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Effect of Skew Angle on Distribution of Bending Moments in Bridge Slabs

Maher Shaker Qaqish
College of Engineering, Al-Balqa Applied University, Salt, Jordan

Abstract: A finite element model was carried out for prestressed precast beams and cast *in situ* slab bridge. This structural model was subjected to 1.8 AASHTO truck loading, 1.8 AASHTO equivalent distributed loading and abnormal loading. The results for transverse and longitudinal moments were compared with the results obtained from AASHTO specifications. This comparison shows that applying AASHTO specification for slab bridge deck is safe and economical.

Key words: Bridge, skew angle, finite element, transverse and longitudinal moments, AASHTO loading

INTRODUCTION

The design of skew bridges has special consideration particularly in bridges up to 20 m span. The skew is defined as the inclination of the abutment to the perpendicular between the free edges.

The effect of skew angle up to 20° can be neglected on the variation of bendings and shears in slabs and beams. Bridges with skew angles up to 20° can be analyzed as straight bridges.

Generally, in skew bridges the behavior of the structure near the bearings particularly at the obtuse corner requires special consideration.

The special characteristics of skew of a slab deck are, torsional moments in the deck slab, concentration of reaction forces and negative moments at obtuse corner, low reactions and a possibility of uplift reaction forces at acute corner, bending moments at the edge of concrete slab bridge adjacent to the abutment and in the direction normal to it, distribution of bending moments in cast *in situ* slabs.

Many methods used in analyzing skew bridges such as grillage and finite element method. Generally, grillage analysis is the most common method used in bridge analysis. In this method the deck is represented by an equivalent grillage of beams. The finer grillage mesh, provide more accurate results. It was found that the results obtained from grillage analysis compared with experiments and more rigorous methods are accurate enough for design purposes. If the load is concentrated on an area which is much smaller than the grillage mesh, the concentration of moments and torque cannot be given by this method and the influence charts described in Puncher[1] can be used. The orientation of the longitudinal members should be always parallel to the free edges while the orientation of transverse members can be either

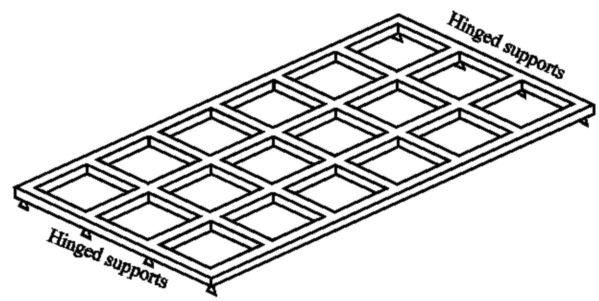

Fig. 1: Grillage mesh showing transverse beams parallel to supports

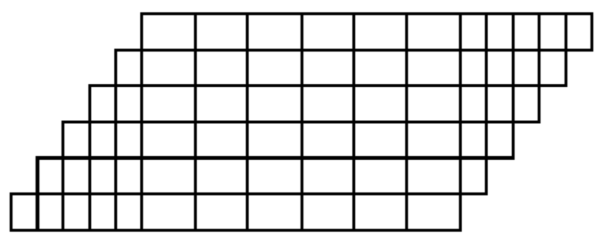

Fig. 2: Grillage mesh showing transverse beams orthogonal to the longitudinal directions

parallel to the supports as shown in Fig. 1 or orthogonal to the longitudinal beams as shown in Fig. 2. According to CCA[2] the orthogonal mesh is cumbersome in input data but the output moments result Mx, My and Mxy can be used directly in the Wood-Armer equations as in Hambly[3] to calculate the steel required in any direction.

Therefore, it is preferable to have orthogonal grillage unless the transverse steel is in the skew direction.

The grillage method involves a plane grillage of discrete interconnected beams.

The grillage analogy has become popular because of the following reasons:

- It can be used in such cases where the bridges exhibits complicating features such as a heavy skew, edge stiffening and deep hunches over supports.
- The representation of a bridge as a grillage is ideally suited to carrying out the necessary calculations associated with analysis and design on a digital computer.
- The grillage representation is conductive to giving the designer an idea about the structure behavior of the bridge and the manner.

