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Computer-aided Deflection and Slope Analyses of Beams

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Abstract: An analyses program of deflection and slope of beams is presented. Mechanical, mathematical and computer methods, techniques and models are included in the program. We can quickly and accurately obtain the deflections and slopes of cantilever and simply supported beams from this program. Moreover, the program can be executed in PC machine, also the design algorithm in this study can be performed in other computer systems. Two critical examples are given to illustrate this program.

Key words: Beam, deflection, slope, CAD, programming, simulation

INTRODUCTION

In constructing the structure, people usually need to predict and control the behaviors and properties of the structure, such as the deflection and slope, to ensure the safety of the structure. So the studies of material and structure are important. Fortunately, the relations between behaviors, material and structure have been carried out by engineers, and written in some literatures[1-4]. They are the exact forms. However, it is a troublesome computation process and easy to mistake. Therefore, we have to develop some general algorithms and design an auto-analysis program to help us to accomplish these works.

This study systematically and modularly combines the theories of solid mechanics, superposition, coordinate transformation, functional mapping and computer techniques[5-8], etc., to develop a computer-aided analyses program of deflection and slope for the cantilever and simply supported beams. With the help of this program, it is found that a great advantage, in computing and solving the solutions, is obtained; the necessary values of deflection and slope will be determined quickly and correctly from this program.

FORMULATION AND PROGRAMMING

Many structures, such as bridge, frame, springboard and so on, use the fundamental types of beams to build them. We have two basic types of beams called cantilever and simply supported beams can be applied to construct the structures. There were some principles and formulas to describe the relations of deflection and slope for these beams. In this study, we take six necessary fundamental types and apply the superposition method to extend to any complex situations of forced beams. Table 1 show the lists the six necessary fundamental types and graphs of forced beams and their deflection and slope equations.

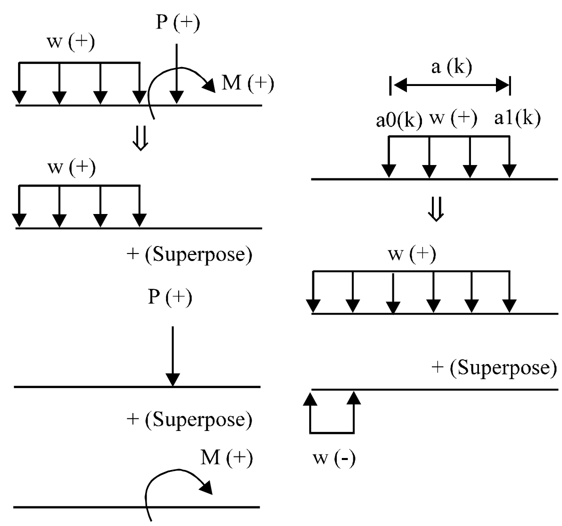


Fig. 1: Superposition method applied to multiple and non-standard forced beams

Using the six fundamental types of forced beams, we can superpose any complex cases of multiple and non-standard forced beams as shown in Fig. 1. These combinations can be finished just by the six fundamental formulas provided in Table 1. For the case of non-standard forced beam in Fig. 1b, we give the input as XXXXXX,XX where, the front number XXXXXX separated from “,” means the final position a1(k) and another number XX means the start position a0(k). In other words, we can regard “,” as “from”. Then we develop the following compiler algorithm to finish the computation of this case:

Table 1: The deflection and slope equations of cantilever and simply supported beams

