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A Note on the Deterministic Inventory Models with Shortage and Defective Items Derived Without Derivatives

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Abstract: Huang studied the EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) models with backlogging and defective items using the algebraic approach. Huang assumed 100% inspection policy and the known proportion of defective items was removed prior to storage or use after the screening process. This study will offer a simple algebraic approach to improve his algebraic skill to find the optimal solution under the total relevant cost per unit time minimized.

Key words: EOQ, EPQ, shortage, defective item, algebraic approach

INTRODUCTION

The Economic Order Quantity (EOQ) model with/without shortages and Economic Production Quantity (EPQ) model with/without shortages are widely used by practitioners as a decision-making tool for the control of inventory. However, the assumptions of the EOQ/EPQ model are rarely met. This has led many researchers to study the EOQ/EPQ extensively under realistic situations.

A common unrealistic assumption of the EOQ/EPQ is that all units produced or purchased are of good quality. We know that it is difficult to produce or purchase items with 100% good quality. Recently, Huang[1] studied the EOQ and EPQ models with backlogging and defective items. He assumed 100% inspection policy and the known proportion of defective items was removed prior to storage or use after the screening process. In addition, Huang[1] used the algebraic method to determine the optimal solution under minimizing the annual relevant cost. In previous several studies, the EOQ and EPQ formulae for the shortage case, have been derived using differential calculus and solving two simultaneous equations with the need to prove optimality conditions with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. Grubbström and Erdem[2] and Cárdenas-Barrón[3] showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. But Ronald *et al.*[4] thought that their algebraic procedure is too sophisticated to be absorbed by ordinary readers. Hence, Ronald *et al.*[4] derived a procedure to transform a two-variable problem into two steps and then, in each step, they solve a one-variable problem using only the algebraic method without referring to calculus. Recently, Chang *et al.*[5] rewrote the objective function of Ronald *et al.*[4] such that the usual skill of completing the square can handle the problem without using their sophisticated method.

Although, Huang[1] used the easily algebraic approach to find the optimal solution. However, his method had the same problem as Grubbström and Erdem[2] and Cárdenas-Barrón[3]. Therefore, in this study, we will offer a simple algebraic approach same as Chang *et al.*[5] to replace his complicated algebraic skill. This method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus as Grubbström and Erdem[2] and Cárdenas-Barrón[3] stated.

Algebraic improvement in the two models: The following notation and assumptions the same as Huang[1] will be used in this note.

Notation

Q = Order Quantity (EOQ model)/Production Quantity (EPQ model) (including defective items)

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V = Maximum on-hand inventory level (including defective items)
L = Maximum shortage (backorder) level (including defective items)
D = Demand rate for nondefective items, units per time
P = Production rate for nondefective items, units per time (P>D)
A = Ordering cost/setup cost
I = The fixed inspection cost incurred with each lot
i = Unit inspection cost
h = Unit stock holding cost per unit per time
b = Unit shortage cost per unit short per time
k = The known percentage of defective items in Q
TAC (V, L) = Total relevant cost per unit time

Assumptions

- Demand rate is known and constant.
- Production rate is known and constant.
- Time period is infinite.
- Each lot purchased/produced contains a known proportion of defectives that removed prior to storage or use.
- The screening time for items is so fast that we can neglect it. The inspection cost consists of a fixed per lot inspection cost and a fixed unit inspection cost.

Model I: EOQ model (shown in Fig. 1): From Eq. 2 in Huang[1], we know the total relevant cost per unit time, TAC (V, L), can be expressed as:

$$\mathrm{TAC}(V,L)=\frac{D}{(1-k)(V+L)}\left[A+I+(V+L)i+\frac{(1-k)^2V^2h}{2D}+\frac{(1-k)^2L^2b}{2D}\right] \quad (1)$$

For convenience, we let Q = V + L and substitute into Eq. 1.

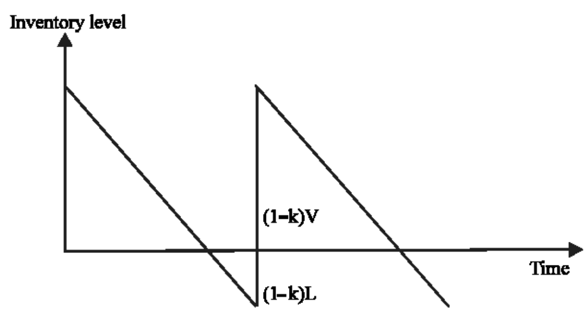

Fig. 1: The inventory level of model I

Then, we can obtain

$$\mathrm{TAC}(Q,L)=\frac{D}{(1-k)Q}\left[A+I+Qi+\frac{(1-k)^2(Q-L)^2h}{2D}+\frac{(1-k)^2L^2b}{2D}\right] \quad (2)$$

Our goal is to find the minimum solution of TAC (Q, L) by algebraic approach. Then we rewrite Eq. 2 as

$$\mathrm{TAC}(Q,L)=\frac{(1-k)(h+b)}{2Q}\left[L-\frac{h}{h+b}Q\right]^2+\frac{(1-k)hbQ}{2(h+b)}+\frac{D(A+I)}{(1-k)Q}+\frac{Di}{1-k} \quad (3)$$

