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Effect of Radiation on the Flow of a Visco-Elastic Fluid and Heat Transfer in a Porous Medium over a Stretching Sheet

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Abstract: A numerical study is carried out on the flow and heat transfer characteristics of a visco-elastic fluid (Walters' liquid-B model) past a stretching sheet in a porous medium considering the influence of radiation. The boundary layer equations together with the appropriate boundary conditions are first transformed by a similarity transformation into a system of ordinary differential equations. The similarity equations are then solved numerically by using the fourth order Runge-Kutta scheme with the help of shooting method. Two cases of heat transfer are considered- (1) The sheet with Prescribed Surface Temperature (PST case) and (2) The sheet with Prescribed Heat Flux (PHF case). The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The velocity and temperature profiles are graphically presented for various influencing parameters in non-dimensional form such as radiation number, Prandtl number, visco-elastic parameter, permeability parameter and heat source/sink parameter. The study has shown that the numerical technique used herein, can be advantageously used to solve the boundary value problem of a fluid flow and to assess temperature variation within the boundary.

Key words: Heat transfer, visco-elasticity, porosity, radiation, Prandtl number, boundary layer

INTRODUCTION

The study of laminar flow in a boundary layer caused by a moving rigid surface was initiated by Sakiadis (1961). Later the work was extended by Crane (1970) to the flow due to stretching of a sheet. Heat transfer in the boundary layer of an incompressible fluid past a continuous moving surface has several industrial applications in textile and paper industries, extrusion process of plastic sheets, spinning of fibers etc. Non-Newtonian fluids have since gained considerable importance due to their extensive use in industry. Visco-elastic boundary layer flow over a stretching sheet has been the main subject of a number of researchers in the past (Rajagopal and Gupta, 1984, Rollins and Vajravelu, 1992). It has been generally found that heat transfer in the visco elastic fluid is less compared to that of a Newtonian fluid. Viscoelastic fluids are therefore more popularly used in industry than Newtonian fluids.

The transport of heat in a porous medium has also considerable practical applications in geothermal systems, crude oil extraction, ground water pollution and also in a wide range of biomechanical problems.

In the earlier investigations involving heat transfer, two different heating processes have been considered namely 1. Prescribed Surface Temperature (PST case) and 2. Prescribed Heat Flux (PHF case). Vajravelu (1994) studied the flow of a steady viscous fluid and heat transfer characteristics in a porous medium by considering both these cases. Abel *et al.* (1998) studied the visco-elastic fluid flow and heat transfer characteristics in a saturated porous medium over an impermeable stretching surface. Abel *et al.* (2004) carried out the study of the momentum, mass and heat transfer characteristics on the flow of a visco-elastic fluid past a stretching sheet in the presence of a transverse magnetic field.

The influence of radiation on hydrodynamic flow and heat transfer characteristics is also recognized by some investigators. Raptis (1999) studied the effect of radiation on viscoelastic flows. The magneto hydrodynamic flow and heat transfer of a viscoelastic fluid over a stretching sheet considering the radiation effects has been studied by Siddheshwar and Mahabaleswar (2005). The study of the effects of variable viscosity and variable thermal conductivity on heat transfer from a moving surface in a micropolar fluid through a porous medium considering radiation is done by Elsayed Elbarbary *et al.* (2004).

In all the above cases involving radiation, the studies were essentially based on closed form analytical solutions. The aim of the present study is to use a

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numerical technique to study the effect of radiation on the visco-elastic flow and heat transfer in a porous medium over a stretching sheet and compare the results with those of earlier researchers. For the heat transfer category the two cases namely PST and PHF have been considered. The governing partial differential equations have been reduced to a system of ordinary differential equations by similarity transformation. This equation set is then solved numerically by the Runge-Kutta method using the shooting technique for the relevant boundary conditions and flow parameters. The results obtained for typical values of the influencing parameters in non-dimensional form which include the visco-elastic parameter (K_1), the permeability parameter (K_2), radiation number (N_R), the Prandtl number (Pr) and heat source/sink parameter (β) are graphically represented to show the degree of influence of these variables on the flow and the heat transfer characteristics. These parameters will be defined in subsequent sections.

MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible visco-elastic fluid (Walter's liquid B model) through a porous medium past a semi-infinite stretching sheet. The following notations are used in the mathematical treatment that follows:

Let x and y represent the tangential (along flow) and transverse (across flow) directions respectively; let u and v denote the velocity components of the fluid in the x and y directions. T the temperature, v the kinematic viscosity, k_0 is the coefficient of viscoelasticity, k' is the permeability coefficient of the porous medium and ρ is the fluid density. Further, let k denote the thermal conductivity, c_P, the specific heat at constant pressure and q_r, the radiative heat flux. Let Q represent the volumetric rate of heat generation, T_w the sheet surface temperature, T_∞ the temperature at the farthest location representing the edge of the boundary and q_w denote the heat transfer rate per unit area. λ is the stretching rate of the sheet (assumed constant) along x axis.

The fluid flow is confined to the half space y>0. By applying two equal and opposite forces along the x-axis, the sheet gets stretched with a speed proportional to the distance from the fixed origin i.e., u = λx. The continuity equation and the basic boundary layer equations governing the flow and heat transfer including radiation take the form of Eq. 1 to 3.

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equation governing velocity of flow:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{v}{k'}u \tag{2}$$

Equation governing heat transfer:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho\, c_P}\left(T-T_\infty\right)-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y} \tag{3}$$

Boundary conditions: For the PST case, the surface temperature and for the PHF case, the heat flux are considered as a power series in x. The expressions for temperature T involve the characteristic length l and two distinct constants A and D for the PST and PHF cases, respectively.

(i) At y - 0: $\quad u = \lambda x, \quad v = 0\ (\lambda>0)$

$$T=T_w\left(=T_\infty+A\left(\frac{x}{l}\right)^2\right) \quad \text{(PST case)}$$

$$-k\frac{\partial T}{\partial y}=q_w=D\left(\frac{x}{l}\right)^2 \quad \text{(PHF case)} \tag{4}$$

(ii) As $y\to\infty$: $u\to 0$, $u_y\to 0$

$$T\to T_\infty \tag{5}$$

Transformation of equations: The continuity Eq. 1 is satisfied by the stream function defined by

$$u=\frac{\partial\psi}{\partial y},\qquad v=\frac{-\partial\psi}{\partial x} \tag{6}$$

The similarity transformation is introduced in the form

$$\eta=y\sqrt{\frac{\lambda}{v}} \qquad \psi(x,y)=(\lambda\, v)^{\frac{1}{2}}\,x\,f(\eta) \tag{7}$$

The non-dimensional temperature θ and g for the two cases PST and PHF are as follows:

PST case:
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

where

$$T - T_\infty = A\left(\frac{x}{1}\right)^2 \theta(\eta) \quad \text{and} \quad T_w - T_\infty = A\left(\frac{x}{1}\right)^2$$

PHF case:
$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

where

$$T - T_\infty = \frac{D}{k}\left(\frac{x}{1}\right)^2 \sqrt{\frac{\nu}{\lambda}}\, g(\eta) \text{ and } T_w - T_\infty = \frac{D}{k}\left(\frac{x}{1}\right)^2 \sqrt{\frac{\nu}{\lambda}}$$

The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. The radiative heat flux q_r is employed according to Rosseland approximation given by Siddheshwar and Mahabaleswar (2005) in the form

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{10}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively.
Expanding T^4 in a Taylor series about T_∞ we get

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \cdots\cdots\infty \tag{11}$$

Neglecting terms beyond first degree in $(T-T_\infty)$, we get

$$T^4 \cong -3T_\infty^4 + 4T_\infty^3 T \tag{12}$$

Substituting (12) in (10)

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial}{\partial y}\left(-3T_\infty^4 + 4T_\infty^3 T\right)$$

$$\Rightarrow q_r = \frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \tag{13}$$

We use the following non-dimensional influencing parameters in further steps.

$$K_1 = \frac{K_0 \lambda}{\nu} \quad \text{(Visco-elastic parameter)}$$

$$K_2 = \frac{\nu}{k' \lambda} \quad \text{(Permeability parameter)}$$

$$N_R = \frac{16\sigma^* T_\infty^3}{3kk^*} \quad \text{(Radiation Number)}$$

$$\Pr = \frac{\mu\, c_P}{k} \quad \text{(Prandtl number)}$$

$$\beta = \frac{Q}{\lambda \rho c_p} \quad \text{(Heat source/sink parameter)}$$

Using expressions (7) to (9), (13) and the above non-dimensional parameters, Eq. 2 and 3, representing velocity and temperature variation within the boundary layer can be reduced to the following Eq. 14 to 16.

