rows of a non-zero values are 3, 4, 7 and 8, this means that after branch 3 we count nodes 3, 4, 7 and 8.

On the basis of what have been just said and from the G matrix we can construct the matrix BR the rows of which indicate the branches number and the generic elements indicate the nodes belonging each branch. For the line example of Fig. 1 , the construction of BR gives:

$$BR = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 \\ 3 & 4 & 7 & 8 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The total number of nodes after branch i is noted by M(i). M(i) is of a great utility owing to the fact that it allows a saving in computing time by avoiding calculation for the BR(i,j) equal to zero.

MATHEMATICAL FORMULATION

Assumptions: It is assumed that the three-phase radial distribution network is balanced and thus can be represented by its one-line diagram. Distribution lines are of medium level voltage then, the shunt capacitance are small and thus ignored. The single-line equivalent diagram of a such line is shown in Fig.1.

Mathematical models: The load flow of radial distribution network can be solved iteratively from two sets of recursive equations. The first set concern the determination of the branches current by going up the line (backward sweep). The second one allow us to determine the nodes voltage by going down the line (forward sweep).

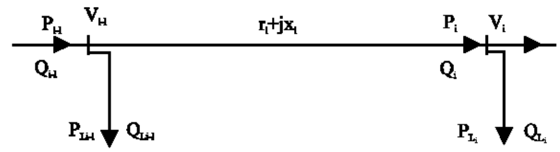

Fig. 2: Branch One-line diagram

Branches power and current: From the branch electric equivalent shown in Fig. 2 we can write the set of the above equations.

$$P_i = \sum_{k=BR(i,M(i))}^{BR(i,1)} P_{Lk} + \sum_{k=BR(i;M(i))}^{BR(i,2)} ploss_k$$
$$Q_i = \sum_{k=BR(i,M(i))}^{BR(i,1)} Q_{Lk} + \sum_{k=BR(i;M(i))}^{BR(i,2)} qloss_k \quad (1)$$

where:

- M(i): is the total number of nodes belonging branch i.
- BR(i,j); $1 \le j \le M(i)$: is the set of nodes beyond branch i.
- P_{LK} and Q_{LK}: are the active and reactive power of the load at node k.
- P_i: is the active power fed through bus i. It is equal to the sum of the active power of all the loads beyond node i (node i included) plus the sum of the active power loss in the branches beyond node i (branch i not included).
- Q_i: is the reactive power fed through bus i. It is equal to the sum of the reactive power of all the loads beyond node i (node i included) plus the sum of the reactive power loss in the branches beyond node i (branch i not included).
- $ploss_k$: is the active power loss in the kth branch.
- $qloss_k$: is the reactive power loss in the kth branch.

The active and reactive power loss are given by:

$$ploss_k = \frac{P_k^2 + Q_k^2}{V_k^2} r_k$$
$$qloss_k = \frac{P_k^2 + Q_k^2}{V_k^2} X_k \quad (2)$$

where :

- $r_k(X_k)$: is the resistance (reactance) of the kth branch.

The current flowing through the ith branch is given by:

$$\bar{F}_i = \frac{P_i - jQ_i}{\bar{V}_i^*} \quad (3)$$

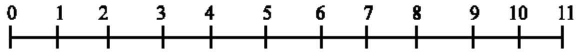

Fig. 3: One-line 12-node feeder (Rajan and Das, 2003)

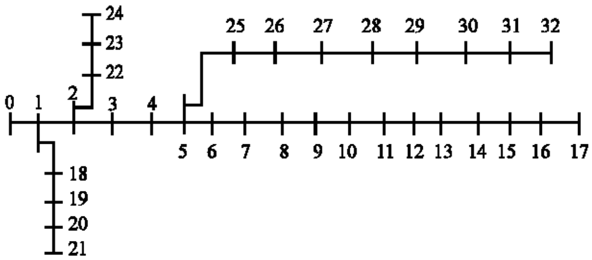

Fig. 4: one-line 33-node feeder (Rajan *et al.*, 2003)