The other method used in modeling the bridges is the finite element method. This method needs more time and efforts in modeling than the grillage. The results obtained from the finite element method depends on the mesh size but by using optimization of the mesh the results of this method are considered more accurate than grillage. Figure 3 and 4 shows the finite element mesh for the deck slab and also for three-dimensional model of bridge.

The finite element method is a well-known tool for the solution of complicated structural engineering problems, as it is capable of accommodating many complexities in the solution. In this method, the actual continuum is replaced by an equivalent idealized structure composed of discrete elements, referred to as finite elements, connected together at a number of nodes.

The finite element method was first applied to problems of plane stress, using triangular and rectangular elements. The method has since been extended and we can now use triangular and rectangular elements in plate bending, tetrahedron and hexahedron in three-dimensional stress analysis and curved elements in singly or doubly curved shell problems. Thus the finite element method may be seen to be very general in application and it is sometimes the only valid form of analysis for difficult deck problems.

Tiedman[4] shows the finite element method is a numerical method with powerful technique for solution of complicated structural engineering problems. It most accurately predicted the bridge behavior under the truck axle loading.

The finite element method involves subdividing the actual structure into a suitable number of subregions that are called finite elements. These elements can be in the form of line elements, two dimensional elements and three dimensional elements to represent the structure. The intersection between the elements are called nodal points in one dimensional problems where in two and three dimensional problems are called nodal lines and nodal planes respectively. At the nodes, degrees of freedom (which are usually in the form of the nodal displacements and/or their derivatives, stresses, or combinations of these) are assigned. Models which use displacements are called displacement models and some models use stresses defined at the nodal points as unknown. Models based on stresses are called force or equilibrium models, while those based on combinations of both displacements and stresses are termed mixed models or hybrid models. Displacements are the most commonly used nodal variables, with most general purpose programs limiting their nodal degree of freedom to displacements. A number of displacement functions such as polynomials and trigonometric series can be assumed, especially polynomials because of the ease and simplification they provide in the finite element formulation.

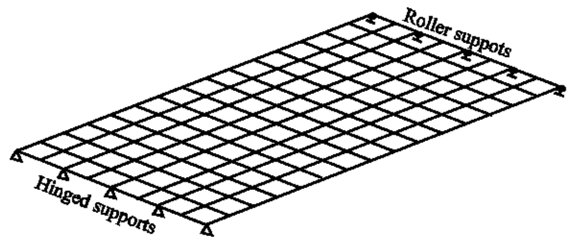

Fig. 3: Finite element model for plate bending of slab deck

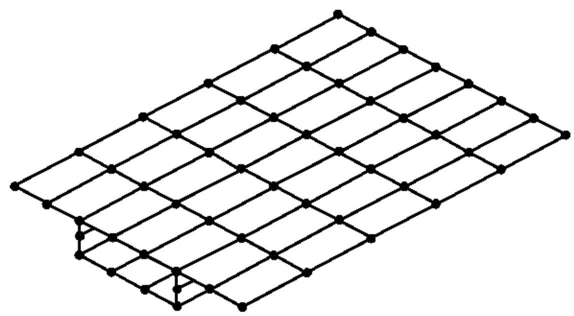
Fig. 4: Three-dimensional structures composed of finite plate elements

To develop the element matrix, it is much easier to apply a work or energy method. The principle of virtual work, the principle of minimum potential energy and castigliano's theorem are methods frequently used for the purpose of derivation of element equations.

The finite element method has a number of advantages, they include the ability to:

- Model irregularly shaped bodies and composed of several different materials.
- Handle general load condition and unlimited numbers and kinds of boundary conditions.
- Include dynamic effects.
- Handle nonlinear behavior existing with large deformation and non-linear materials.

Al Foqaha'a[5] indicates that 50% of the total number of the existing bridges are T-beam bridges. Furthermore, many of these bridges are skew ones due to the alignment of the highway.

Alasa'd[6] sludied the effect of skew on cast *in situ* bridges with spans 12, 14, 16, 18 and 20 and skew angles 10° through 60°, inclusively with an increment of 10°. It was concluded that additional steel should be provided at certain locations on the superstructure especially at the edge of concrete slab adjacent to the abutment to compensate these stresses. Elastomeric Bearing pads at corner of bridges should also have special consideration.