$$\left(f'\right)^2 - ff'' = f''' - k_1\left\{2f'f''' - ff^{iv} - \left(f''\right)^2\right\} - K_2 f' \tag{14}$$

PST case:
$$(1 + N_R)\theta'' + \Pr f\theta' - \Pr\left(2f' - \beta\right)\theta = 0 \tag{15}$$

PHF case:
$$(1 + N_R)g'' + \Pr fg' - \Pr\left(2f' - \beta\right)g = 0 \tag{16}$$

The superscript ′ denotes differentiation with respect to η

The boundary conditions (4) and (5) are rewritten in non-dimensional form as follows:

(i) $\eta = 0$: $f = 0$, $f' = 1$

$\theta = 1$ (PST case)

$g' = -1$ (PHF case) (17)

(ii) $\eta \to \infty$: $f' \to 0$, $f'' \to 0$

$\theta \to 0$ (PST case)

$g \to 0$ (PHF case) (18)

Numerical procedure: The shooting method for the solution of non-linear differential equations basically involves choosing initial values for the concerned derivatives in such a way that the end boundary conditions are satisfied with in a prescribed numerical tolerance value. In this study, the numerical tolerance value is chosen 10^{-6} which is very close to zero for computational purpose. The sequence of initial values is given by the secant method. The initial value problem is solved using the fourth order Runge-Kutta scheme. The value of η at ∞ i.e., η_{max} is so chosen that the solution shows little further change for η larger than η_{max}. The system of differential Eq. (14) to (16) together with the boundary conditions (17) and (18) have been solved numerically using the Runge-Kutta algorithm starting a systematic guessing of values for $g''(0)$ $f'''(0)$, $\theta'(0)$ and $g(0)$ and with the help of shooting technique until the boundary conditions f', f'', θ and g at η_{max} are satisfied at the far end of the boundary. If the boundary conditions at η_{max} are not satisfied, then the numerical technique uses a half-interval method to calculate the corrections to be applied to the initially estimated values of $f''(0)$, $f'''(0)$, $\theta'(0)$ and $g(0)$. This process is repeated iteratively until the prescribed end values for f', f'', θ and g are obtained finally. The intermediate values at the end of each chosen interval within the boundary are evaluated stage by stage.

RESULTS AND DISCUSSION

Figure 1 and 2 show the effects of viscoelastic parameter (k_1) and permeability parameter (k_2) on the flow velocity for two chosen values of k_2 and k_1, respectively. The main effect of viscoelasticity is to gradually reduce the flow velocity within the boundary layer, as may be seen from Fig. 1. It is clear from Fig. 2 that the flow velocity also decreases gradually with increase in porosity (k_2). It is observed however that the influence of k_1 is marginal as in Fig. 1 while the influence of k_2 is relatively appreciable at low values of k_2 (<1).

The appropriate boundary conditions corresponding to PST and PHF cases are used while solving the respective heat transfer Eq. (15) and (16). The Rosseland approximation given by Siddheshwar and Mahahbaleswar (2005) is used here to describe the radiative heat flux (q_r) in the energy equation.

Figure 3-6 show the results of the temperature variation under varying values of k_1, k_2, N_R and Pr. It may be seen from these figures that the effect of increasing k_1 and k_2 is to increase the temperature at any point within the boundary. The influences of k_1 and k_2 are, however, seen to be marginal (Fig. 3 and 4). However, lower values

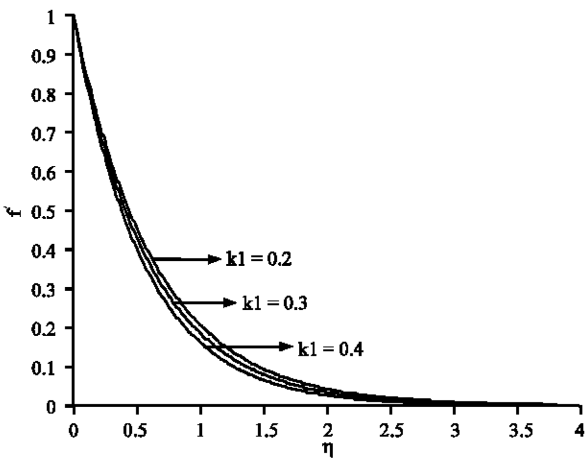

Fig. 1: Effect of the viscoelastic parameter k_1 on the velocity distribution (k_2 = 1.0)