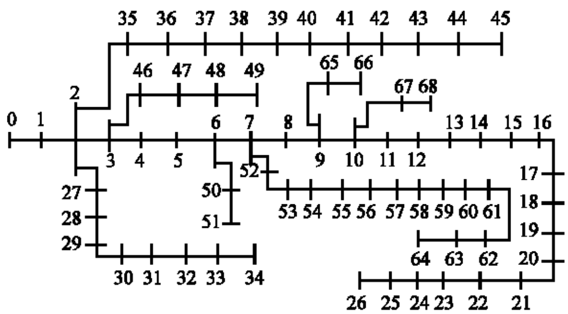

Fig. 5: One-line 69-node feeder (Rajan and Das, 2003)

If the complex voltage at the node i is: $\overline{V}_i = V_i(\cos\varphi_i + j\sin\varphi_i)$, (3) can be expressed as:

$$\overline{F}_i = \frac{P_i\cos\varphi_i + Q_i\sin\varphi_i}{V_i} - j\frac{Q_i\cos\varphi_i - P_i\sin\varphi_i}{V_i} \quad (4)$$

The d and q components of the current (4) are:

$$\begin{aligned} F_{di} &= \frac{P_i\cos\varphi_i + Q_i\sin\varphi_i}{V_i} \\ F_{qi} &= \frac{Q_i\cos\varphi_i - P_i\sin\varphi_i}{V_i} \end{aligned} \quad (5)$$

Nodes voltage: For the voltage and regarding our numbering scheme, we can write the following complex expression.

$$\begin{aligned} \overline{V}(RE(i)) = \overline{V}(SE(i)) - [r(RE(i)) + jX(RE(i))]. \\ [F_d(RE(i)) - jF_q(RE(i))] \end{aligned} \quad (6)$$

the d and q components of which are:

$$\begin{aligned} V_d(RE(i)) &= V_d(SE(i)) - r(RE(i))F_d(RE(i)) \\ &\quad - X(RE(i))F_q(RE(i)) \\ V_q(RE(i)) &= V_q(SE(i)) - X(RE(i))F_d(RE(i)) \\ &\quad + r(RE(i))F_q(RE(i)) \end{aligned} \quad (7)$$

where:

- RE(i): is the receiving-end of the branch i.
- SE(i): is the sending-end of the branch i.

For the first branch the d and q components of sending-end of the branch one are respectively equal to 1.0 p.u and zero. This correspond to the source node (node 0) which is also the reference node.

The voltage rms value and phase-angle of the receiving-end of the branch i are given by:

$$\begin{aligned} V(RE(i)) &= \sqrt{V_d^2(RE(i)) + V_q^2(RE(i))} \\ \varphi(RE(i)) &= a\tan\frac{V_q(RE(i))}{V_d(RE(i))} \end{aligned} \quad (8)$$

SOLUTION METHODOLOGY

To determine the voltage at each node of radial distribution networks, the proposed method can be summarized in the following algorithm.

Step 1: Read the line data.
Step 2: Determine the nodes beyond each branch and their total number (matrix BR and M(i)).
Step 3: Initialize the voltage of all the nodes to 1p.u and phase-angle to zero.
Step 4: Perform the backward sweep to obtain the current in each branch by using Eq. (1) to (5).
Step 5: Perform the forward sweep to calculate the voltage rms value and phase-angle at each node by using Eq. (7) and (8).
Step 6: If the voltage at each node for two successive iterations is within a certain tolerance (10^{-4}p.u) the solution is reached go to step 7 else, repeat step 4 to 6 until the convergence criterion is reached.
Step 7: Read the results.

APPLICATION

To check the validity of the proposed method, an algorithm was implemented in Matlab. Several tests were carried out to verify its accuracy and convergence behaviour. Three sample radial lines; 12-node system (Das, 1994) of Fig. 3, 33-node system (Ranjan and Das, 2003) shown in Fig. 4 and 69-node system (Ranjan *et al.*, 2003) represented in Fig. 5.

CONCLUSIONS

In this study, a simple and efficient load flow solution has been proposed for determining voltage rms values and phase-angles of radial distribution feeders. A simple to understand method to number the nodes beyond each branch was put ahead. If the number of iterations is relatively great in our case, the time to reach the final solution is weak although compared to authors cited in this study, in our study we have considered the phase-angles calculation (Table 6). From the results point of view (Table 2-5), the values obtained for the rms values are comparable to those given in references (Das, 1994) for the 12-node system (Table 7) (Ranjan and Das, 2003) for the 33-node system (Ranjan *et al.*, 2003) (Table 8) and for 69-node system (Table 9). The deviation in the results are between [0; 29 10^{-4} %] if the tolerance is of 10^{-4} p.u and [0 ; 9 10^{-4} %] if the tolerance is of 10^{-7}p.u for the 12-node system. The results deviation is between [0 ; 10.62 10^{-3}%] for the 33-node system and [0; 2.18 10^{-2} %] for the 69-node system.