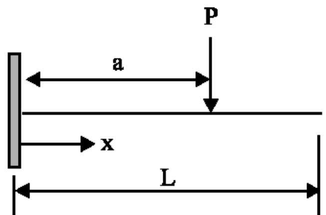
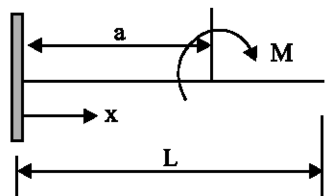
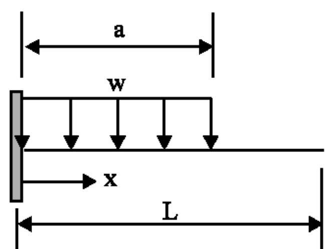
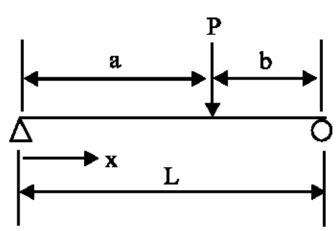
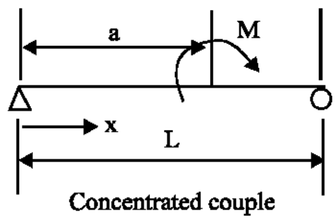
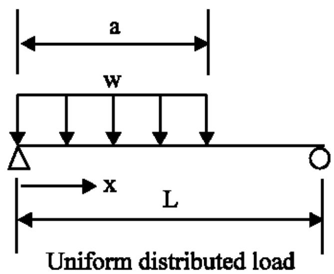
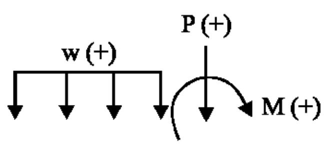
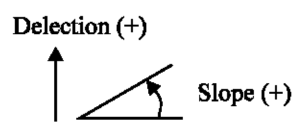
Types and graphs		Equations of deflection and slope
Cantilever beam	 Concentrated load	$0 \le x \le a$: $\text{Slope} = \frac{P}{2EI}(x^2 - 2ax)$ $\text{Deflection} = \frac{P}{6EI}(x^3 - 3ax^2)$ $a \le x \le L$: $\text{Slope} = -\frac{Pa^2}{2EI}$ $\text{Deflection} = \frac{Pa^2}{6EI}(a - 3x)$
Cantilever beam	 Concentrated couple	$0 \le x \le a$: $\text{Slope} = -\frac{Mx}{EI}$ $\text{Deflection} = -\frac{Mx^2}{2EI}$ $a \le x \le L$: $\text{Slope} = -\frac{Ma}{EI}$ $\text{Deflection} = \frac{Ma}{2EI}(a - 2x)$
Cantilever beam	 Uniform distributed load	$0 \le x \le a$: $\text{Slope} = \frac{w}{6EI}(3ax^2 - 3a^2x - x^3)$ $\text{Deflection} = \frac{w}{24EI}(4ax^3 - 6a^2x^2 - x^4)$ $a \le x \le L$: $\text{Slope} = -\frac{wa^3}{6EI}$ $\text{Deflection} = \frac{wa^3}{24EI}(a - 4x)$
Simply supported beam	 Concentrated load	$0 \le x \le a$: $\text{Slope} = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$ $\text{Deflection} = \frac{Pb}{6EIL}(x^3 + b^2x - L^2x)$ $a \le x \le L$: $\text{Slope} = \frac{Pa}{6EIL}[L^2 - a^2 - 3(L - x)^2]$ $\text{Deflection} = \frac{Pa(L - x)}{6EIL}(x^2 + a^2 - 2Lx)$

Table 1: Continue

Types and graphs		Equations of deflection and slope
Simply supported beam		$0 \le x \le a:$ $\text{Slope} = \frac{M}{6EIL}(-3x^2 + 6aL - 3a^2 - 2L^2)$ $\text{Deflection} = \frac{M}{6EIL}(-x^3 + 6aLx - 3a^2x - 2L^2x)$ $a \le x \le L:$ $\text{Slope} = \frac{M}{6EIL}(-3x^2 + 6Lx - 3a^2 - 2L^2)$ $\text{Deflection} = \frac{M}{6EIL}(-x^3 + 3Lx^2 - 3a^2x - 2L^2x + 3La^2)$
Simply supported beam		$0 \le x \le a:$ $\text{Slope} = -\frac{w}{24EIL}[4Lx^3 - 6a(2L-a)x^2 + a^2(2L-a)^2]$ $\text{Deflection} = -\frac{w}{24EIL}[Lx^4 - 2a(2L-a)x^3 + a^2(2L-a)^2x]$ $a \le x \le L:$ $\text{Slope} = -\frac{wa^2}{24EIL}(6x^2 - 12Lx + a^2 + 4L^2)$ $\text{Deflection} = -\frac{wa^2}{24EIL}(L-x)(-2x^2 + 4Lx - a^2)$
Coordinate system		