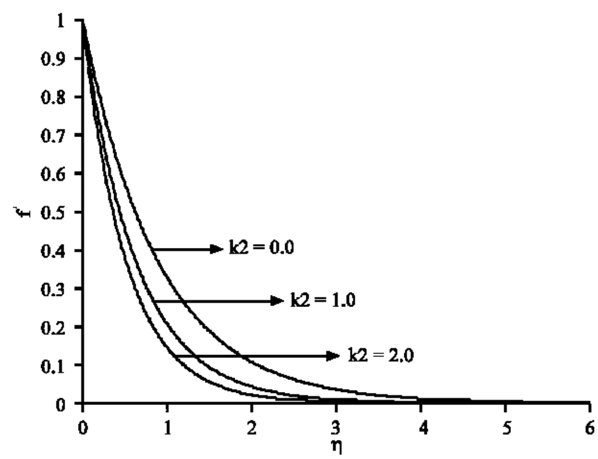

Fig 2: Effect of the permeability parameter k_2 on the velocity distribution (k_1 = 0.2)

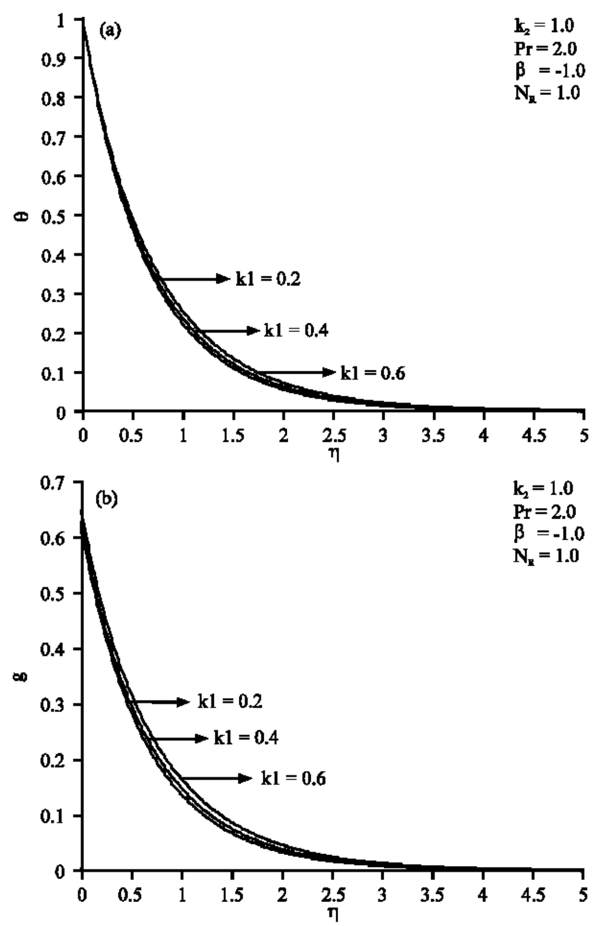

Fig 3: Effect of viscoelastic parameter k_1 on the temperature distribution. (a) PST case (b) PHF case