Alasa'd[6] used computer program SAP 90[7] for the analysis of the finite element mesh of the bridge superstructure. For Design purposes winter[8] and Ministry of public works, code of practice for plain and reinforced concrete[9] are used.

Al Mubaydeen[10] find that increasing the skew angle will move the point of max. positive moment near to the middle support of the bridge and also increasing the value of Max. positive moment of the girder for the same span length. In addition to, the shear force near the obtuse corner was decreasing for increasing of the skew angle while near the acute corner it was increasing for all spans. The vertical reaction at the support near the obtuse corner was increasing for increasing the skew angle for all spans with a percent of 8.5%.

Shattarat[11] and Jabr[12] studied the trends in design methods of bridges either simply supported or continuous.

In this study the finite element model was carried out by using STAADPRO 2003[13].

LIVE LOADS ON BRIDGES

The live loads used for design bridges are the live load adopted by AASHTO specifications[14]. The live loads recommended by AASHTO for bridge design are either truck loading or lane loading as shown in Fig. 5 and 6. To encounter the unexpected traffic loadings, these loadings are multiplied by a factor of 1.8.

Standard truck or lane loading are assumed to occupy a loaded width of 10ft (3.048 m). These loads shall be placed in 12ft (3.65 m) wide design traffic lanes spaced across the entire bridge roadway width in number and position required to produce the maximum stress in the member under consideration. The uniform and concentrated load of a lane loading shall be considered to be uniformly distributed over a 10ft (3.048 m) width on a line normal to the centerline of the lane. In computing stresses, each 10ft lane loading or single standard truck shall be considered as a unit that can occupy position

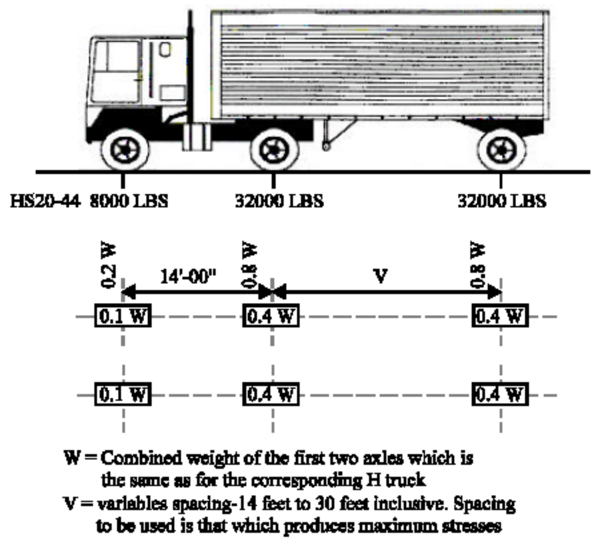

Fig. 5: Truck loading HS 20-44

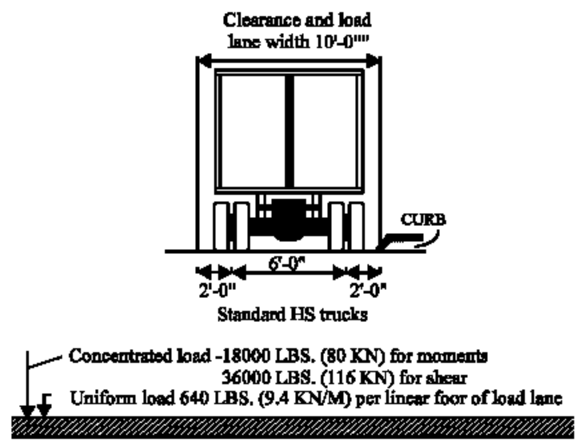

Fig. 6: Lane loading

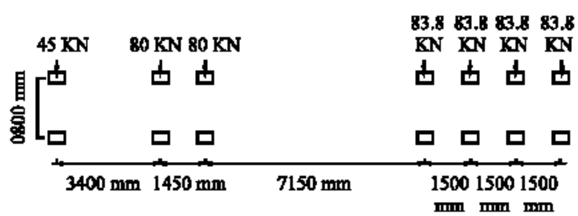

Fig. 7: Abnormal loading

within its individual traffic lane, so as to produce maximum stress. Fractional load lane widths or fractional trucks shall not be used.