Table 2: 12-nodeload flow solution, tolerance 10^{-4}

Node n°	V in p.u	φ in rd	No. of iterations	Cpu in (sec)	Total power losses
0	1.000000	0.000000			
1	0.994332	0.002029			
2	0.989030	0.003899			
3	0.980578	0.007020			
4	0.969823	0.010973			Active:
5	0.966536	0.012181	13	0.031	20.714 kW
6	0.963750	0.013236			
7	0.955311	0.017652			Reactive:
8	0.947281	0.021681			08.041 kVAr
9	0.944470	0.023004			
10	0.943578	0.023417			
11	0.943379	0.023541			

Table 3: 12-nodeload flow solution, tolerance 10^{-7}

Node n°	V in p.u	φ in rd	No. of iterations	Cpu in (sec)	Total power losses
0	1.000000	0.000000			
1	0.994332	0.002029			
2	0.989030	0.003899			
3	0.980578	0.007020			
4	0.969823	0.010973			Active:
5	0.966536	0.012181	21	0.047	20.714 kW
6	0.963749	0.013236			
7	0.955309	0.017652			Reactive:
8	0.947277	0.021681			08.041 kVAr
9	0.944461	0.023003			
10	0.943563	0.023417			
11	0.943354	0.023540			

Table 4: 33-node load flow solution

Node n°	V in p.u	φ in rd	Node n°	V in p.u	φ in rd	No. of iterations	Cpu in (sec)	Total power losses
0	1.000000	0.000000	17	0.903915	-0.012124			
1	0.997014	0.000238	18	0.996486	0.000049			
2	0.982882	0.001673	19	0.992908	-0.001120			
3	0.975373	0.002827	20	0.992204	-0.001458			
4	0.967946	0.003999	21	0.991567	-0.001813			
5	0.949468	0.002356	22	0.979296	0.001132			Active:
6	0.945944	-0.001688	23	0.972625	-0.000417	19	0.062	210.986 kW
7	0.932288	-0.004367	24	0.969300	-0.001179			
8	0.925956	-0.005668	25	0.947539	0.003045			Reactive:
9	0.920100	-0.006789	26	0.944975	0.004025			143.127 kVAr
10	0.919233	-0.006660	27	0.933533	0.005473			
11	0.917722	-0.006456	28	0.925314	0.006833			
12	0.911552	-0.008083	29	0.921758	0.008671			
13	0.909273	-0.009485	30	0.917599	0.007197			
14	0.907862	-0.010157	31	0.916689	0.006793			
15	0.906503	-0.010573	32	0.916414	0.006657			
16	0.904486	-0.011951						

Table 5: 69-node load flow solution

Node n°	V in p.u	φ in rd	Node n°	V in p.u	φ in rd	No. of iterations	Cpu in (sec)	Total power losses
0	1.000000	0.000000	35	0.999919	-0.000052			
1	0.999967	-0.000021	36	0.999747	-0.000164			
2	0.999933	-0.000043	37	0.999589	-0.000206			
3	0.999840	-0.000103	38	0.999543	-0.000218			
4	0.999021	-0.000323	39	0.999541	-0.000219			
5	0.990086	0.000860	40	0.998843	-0.000410			
6	0.980794	0.002113	41	0.998551	-0.000491			
7	0.978578	0.002413	42	0.998512	-0.000502			
8	0.977445	0.002567	43	0.998504	-0.000505			
9	0.972447	0.004045	44	0.998405	-0.000536			
10	0.971347	0.004372	45	0.998405	-0.000536			
11	0.968187	0.005293	46	0.999789	-0.000134			
12	0.965265	0.006101	47	0.998544	-0.000917			
13	0.962368	0.006909	48	0.994699	-0.003344			
14	0.959500	0.007711	49	0.994154	-0.003690			
15	0.958967	0.007861	50	0.978543	0.002418			
16	0.958088	0.008108	51	0.978533	0.002421			Active:
17	0.958081	0.008111	52	0.974659	0.002949	25	0.078	224.946 kW
18	0.957620	0.008260	53	0.971416	0.003396			
19	0.957326	0.008357	54	0.966942	0.004017			Reactive:
20	0.956851	0.008513	55	0.962574	0.004628			102.139 kVAr
21	0.956853	0.008516	56	0.940100	0.011549			
22	0.956792	0.008541	57	0.929041	0.015084			
23	0.956646	0.008593	58	0.924763	0.016498			
24	0.956486	0.008648	59	0.919740	0.018321			
25	0.956428	0.008671	60	0.912344	0.019527			
26	0.956457	0.008699	61	0.912059	0.019574			
27	0.999926	-0.000046	62	0.911676	0.019638			
28	0.999859	-0.000082	63	0.909782	0.019951			
29	0.999762	-0.000037	64	0.909219	0.020045			
30	0.999745	-0.000029	65	0.971290	0.004392			
31	0.999661	0.000011	66	0.971289	0.004393			
32	0.999457	0.000106	67	0.967857	0.005399			
33	0.999228	0.000244	68	0.967856	0.005399			
34	0.999161	0.000262						