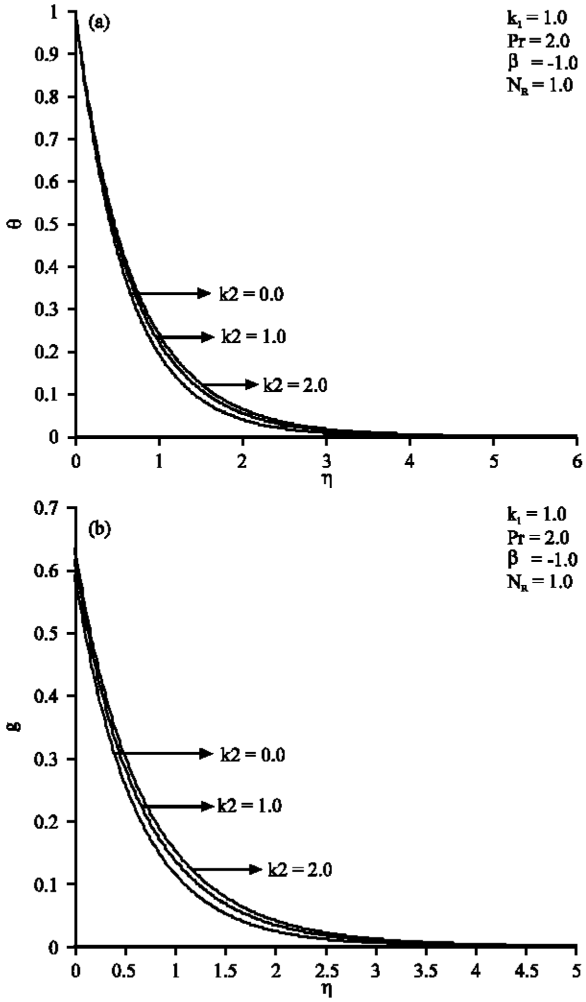

Fig. 4: Effect of permeability parameter k_2 on the temperature distribution (a) PST case (b) PHF case

of k_1 and k_2 are desirable for the fluid to act as an effective coolant in the application for extrusion polymer process.

The effect of the radiation parameter (N_R) is found to be substantial as may be seen from Fig. 5. In contrast to this, influence of Prandtl number (Pr) is seen to be opposite to that of N_R. This means that, as Pr increases, the temperature decreases. This implies that a low value of N_R and a high value of Pr is a good combination for the fluid to perform as an effective cooling medium in applications such as extrusion processes in polymer industry.

Comparing the trend of results between PST and PHF cases for the same set of values of the influencing parameters, it is seen that at any chosen location within the boundary, the temperature is lower in PHF case than in PST case. This leads to the inference that PHF is a better option for the effective coolant action than PST case.

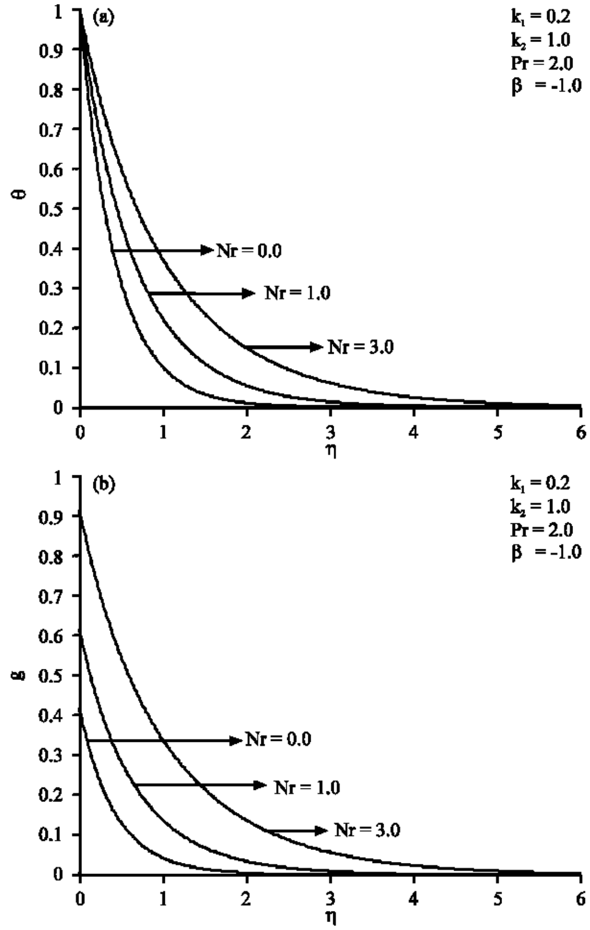

Fig. 5: Effect of the radiation number N_R on the temperature distribution (a) PST case (b) PHF case

Table 1: Values of wall temperature gradient $|\theta'(0)|$ for the PST case and wall temperature g(0) for the PHF case for different values of k_1, k_2, N_R, Pr and β

k_1	k_2	N_R	Pr	β	$\lvert\theta'(0)\rvert$	g(0)
0.2	1.0	1.0	2.0	-0.10	1.661550	0.601848
0.3	1.0	1.0	2.0	-0.10	1.651236	0.605607
0.4	1.0	1.0	2.0	-0.10	1.638395	0.610354
0.2	0.0	1.0	2.0	-0.10	1.707130	0.585778
0.2	1.0	1.0	2.0	-0.10	1.661550	0.601848
0.2	2.0	1.0	2.0	-0.10	1.627064	0.614604
0.2	1.0	0.0	2.0	-0.10	2.395709	0.417413
0.2	1.0	1.0	2.0	-0.10	1.661550	0.601848
0.2	1.0	3.0	2.0	-0.10	1.198929	0.834077
0.2	1.0	1.0	1.0	-0.10	1.198929	0.834077
0.2	1.0	1.0	2.0	-0.10	1.661550	0.601848
0.2	1.0	1.0	3.0	-0.10	2.053065	0.487077
0.2	1.0	1.0	2.0	-1.00	1.661550	0.601848
0.2	1.0	1.0	2.0	0.00	1.313952	0.761063
0.2	1.0	1.0	2.0	0.05	1.294436	0.772537

