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A Deterministic Inventory Model under Quantity-depended Payments Delay Policy Using Algebraic Method

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Abstract: In 1996, Khouja and Mehrez investigated the effect of supplier credit policies depending on the order quantity. The authors assumed that the supplier offers the retailer fully permissible delay in payments if the retailer ordered a sufficient quantity. Otherwise, permissible delay in payments would not be permitted. However, in this article, we want to extend this case by assuming that the supplier would offer the retailer partially permissible delay in payments when the retailer ordered a sufficient quantity. Otherwise, permissible delay in payments would not be permitted. Then, we model the retailer's inventory system and develop three theorems to efficiently determine the optimal lot-sizing decisions for the retailer.

Key words: Deterministic inventory model, quantity-depended, payments delay, optimization, algebraic method

INTRODUCTION

To encourage customers to order a large quantity, the supplier may give the payments delay only for a large order quantity. In other words, the supplier requires immediate payments for a small order quantity. In 1985[1] considering the inventory replenishment problem under permissible delay in payments independent of the order quantity. In 1996[2] investigated the effect of supplier credit policies depending on the order quantity. Then, Chang *et al.*[3] and Chung and Liao[4] established an EOQ model for deteriorating items under supplier credits linked to order quantity. In this regard, Chang[5] extended Chung and Liao[4] by taking into account inf lation and finite time horizon. Recently, Chung *et al.*[6] investigated retailer's lot-sizing policy under permissible delay in payments depending on the ordering quantity. Chang and Dye[7] investigated an inventory model for deteriorating items with time varying demand and deterioration rates when the credit period depends on the retailer's ordering quantity.

However, all above published papers dealing with retailer's lot-sizing policy in the presence of payments delay assumed that the supplier offers the retailer full payments delay when the retailer ordered a sufficient quantity. Otherwise, payments delay would not be permitted. That is, the retailer would obtain 100% payments delay if the retailer ordered a sufficient quantity. We know that this is just an extreme case. In reality, the supplier can relax this extreme case to offer the retailer partial payments delay. That is, the retailer must make a partial payment to the supplier when the order is received. Then, the retailer must pay off the remaining balances at the end of the permissible delay period. In other words, the supplier requires immediate full payments for a small order quantity. That is, payments delay would not be permitted.

Under this condition, we model the retailer's inventory system and develop three theorems for efficiently determination the optimal lot-sizing decisions for the retailer.

Model formulation: The following notation and assumptions are used throughout this article.

Notation

- D = demand rate per year
- W = quantity at which the partially delay payments permitted per order
- A = cost of placing one order
- c = unit purchasing price
- h = unit stock holding cost per year excluding interest charges
- α = the fraction of the total amount owed payable at the time of placing an order, $0<\alpha\leq 1$
- I_e = interest earned per \$ per year
- I_k = interest charges per \$ investment in inventory per year

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M = the length of the payments delay period in years
T = the cycle time in years
TRC(T) = the annual total relevant cost when T>0
T* = the optimal cycle time of TRC(T)
Q* = the optimal order quantity =DT*.

Assumptions

- Demand rate is known and constant.
- Shortages are not allowed.
- Time horizon is infinite.
- Replenishments are instantaneous.
- $I_k \geq I_e$.
- If Q<W, i.e. T<W/D, the payments delay would not be permitted. Otherwise, partially delayed payment is permitted. Hence, if Q≥W, the retailer must make a partial payment, αcDT, to the supplier. Then the retailer must pay off the remaining balances, (1-α)cDT, at the end of the trade credit period. Otherwise, as the order is filled, the retailer must pay the full payments immediately to the supplier.
- During the time period that the account is not settled, generated sales revenue is deposited in an interest-bearing account.

The model: The annual total relevant cost consists of the following elements. There are three situations to occur: (I) M≥W/D, (II) M<W/D≤M/α and (III) M/α<W/D.

Case I: Suppose that M≥W/D

- Annual ordering cost = A/T
- Annual stock holding cost (excluding interest charges) = DTh/2.
- From assumptions (6) and (7), there are four sub-cases in terms of annual opportunity cost of the capital.