Step 1: Read the input XXXXXX,XX by a temporary string Sa(k)
Step 2: Search the character "," from Sa(k)
Step 3: If exists "," then decompose Sa(k) into a0(k) and a1(k) two parts
Step 4: If not exists "," then a0(k) = 0 and a1(k) = Sa(k)
Step 5: Firstly set a(k) = a1(k) to calculate the results by the standard formula
Step 6: Next set a(k) = a0(k) and opposes w sign, then calculates the results again
Step 7: Repeat step 5 and 6 until x = L

On the other hand, we consider the functional mapping relationships between the two different popular unit systems, International System (SI) and British Gravitational System (BG), and monitor scale, and conclude the Table 2.

About the arrows for the force or couple, we use coordinate transformation method to draw it. For example, the coordinate assignments of force and couple shown in Fig. 2, we have the coordinate of (P_{1x}, P_{1y}), (P_{2x}, P_{2y}), (V_x, V_y), (M_{1x}, M_{1y}) and (M_{2x}, M_{2y}) in our assignment are, respectively.

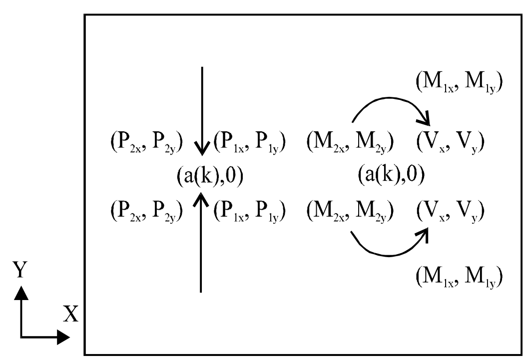


Fig. 2: The coordinate assignments for the force and couple arrows

Table 2: The functional mapping relationships between unit systems and monitor scale

Unit systems		Monitor scale	Mapping equations
w	100 kg m^{-1} ≈ 1 N mm^{-1}	1/20 L	Monitor length = Abs(w)/920 * L *
	30 ton m^{-1} ≈ 300 N mm^{-1}	3/8 L	B2S_ratio + 45/920 * L
	1 lb/in = 0.454 * 9.8/(2.54 * 10) N mm^{-1}		B2S_ratio = 0.454 * 9.8/(2.54 * 10)
P	100 kg ≈ 1e3 N	1/20 L	Monitor length = Abs(P)/920e3 * L *
	30 ton ≈ 300e3 N	3/8 L	B2S_ratio = 45/920 * L
	1 lb = 0.454 * 9.8 N		B2S_ratio = 0.454 * 9.8
M	100 kg-m ≈ 1e6 N mm	1/20 L	Monitor radius = Abs(M)/920e6 * L *
	30 ton-m ≈ 300e6 N mm	3/8 L	B2S_ratio = 0.454 * 9.8 * 2.54 * 10
	1 lb-in = 0.454 * 9.8 * 2.54 * 10 N mm		
	Sign(M) = 1	π/6 ~ 5π/6	Monitor start angle = -pi/2 * Sgn(M) + 2/3 * pi
	Sign(M) = -1	7π/6 ~ 11π/6	Monitor end angle = Monitor start angle + 2/3 * pi