Table 1 represents the values of the wall temperature gradient $|\theta^1(0)|$ for the PST case and the wall temperature g (0) for various values of k_1, k_2, N_R, Pr and β. As all values of $\theta^1(0)$ are obtained negative for the range of parameters studied herein, modulus values ($|\theta'(0)|$) are

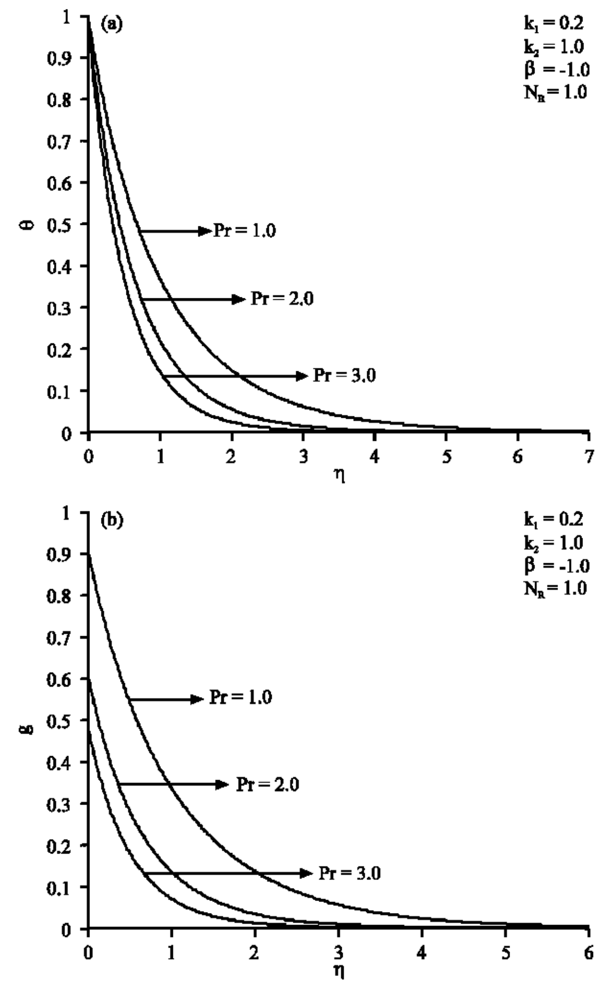

Fig. 6: Effect of the Prandtl number on the temperature distribution (a) PST case (b) PHF case

tabulated. This also implies that in this case, heat flows from the sheet to the fluid region.

It may also be observed from the table that $|\theta^1(0)|$ decreases with increase in k_1, k_2, NR and β Further $|\theta^1(0)|$ increases with increase in Pr.

The wall temperature g(0) is seen to increase with increase in k_1, k_2, N_R and β but it decreases with increase in Pr.

In the absence of radiation, the results deduced from this study are seen to agree well with those obtained by Abel *et al.* (1998).

CONCLUSIONS

From this numerical study, the following conclusions are drawn:

- The velocity of the fluid decreases with increase in viscoelastic parameter k_1 and permeability parameter k_2. The influence of k_2 is however more marked than k_1.
- The temperature at any point within the thermal boundary layer increases with increase in k_1, k_2 as also with increase in radiation parameter N_R. The influence of Prandtl number Pr is however seen to be opposite to the above. Further these results are seen to be valid for both PST and PHF cases.
- In applications where the fluid is to act as an effective coolant such as in extrusion polymer processes, the PHF case may be preferred to PST case since the former gives lower temperatures than in the latter case.
- The wall temperature gradient $|\theta^1(0)|$ in the PST case decreases with increase in k, k_2, N_R and β but increases with increase in Pr. The opposite trend is seen for the influence of these parameters on the wall temperature g(0) in the PHF case.
- The results obtained from this study for the particular case when the effect of radiation is ignored (i.e., $N_R = 0$) agree well with those obtained by Abel *et al.* (1998).

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