Sub-case 1: M/α≤T

Annual opportunity cost of the capital

$$cI_k[\frac{DT^2}{2}-(1-\alpha)DTM]/T-cI_e\left[\frac{(1-\alpha)DM^2}{2}\right]/T$$

Sub-case 2: M≤T<M/α

Annual opportunity cost of the capital

$$=cI_k[\frac{\alpha^2DT^2}{2}+\frac{D(T-M)^2}{2}]/T$$
$$-cI\left\{\frac{(1-\alpha)\alpha^2DT^2}{2}+\frac{[(1-\alpha)\alpha DT+DM](M-\alpha T)}{2}\right\}/T$$
$$=cI_k[\frac{\alpha^2DT^2}{2}+\frac{D(T-M)^2}{2}]/T-cI\left[\frac{DM(M-\alpha^2T)}{2}\right]/T$$

Sub-case 3: W/D≤T<M

Annual opportunity cost of the capital

$$=cI_k(\frac{\alpha^2DT^2}{2})/T$$
$$-cI_e\left\{\begin{array}{c}\frac{(1-\alpha)\alpha^2DT^2}{2}+\frac{[(1-\alpha)\alpha DT+DT](T-\alpha T)}{2}\\+DT(M-T)\end{array}\right\}/T$$
$$=cI_k(\frac{\alpha^2DT^2}{2})/T-cI_eDT\left[M-\frac{(1+\alpha^2)T}{2}\right]/T$$

Sub-case 4: 0 <T<W/D

Annual opportunity cost of the capital = $cI_k(\frac{DT^2}{2})/T$

From the above arguments, the annual total relevant cost for the retailer can be expressed as:
Annual total relevant cost = ordering cost + stock-holding cost + opportunity cost of the capital.

We show that the annual total relevant cost is given by:

$$TRC(T)=\begin{cases}TRC_1(T) & \text{if} \quad T\geq\frac{M}{\alpha} & (1a)\\ TRC_2(T) & \text{if} \quad M\leq T<\frac{M}{\alpha} & (1b)\\ TRC_3(T) & \text{if} \quad W/D\leq T<M & (1c)\\ RC_4(T) & \text{if} \quad 0<T<W/D & (1d)\end{cases}$$

Where:

$$TRC_1(T)=\frac{A}{T}+\frac{DTh}{2}+cI_k[\frac{DT^2}{2}-(1-\alpha)DTM]/T-cI_e\left[\frac{(1-\alpha)DM^2}{2}\right]/T \quad (2)$$

$$TRC_2(T)=\frac{A}{T}+\frac{DTh}{2}+cI_k[\frac{\alpha^2DT^2}{2}+\frac{D(T-M)^2}{2}]/T-cI_e\left[\frac{DM(M-\alpha^2T)}{2}\right]/T \quad (3)$$

$$TRC_3(T)=\frac{A}{T}+\frac{DTh}{2}+cI_k(\frac{\alpha^2DT^2}{2})/T-cI_eDT\left[M-\frac{(1+\alpha^2)T}{2}\right]/T \quad (4)$$

and

$$TRC_4(T)=\frac{A}{T}+\frac{DTh}{2}+cI_k(\frac{DT^2}{2})/T \quad (5)$$

Since, $TRC_1(\frac{M}{\alpha}) = TRC_2(\frac{M}{\alpha})$, $RC_2(M) = TRC_3(M)$, $TRC_3(W/D) < TRC_4(W/D)$, TRC (T) is continuous except at T = W/D. All TRC_1 (T), TRC_2 (T), TRC_3 (T), TRC_4 (T) and TRC (T) are defined on T>0. From Eq. 2-5, we can obtain TRC_4 (T) >TRC_1 (T), TRC_4 (T) > TRC_3 (T) for all T > 0 and TRC_4 (T) > TRC_2 (T) for M < T < M/α. That is, TRC_4 (T) will be higher than all TRC_1 (T), TRC_2 (T) and TRC_3 (T) on suitable domain. From now on, we can neglect TRC_4 (T) when we want to develop the efficient procedure to determine the optimal lot-sizing decisions for the retailer.

Then, we can rewrite

$$TRC_1(T) = \frac{D(h+cI_k)}{2T}\left[T - \sqrt{\frac{2A - c(1-\alpha)DM^2I_e}{D(h+cI_k)}}\right]^2 + \left\{\begin{matrix}\sqrt{D(h+cI_k)[2A - c(1-\alpha)DM^2I_e]} \\ -c(1-\alpha)I_kDM\end{matrix}\right\} \quad (6)$$

From Eq. 6 the minimum of TRC_1 (T) is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_1* is

$$T_1^* = \sqrt{\frac{2A - c(1-\alpha)DM^2I_e}{D(h+cI_k)}} \quad \text{if } 2A - c(1-\alpha)DM^2I_e > 0 \quad (7)$$

Therefore,

$$TRC_1(T_1^*)\left\{\begin{matrix}\sqrt{D(h+cI_k)[2A - c(1-\alpha)DM^2I_e]} \\ -c(1-\alpha)I_kDM\end{matrix}\right\} \quad (8)$$