$$\begin{aligned} P_{1x} &= a(k) + l \times \cos(\mathrm{sgn}(P) \times 60^\circ) \\ P_{1y} &= 0 + l \times \sin(\mathrm{sgn}(P) \times 60^\circ) \\ P_{2x} &= a(k) + l \times \cos(\mathrm{sgn}(P) \times 120^\circ) \\ P_{2y} &= 0 + l \times \sin(\mathrm{sgn}(P) \times 120^\circ) \\ V_x &= a(k) + M\ \mathrm{radius} \times \cos(\mathrm{sgn}(M) \times 30^\circ) \\ V_y &= 0 + M\ \mathrm{radius} \times \sin(\mathrm{sgn}(M) \times 30^\circ) \\ M_{1x} &= V_x + l \times \cos(\mathrm{sgn}(M) \times 90^\circ) \\ M_{1y} &= V_y + l \times \sin(\mathrm{sgn}(M) \times 90^\circ) \\ M_{2x} &= V_x + l \times \cos(\mathrm{sgn}(M) \times 150^\circ) \\ M_{2y} &= V_y + l \times \sin(\mathrm{sgn}(M) \times 150^\circ) \end{aligned} \qquad (1)$$

Where, we take $l = \frac{L}{20} \times \frac{2}{\sqrt{3}} \approx \frac{L}{17.32}$. The other parts, such as the consideration of inputs and outputs (computer and debug techniques), graph and gain of deflection and slope (drawing artifices), etc., can see the source code for details. Now, we give two critical examples to illustrate the program.

EXAMPLES

Example 1: Consider the determinate simply supported beam with multiple loads

As shown in Fig. 3, if we take a determinate simply supported beam with its material properties L, E, I and loads w, P, M as

$$\begin{aligned} &L: 10\ \mathrm{m} = 10\mathrm{e}3\ \mathrm{mm} \\ &E: 200\mathrm{e}3\ \mathrm{MPa} \\ &I: 500\mathrm{e}6\ \mathrm{mm}^4 \\ &w_1: 100\ \mathrm{kg/m} \approx 1\ \mathrm{kN/m} = 1\ \mathrm{N/mm,\ from\ 0\ to\ 2e3\ mm} \\ &w_2: 5\ \mathrm{ton/m} \approx 50\ \mathrm{kN/m} = 50\ \mathrm{N/mm,\ from\ 7e3\ to\ 10e3\ mm} \\ &P: 30\ \mathrm{ton} \approx 300\mathrm{e}3\ \mathrm{N,\ at\ 5e3\ mm} \\ &M: 5\ \mathrm{ton\text{-}m} \approx 50\ \mathrm{kN\text{-}m} = 50\mathrm{e}6\ \mathrm{N\text{-}mm,\ at\ 5e3\ mm} \end{aligned} \qquad (2)$$

Substituting the above values into the program, we obtain the results of deflection, slope in each x (we takes dx = 1 cm = 10 mm for S.I. system, dx = 1 in for B.G. system), maximum deflection, maximum slope and its graph (the gain of deflection and slope in graph is 10).

Example 2: Consider the indeterminate cantilever beam with superposition method

Figure 4 is an indeterminate cantilever beam with superposition method. According to Table 1 and Fig. 4,we have

$$\begin{aligned} \delta_{A,w} &= \frac{wL^3}{24EI}(L - 4L) = -\frac{wL^4}{8EI} \\ \delta_{A,P} &= \frac{PL^2}{6EI}(L - 3L) = -\frac{PL^3}{3EI} \\ &\text{Geometric constraint: } \delta_{A,w} + \delta_{A,P} = 0 \Rightarrow P = -\frac{3}{8}wL \end{aligned} \qquad (3)$$

For the following equations, including the material properties L, E, I and load w, we obtain the auxiliary force P = -15000 lb

$$\begin{aligned} &L: 200\ \mathrm{in} \\ &E: 29000\mathrm{e}3\ \mathrm{Psi} \\ &I: 4000\ \mathrm{in}^4 \\ &w: 200\ \mathrm{lb/in,\ from\ 0\ to\ 200\ in} \\ &P: -\frac{3}{8} \times 200 \times 200 = -15000\ \mathrm{lb,\ at\ 200\ in} \end{aligned} \qquad (4)$$

Similarly, substituting the above values into the program, we get the results of deflection, slope in each x, maximum deflection, maximum slope and its graph (the gain of deflection and slope in this graph is 1000) in Fig. 5.