It implies that when Q is given, we can set L as: L = [h/h + b] Q to get the minimum value of TAC (Q, L) as follows:

$$\mathrm{TAC}[Q,L(Q)]=\frac{(1-k)hbQ}{2(h+b)}+\frac{D(A+I)}{(1-k)Q}+\frac{Di}{1-k} \quad (4)$$

Then, we rewrite Eq. 4 as:

$$\mathrm{TAC}[Q,L(Q)]=\left[\sqrt{\frac{(1-k)hbQ}{2(h+b)}}-\sqrt{\frac{D(A+I)}{(1-k)Q}}\right]^2+\sqrt{\frac{2hbD(A+I)}{(h+b)}}+\frac{Di}{1-k} \quad (5)$$

Then, we can find the optimal ordering quantity

$$\mathrm{EOQ}(Q^*)=\frac{1}{1-k}\sqrt{\frac{2(A+I)D}{h}}\sqrt{\frac{h+b}{b}}=\frac{1}{1-k}\sqrt{\frac{2(A+I)Db}{h(h+b)}}+\frac{1}{1-k}\sqrt{\frac{2(A+I)Dh}{b(h+b)}} \quad (6)$$

and the optimal allowable backorder level

$$L^*=\frac{1}{1-k}\sqrt{\frac{2(A+I)Dh}{b(h+b)}} \quad (7)$$

Therefore, the minimum value of total relevant cost per unit time, TAC (Q*, L*) is

$$\mathrm{TAC}(Q^*,L^*)=\sqrt{\frac{2hbD(A+I)}{(h+b)}}+\frac{Di}{1-k} \quad (8)$$

Equation 6-8, in this note, are the same as Eq. 5-7 in Huang[1], respectively.

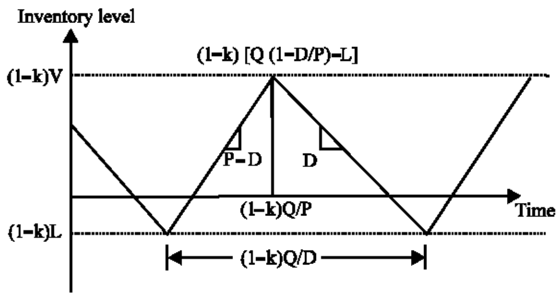

Fig. 2: The inventory level of mogel II

Model II: EPQ model (shown in Fig. 2): For convenience, let $\rho = (1 - D/P)$, $Q = V + L/\rho$ and Eq. 9 in Huang[1], we know the total relevant cost per unit time, TAC (Q, L), can be expressed as:

$$TAC(Q,L) = \frac{D}{(1-k)Q\rho}\left[A + I + \frac{(1-k)^2(Q\rho - L)^2 h}{2D\rho} + \frac{(1-k)^2 L^2 b}{2D\rho}\right] + \frac{Di}{1-k} \tag{9}$$

Our goal is to find the minimum solution of TAC (Q, L) by algebraic approach. Then we rewrite Eq. 9 as

$$TAC(Q,L) = \frac{(1-k)(h+b)}{2Q\rho^2}\left[L - \frac{h\rho}{h+b}Q\right]^2 + \frac{(1-k)hbQ}{2(h+b)} + \frac{D(A+I)}{(1-k)Q\rho} + \frac{Di}{1-k} \tag{10}$$

It implies that when Q is given, we can set L as $L = [h\rho/h+b]Q$ to get the minimum value of TAC (Q, L) as follows:

$$TAC[Q,L(Q)] = \frac{(1-k)hbQ}{2(h+b)} + \frac{D(A+I)}{(1-k)Q\rho} + \frac{Di}{1-k} \tag{11}$$

Then, we rewrite Eq. 11 as

$$TAC[Q,L(Q)] = \left[\sqrt{\frac{(1-k)hbQ}{2(h+b)}} - \sqrt{\frac{D(A+I)}{(1-k)Q\rho}}\right]^2 + \sqrt{\frac{2hbD(A+I)}{(h+b)\rho}} + \frac{Di}{1-k} \tag{12}$$

Then, we can find the optimal production quantity

$$EPQ(Q^*) = \frac{1}{1-k}\sqrt{\frac{2(A+I)D}{h\rho}}\sqrt{\frac{h+b}{b}} \tag{13}$$

and the optimal allowable backorder level

$$L^* = \frac{1}{1-k}\sqrt{\frac{2(A+I)Dh\rho}{b(h+b)}} \tag{14}$$

Therefore, the minimum value of total relevant cost per unit time, TAC (Q*, L*) is

$$TAC(Q^*,L^*) = \sqrt{\frac{2hbD(A+I)}{(h+b)\rho}} + \frac{Di}{1-k} \tag{15}$$

Equation 13-15, in this note, are the same as Eq. 15-17 in Huang[1], respectively.

CONCLUSIONS

This note improves Huang's[1] algebraic procedure to find the optimal ordering quantity (EOQ model) and the optimal production quantity (EPQ model) with shortages and defective items. Using this improved approach presented in this study, we can find the optimal ordering quantity and the optimal production quantity without using differential calculus. This should also mean that this improved algebraic approach is a more accessible approach than Huang to ease the learning of basic inventory theories for younger students who lack the knowledge of calculus.

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