Similarly, we can derive TRC_2 (T) without derivatives as follows:

$$TRC_2(T) = \frac{D\,[h + cI_k(1+\alpha^2)]}{2T}\left[T - \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D[h + cI_k(1+\alpha^2)]}}\right]^2 + \left\{\begin{matrix}\sqrt{D\,[h + cI_k(1+\alpha^2)][2A + cDM^2(I_k - I_e)]} \\ -cDM(2I_k + \frac{\alpha^2}{2}I_e)\end{matrix}\right\} \quad (9)$$

From Eq. 9 the minimum of TRC_2 (T) is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_2* is

$$T_2^* = \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D\,[h + cI_k(1+\alpha^2)]}} \quad (10)$$

Therefore,

$$TRC_2(T_2^*)\left\{\begin{matrix}\sqrt{D[h + cI_k(1+\alpha^2)][2A + cDM^2(I_k - I_e)]} \\ -cDM(2I_k + \frac{\alpha^2}{2}I_e)\end{matrix}\right\} \quad (11)$$

Likewise, we can derive TRC_3 (T) algebraically as follows:

$$TRC_3(T) = \frac{D[h + \alpha^2cI_k + (1+\alpha^2)cI_e]}{2T}\left[T - \sqrt{\frac{2A}{D[h + \alpha^2cI_k + (1+\alpha^2)cI_e]}}\right]^2 + \left\{\sqrt{2AD[h + \alpha^2cI_k + (1+\alpha^2)cI_e]} - DMcI_e\right\} \quad (12)$$

From Eq. 12 the minimum of TRC_3 (T) is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_3* is

$$T_3^* = \sqrt{\frac{2A}{D[h + c\alpha^2I_k + c(1+\alpha^2)I_e)]}} \quad (13)$$

Therefore,

$$TRC_3(T_3^*) = \left\{\sqrt{2AD[h + \alpha^2cI_k + (1+\alpha^2)cI_e]} - DMcI_e\right\} \quad (14)$$

Case II: Suppose that M<W/D≤M/α: If M<W/D≤M/α, Eq. 1(a, b, c, d) will be modified as

$$TRC(T) = \begin{cases} TRC_1(T) \text{ if } T \geq \frac{M}{\alpha} & (15a) \\ TRC_2(T) \text{ if } W/D \leq T < \frac{M}{\alpha} & (15b) \\ TRC_4(T) \text{ if } 0 < T < W/D & (15c) \end{cases}$$

Since TRC_1 (M/α) = TRC_2 (M/α), TRC_2 (W/D)<TRC_4 (W/D), TRC (T) is continuous except at T = W/D. All TRC_1 (T), TRC_2 (T), TRC_4 (T) and TRC (T) are defined on T>0. Similar to Case I discussion, TRC_4 (T) will be higher than both TRC_1 (T) and TRC_2 (T) on suitable domain. Hence, we can neglect TRC_4 (T) when we want to develop the efficient procedure to determine the optimal lot-sizing decisions for the retailer.

Case III: Suppose that M/α<W/D: If M/α<W/D, Eq. 1 (a, b, c, d) will be modified as

$$TRC(T) = \begin{cases} TRC_1(T) \text{ if } T \geq W/D & (16a) \\ TRC_4(T) \text{ if } 0 < T < W/D & (16c) \end{cases}$$

Since TRC_1 (W/D) < TRC_4 (W/D), TRC (T) is continuous except at T = W/D. All TRC_1 (T), TRC_4 (T) and TRC (T) are defined on T>0. We can easily obtain that TRC_4 (T)>TRC_1(T) for all T>0. Hence, we can neglect TRC_4 (T) when we want to develop the efficient procedure to determine the optimal lot-sizing decisions for the retailer.

Determination of the optimal cycle time T*

Case I: Suppose that M≥W/D: Equation 7 gives that the optimal value of T* for the case when T≥M/α so that $T_1^* \geq M/\alpha$. We substitute Eq. 7 into $T_1^* \geq M/\alpha$, then we can obtain that:

$M/\alpha \leq T_1^*$ if and only if

$$-2A + DM^2[c(1-\alpha)I_e + \frac{h+cI_k}{\alpha^2}] \leq 0$$

Similar disscussion, we can obain following results:

$M \leq T_2^* < M/\alpha$ if and only if

$$-2A + DM^2(cI_e + \frac{h+cI_k}{\alpha^2}) > 0 \quad \text{and}$$

if and only if,

$$-2A + DM^2[h + c(\alpha^2 I_k + I_e)] \leq 0$$

$W/D \leq T_3^* < M$ if and only if

$$-2A + DM^2[h + c\alpha^2 I_k + c(1+\alpha^2)I_e)] > 0 \quad \text{and}$$

if and only if

$$-2A + D\left(\frac{W}{D}\right)^2[h + c\alpha^2 I_k + c(1+\alpha^2)I_e] \leq 0$$