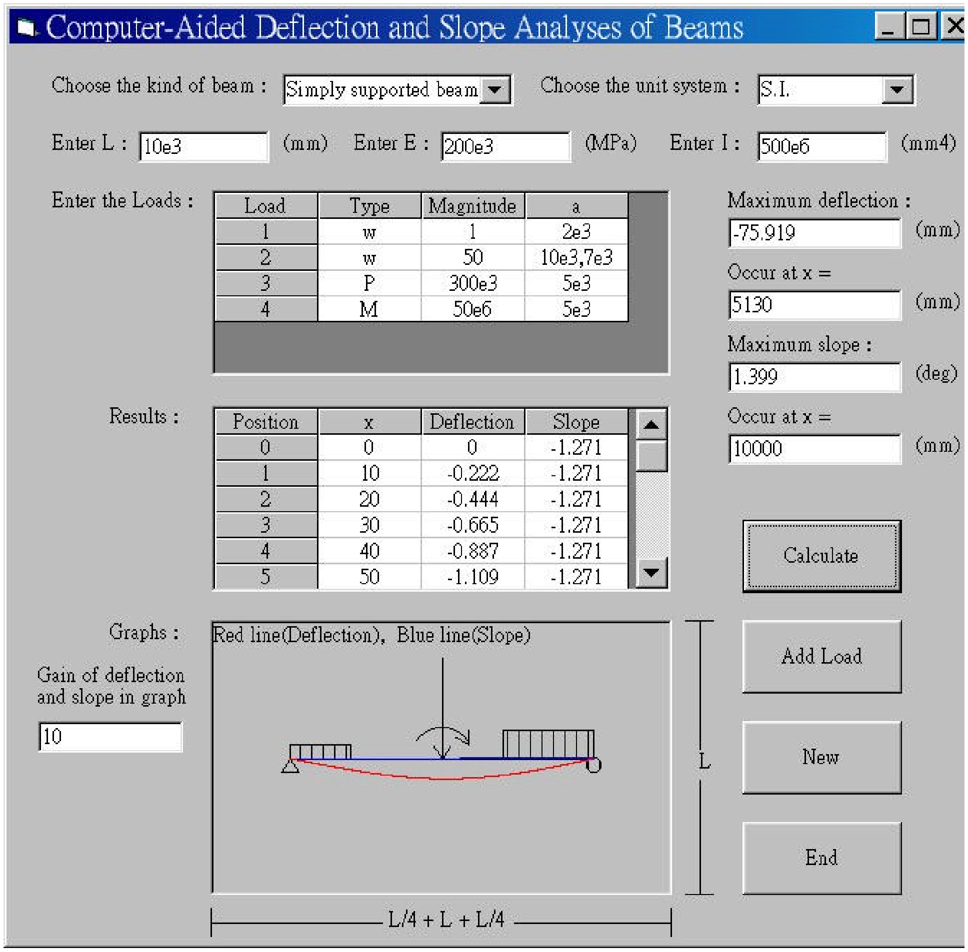


Fig. 3: The computer results of determinate simply supported beam with multiple loads