For continuous spans, only one standard H or HS truck per lane shall be considered and placed so as to produce maximum positive or negative moments.

The type of loading used, whether lane loading or truck loading and whether the spans are simple or

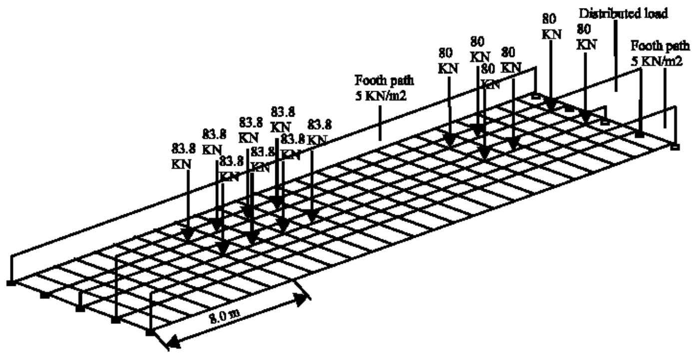

Fig. 8: Abnormal loading in one lane and distributed equivalent AASHTO loading on another lane

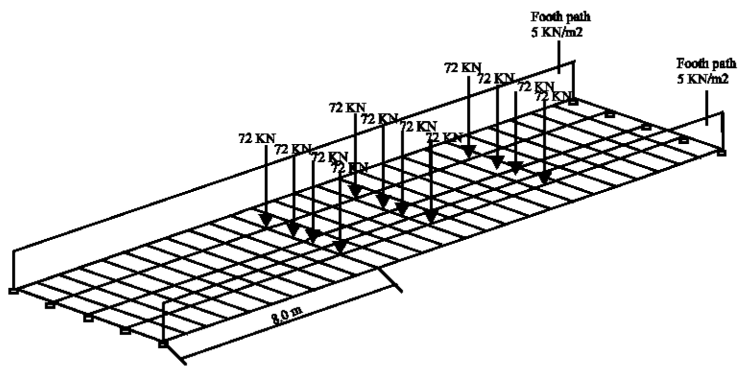

Fig. 9: AASHTO loading on two lanes

continuous, shall be the loading, which produces the maximum tress. Where maximum stresses are produced in any member by loading any member of traffic lanes simultaneously, the following percentages of the resultant live load stresses shall be in view of improbable coincident maximum loading:

One or two lanes	100%
Three lanes	90%
Four lanes or more	75%

For the purpose of this study abnormal loading is considered as shown in Fig. 7 to encounter the unexpected traffic loading. Both abnormal loading and 1.8 AASHTO Loading are considered and the most critical results from both of them are taken for design purposes. The locations of these loadings are considered in Fig. 8 and 9.

Live load stresses due to truck loading and equivalent lane loading are increased to allow for vibration and the sudden application of the load.

The increase is computed by the formula:

$$I = \frac{15.24}{L + 38} \leq 0.3$$

Where, I = the impact fraction of the live load stress
L = the loaded length in meters.

RESEARCH STUDY

This research is concerned of studying the variation of bending moments in the longitudinal and transverse directions in the concrete slab of skew bridges.

The superstructure used is composed of prestressed concrete beams 26 m length spaced at 2.2 m center to center with cast *in situ* concrete slab 20 cm depth supported by 5 cm precast concrete slab to save scafolding as shown in Fig. 11.

The total width of deck slab is 10.5 m with 5 prestressed concrete elements AASHTO Type IV as shown in Fig. 11.

The skew angle used is 35° and the method used in the analysis of deck slab is the finite element method.