Table 6: Speed comparison (results from (Ghosh and Das, 1999)

Method	CPU time (s)	No. of iterations
Mekhamer *et al*. (2002)	0.05 (9-node network)	11
Ghosh and Das (1999)	0.09 (33-node network)	3
	0.16 (69-node network)	3
Renato load flow	0.14 (33-node network)	4
using forward sweep	0.33 (69-node network)	4
Kersting load flow	0.16 (33-node network)	4
using ladder technique	0.37 (69-node network)	4
Proposed method	0.047 (9-node network)	13
	0.062 (33-node network)	9
	0.078 (69-node network)	25

Table 7: 12-node line data (Das, 1994)

Branch n°	Sending- end	Receiving-end	r (ohms)	X (ohms)	P_L at RE(i) kW	Q_L at RE(i) KVAr
1	0	1	1.093	0.455	60	60
2	1	2	1.184	0.494	40	30
3	2	3	2.095	0.873	55	55
4	3	4	3.188	1.329	30	30
5	4	5	1.093	0.455	20	15
6	5	6	1.002	0.417	55	55
7	6	7	4.403	1.215	45	45
8	7	8	5.642	1.597	40	40
9	8	9	2.890	0.818	35	30
10	9	10	1.514	0.428	40	30
11	10	11	1.238	0.351	15	15

Table 8: 33-node line data (Ranjan and Das, 2003)

Branch n°	Sending- end	Receiving-end	r (ohms)	X (ohms)	P_L at RE(i) kW	Q_L at RE(i) KVAr
1	0	1	0.0922	0.0477	100	60
2	1	2	0.4930	0.2511	90	40
3	2	3	0.3660	0.1864	120	80
4	3	4	0.3811	0.1941	60	30
5	4	5	0.8190	0.7070	60	20
6	5	6	0.1872	0.6188	200	100
7	6	7	1.7114	1.2351	200	100
8	7	8	1.0300	0.7400	60	20
9	8	9	1.0400	0.7400	60	20
10	9	10	0.1966	0.0650	45	30
11	10	11	0.3744	0.1238	60	35
12	11	12	1.4680	1.1550	60	35
13	12	13	0.5416	0.7129	120	80
14	13	14	0.5910	0.5260	60	10
15	14	15	0.7463	0.5450	60	20
16	15	16	1.2890	1.7210	60	20
17	16	17	0.7320	0.5740	90	40
18	1	18	0.1640	0.1565	90	40
19	18	19	1.5042	1.3554	90	40
20	19	20	0.4095	0.4784	90	40
21	20	21	0.7089	0.9373	90	40
22	2	22	0.4512	0.3083	90	50
23	22	23	0.8980	0.7091	420	200
24	23	24	0.8960	0.7011	420	200
25	5	25	0.2030	0.1034	60	25
26	25	26	0.2842	0.1447	60	25
27	26	27	1.0590	0.9337	60	20
28	27	28	0.8042	0.7006	120	70
29	28	29	0.5075	0.2585	200	600
30	29	30	0.9744	0.9630	150	70
31	30	31	0.3105	0.3619	210	100
32	31	32	0.3410	0.5302	60	40

Table 9: 69-node line data (Ranjan *et al*., 2003)