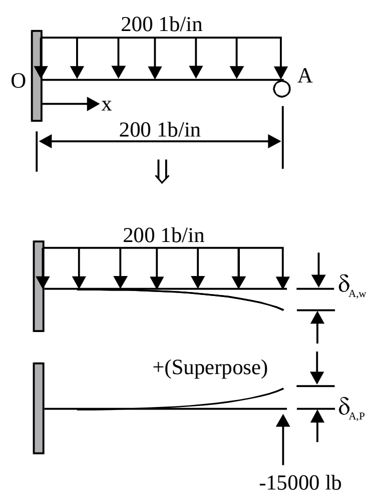


Fig. 4: The indeterminate cantilever beam with superposition method

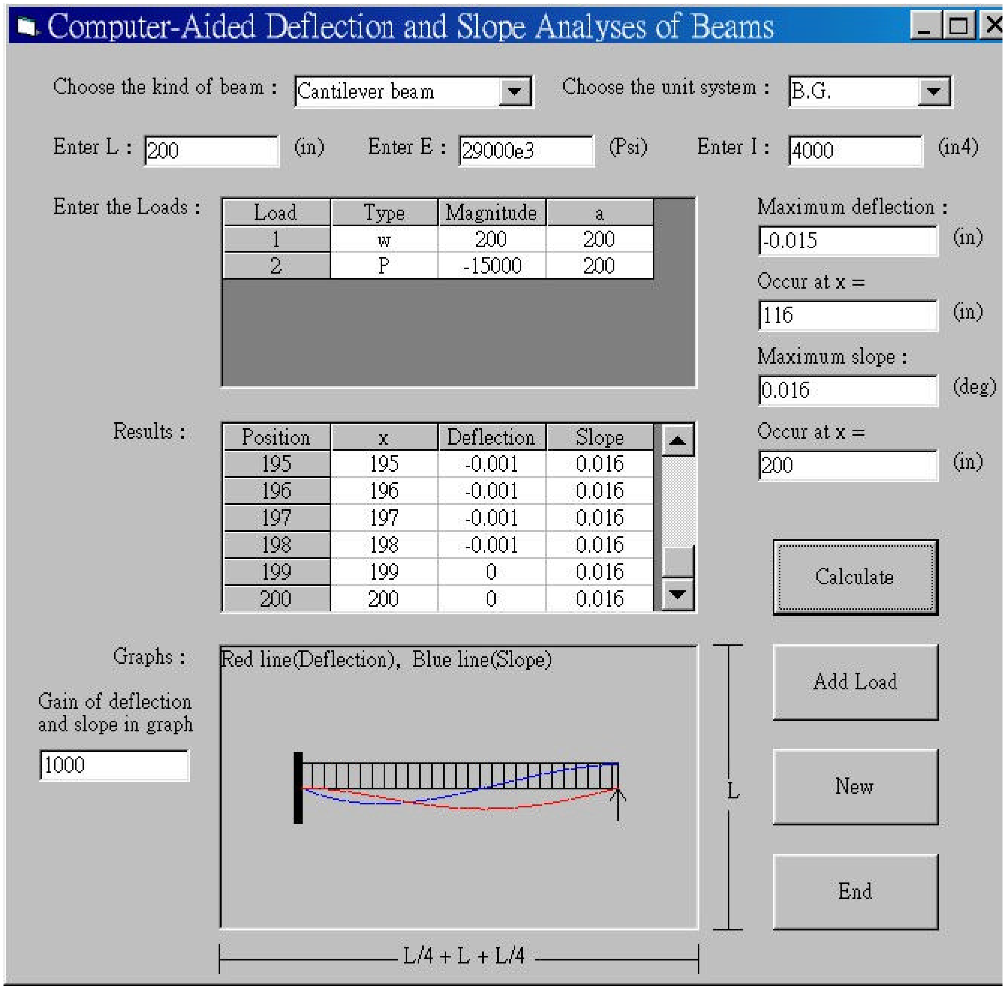


Fig. 5: The computer results of indeterminate cantilever beam with superposition method

CONCLUSIONS

Whenever scientists or engineers develop some scientific or engineering problems, the computation processes are needed. However, the computation processes are usually systematically and routinely in mathematics. So the mistakes are also likely to happen. To avoid it, the repeated checks in our question are very important. It will be a troublesome and time-consuming work for us.

Since the invention of the calculator, scientists or engineers efficiently economize much tasks and unnecessary time-consuming, also avoid some hateful errors. In 20 century, the computer technological development has brought about a revolution in the present-day world. It becomes more convenient, quick and correct in calculating even simulating the scientific or engineering problems, because of some problems can be computerized. Computer program helps us to deal with these problems by batches and automations.

This study applies some mechanical, mathematical and computer methods, techniques and models to develop the design algorithm and a program for the analyses of deflection and slope of beams. From this program, we find the analysis processes are so easy and efficient.

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