STRUCTURAL MODEL

The structural model is composed of finite element model where the deck slab is plate element and the

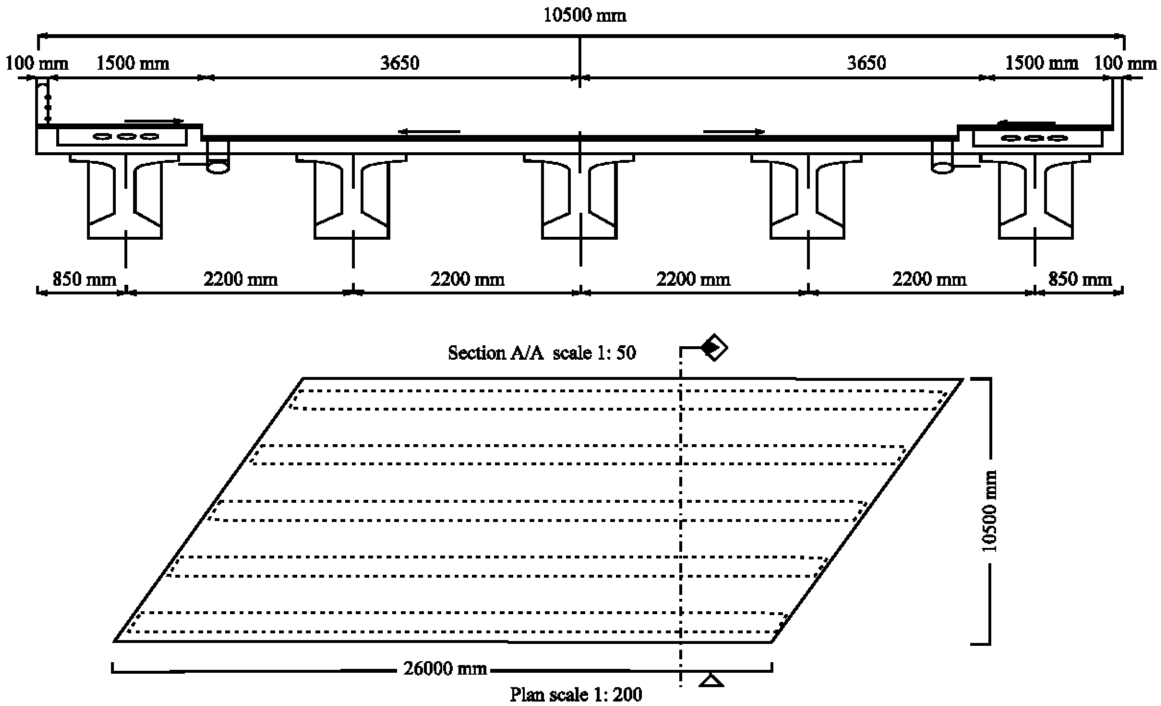

Fig. 10: Plan and section in deck slab

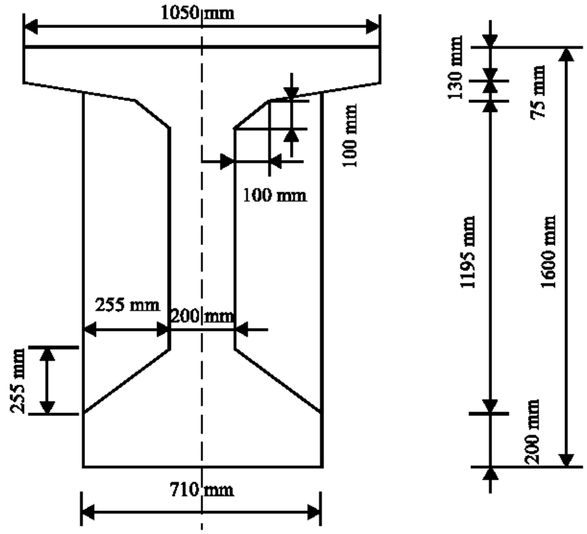

Fig. 11: AASHTO type IV

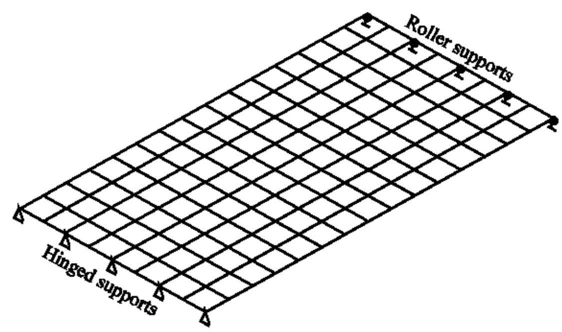

Fig. 12: Finite element model of deck slab showing supports at abutments

prestressed concrete beams are beam elements. The supports are hinged at one end and roller at the other end as shown in Fig. 12 The applied loads are as shown in Fig. 8 and 9.