Branch n°	Sending- end	Receiving-end	r (ohms)	X (ohms)	P_L at RE(i) kW	Q_L at RE(i) KVAr
1	0	1	0.0005	0.0012	0.0	0.0
2	1	2	0.0005	0.0012	0.0	0.0
3	2	3	0.0015	0.0036	0.0	0.0
4	3	4	0.0251	0.0294	0.0	0.0
5	4	5	0.3660	0.1864	2.6	2.2
6	5	6	0.3811	0.1941	40.4	30.0
7	6	7	0.0922	0.0470	75.0	54.0
8	7	8	0.0493	0.0251	30.0	22.0
9	8	9	0.8190	0.2707	28.0	19.0
10	9	10	0.1872	0.0619	145.0	104.0
11	10	11	0.7114	0.2351	145.0	104.0
12	11	12	1.0300	0.3400	8.0	5.0
13	12	13	1.0440	0.3450	8.0	5.5
14	13	14	1.0580	0.3496	0.0	0.0
15	14	15	0.1966	0.0650	45.5	30.0
16	15	16	0.3744	0.1238	60.0	35.0
17	16	17	0.0047	0.0016	60.0	35.0
18	17	18	0.3276	0.1083	0.0	0.0
19	18	19	0.2106	0.0690	1.0	0.6
20	19	20	0.3416	0.1129	114.0	81.0
21	20	21	0.0140	0.0046	5.0	3.5
22	21	22	0.1591	0.0526	0.0	0.0
23	22	23	0.3463	0.1145	28.0	20.0
24	23	24	0.7488	0.2475	0.0	0.0
25	24	25	0.3089	0.1021	14.0	10.0
26	25	26	0.1732	0.0572	14.0	10.0
27	2	27	0.0044	0.0108	26.0	18.6
28	27	28	0.0640	0.1565	26.0	18.6
29	28	29	0.3978	0.1315	0.0	0.0
30	29	30	0.0702	0.0232	0.0	0.0
31	30	31	0.3510	0.1160	0.0	0.0
32	31	32	0.8390	0.2816	14.0	10.0
33	32	33	1.7080	0.5646	9.5	14.0

Table 9: Continued

Branch n°	Sending-end	Receiving-end	r (ohms)	X (ohms)	P_L at RE (i) kW	Q_L at RE (i) KVAr
34	33	34	1.4740	0.4873	6.00	4.00
35	2	35	0.0044	0.0108	26.00	18.55
36	35	36	0.0640	0.1565	26.00	18.55
37	36	37	0.1053	0.1230	0.00	0.00
38	37	38	0.0304	0.0355	24.00	17.00
39	38	39	0.0018	0.0021	24.00	17.00
40	39	40	0.7283	0.8509	1.20	1.00
41	40	41	0.3100	0.3623	0.00	0.00
42	41	42	0.0410	0.0478	6.00	4.30
43	42	43	0.0092	0.0116	0.00	0.00
44	43	44	0.1089	0.1373	39.22	26.30
45	44	45	0.0009	0.0012	39.22	26.30
46	3	46	0.0034	0.0084	0.00	0.00
47	46	47	0.0851	0.2083	79.00	56.40
48	47	48	0.2898	0.7091	384.70	274.50
49	48	49	0.0822	0.2011	384.70	274.50
50	6	50	0.0928	0.0473	40.50	28.30
51	50	51	0.3319	0.1114	3.60	2.70
52	7	52	0.1740	0.0886	4.35	3.50
53	52	53	0.2030	0.1034	26.40	19.00
54	53	54	0.2842	0.1447	24.00	17.20
55	54	55	0.2813	0.1433	0.00	0.00
56	55	56	1.5900	0.5337	0.00	0.00
57	56	57	0.7837	0.2630	0.00	0.00
58	57	58	0.3042	0.1006	100.00	72.00
59	58	59	0.3861	0.1172	0.00	0.00
60	59	60	0.5075	0.2585	1244.00	888.00
61	60	61	0.0974	0.0496	32.00	23.00
62	61	62	0.1450	0.0738	0.00	0.00
63	62	63	0.7105	0.3619	227.00	162.00
64	63	64	1.0410	0.5302	59.00	42.00
65	9	65	0.2012	0.0611	18.00	13.00
66	65	66	0.0047	0.0014	18.00	13.00
67	10	67	0.7394	0.2444	28.00	20.00
68	67	68	0.0047	0.0016	28.00	20.00

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