According to AASHTO[14] the design of bridge slab can be determined according to clause 3.24.3.1 and as follows:

The live load moment for simple spans shall be determined by the following formulas for HS 20-44 laoding.

$$\left(\frac{S+2}{32}\right)P_{20} = \text{Moment in foot-pounds}$$

per foot-width of lane

In slabs continuous over three or more supports, a continuity factor of 0.8 shall be applied to the above formulas for both positive and negative moments.
The impact factor is equal to 24%.
So the AASHTO wheel loading is equal to:

$$P = 7.27 \times 1.8 \times 1.24 = 16.22$$

And S = the distance center to center of supports but should not exceed clear span plus thickness of slab.

The effect of gravity loading will be added to the effect of live load.

The bending moments for the different loadings are as follows:

AASHTO loading: HS20 - 44 × 1.8

$$\text{Moment} = \left(\frac{3.362+2}{32}\right) \times 16.22 \times 1000 \times 2.2$$
$$= 5979.3 \text{ foot.pounds/foot}$$
$$= 2717.86 \text{ kg m m}^{-1}$$

$$\text{Due to continuity } M = 0.8 \times 2717.86$$
$$= 2174.3 \text{ kg m m}^{-1}$$

According to AASHTO[14] the amount of distribution reinforcement shall be certain percentage of the main reinforcement steel required for positive moment.

For main reinforcement perpendicular to traffic

$$\text{Percentage} = \frac{220}{\sqrt{S}} \text{ maximum } 67\%$$

Where: S = the effective span length in feet

So My = 67% Mx

My = 1739.44 kg m m⁻¹

Abnormal loading: The wheel load is:

$$P = 8.38 \times 1.24 = 10.4 \text{ ton}$$

$$\text{Moment} = \left(\frac{3.362+2}{32}\right) \times 10.4 \times 1000 \times 2.2$$
$$= 3833.83 \text{ foot.pounds/foot}$$
$$= 1742.65 \text{ kg m m}^{-1}$$

For Continuity the Moment = 1394.12 kg m m^{-1}

The Moment My = 0.67 ×1394.12 = 1243.91 kg m^{-1}

AASHTO distributed load plus one concentrated load for moment

Bending moment = 1995.875 kg m m^{-1}

Due to continuity M = 0.8×1995.875

= 1596.7 kg m m^{-1}

Bending moment in the other direction (My) = 1069.8 kg m m^{-1}

From the results obtained from the structural model:

- For AASHTO truck loading with 80% increase in this loading:
  Mx = 2107 kg m My = 1434.5 kg m
- For Abnormal loading
  Mx = 1329 kg m My = 977.4 kg m
- For AASHTO lane loading with 80% increase in this loading
  Mx = 1510 kg m My = 891 kg m

So it can be seen for each load case the following results:

AASHTO truck loading: The results obtained for both Mx and My according to AASHTO specification is higher than the results obtained by the structural model for the same moments. This is due to the fact that the AASHTO specification assumes rigid supports while the structural model take the deflection of the beam into consideration, which is equivalent to slab on elastic supports.

Abnormal loading: The results obtained for Mx, My from the structural model is smaller than the stresses calculated according to AASHTO. This is due to the same reason mentioned previously.

For lane loading: The results obtained for Mx and My for lane loading and one concentrated load at the middle of the span according to AASHTO specification is higher than the moments Mx and My obtained from the same loading on the structural model. This is due to the same reason mentioned previously.

This means applying AASHTO Equation under clause 3.24.3.1 for the design of slab bridge is safe and also economical as the differences between the results obtained from the AASHTO equation compared to computer model are close to each other.

CONCLUSIONS

The design of bridge slab according to AASHTO specification is considered safe and economical.

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