Furthermore, we let

$$\Delta_1 = -2A + DM^2[c(1-\alpha)I_e + \frac{h+cI_k}{\alpha^2}] \quad (17)$$

$$\Delta_2 = -2A + DM^2(cI_e + \frac{h+cI_k}{\alpha^2}) \quad (18)$$

$$\Delta_3 = -2A + DM^2[h + c(\alpha^2 I_k + I_e)] \quad (19)$$

$$\Delta_4 = -2A + DM^2[h + c\alpha^2 I_k + c(1+\alpha^2)I_e)] \quad (20)$$

and

$$\Delta_5 = -2A + D\left(\frac{W}{D}\right)^2[h + c\alpha^2 I_k + c(1+\alpha^2)I_e] \quad (21)$$

From Eq. 17-21, we can easily obtain $\Delta_2>\Delta_1$, $\Delta_2 \geq \Delta_3$, $\Delta_4>\Delta_3$ and $\Delta_4 \geq \Delta_5$. Summarized above arguments, the optimal cycle time T* can be obtained as follows.

Theorem 1

(A) If $\Delta_1>0$, $\Delta_3>0$ and $\Delta_5>0$, then TRC (T*) = TRC_3 (W/D). Hence T* is W/D.

(B) If $\Delta_1>0$ $\Delta_3>0$ and $\Delta_5 \leq 0$, then TRC (T*) = TRC_3 (T_3^*). Hence T* is T_3^*.

(C) If $\Delta_1>0$, $\Delta_3 \leq 0$ and $\Delta_5>0$, then TRC (T*) = TRC_2 (T_2^*). Hence T* is T_2^*.

(D) If $\Delta_1>0$, $\Delta_3 \leq 0$, $\Delta_4>0$ and $\Delta_5 \leq 0$, then TRC (T*) = min{TRC_2 (T_2^*), TRC_3 (T_3^*)}. Hence T* is T_2^* or T_3^* associated with the least cost.

(E) If $\Delta_1>0$ and $\Delta_4 \leq 0$, then TRC (T*) = TRC_2 (T_2^*). Hence T* is T_2^*.

(F) If $\Delta_1 \leq 0$, $\Delta_3>0$ and $\Delta_5>0$, then TRC (T*) = TRC_1 (T_1^*). Hence T* is T_1^*.

(G) If $\Delta_1 \leq 0$, $\Delta_3>0$ and $\Delta_5 \leq 0$, then TRC (T*) = min{TRC_1 (T_1^*), TRC_3 (T_3^*)}. Hence T* is T_1^* or T_3^* associated with the least cost.

(H) If $\Delta_1 \leq 0$, $\Delta_2>0$, $\Delta_3 \leq 0$ and $\Delta_5>0$, then TRC(T*) = min{TRC_1 (T_1^*), TRC_2 (T_2^*)}. Hence T* is T_1^* or T_2^* associated with the least cost.

(I) If $\Delta_1 \leq 0$, $\Delta_2>0$, $\Delta_3 \leq 0$, $\Delta_4>0$ and $\Delta_5 \leq 0$, then TRC (T*) = min{TRC_1 (T_1^*), TRC_2 (T_2^*), TRC_3 (T_3^*)}. Hence T* is T_1^*, T_2^* or T_3^* associated with the least cost.

(J) If $\Delta_1 \leq 0$, $\Delta_2>0$ and $\Delta_4 \leq 0$, then TRC (T*) = min{TRC_1 (T_1^*), TRC_2 (T_2^*)}. Hence T* is T_1^* or T_2^* associated with the least cost.

(K) If $\Delta_2 \leq 0$ and $\Delta_5>0$, then TRC (T*) = TRC_1 (T_1^*). Hence T* is T_1^*.

(L) If $\Delta_2 \leq 0$, $\Delta_4>0$ and $\Delta_5 \leq 0$, then TRC (T*) = min{TRC_1(T_1^*), TRC_3 (T_3^*)}. Hence T* is T_1^* or T_3^* associated with the least cost.

(M) If $\Delta_2 \leq 0$ and $\Delta_4 \leq 0$, then TRC (T*) = TRC_1(T_1^*). Hence T* is T_1^*.

Case II: Suppose that M<W/D≤M/α: If M<W/D≤M/α, Eq. 1 (a, b, c and d) will be modified as Eq. 15 (a, b and c). Similar to above Case I discussion, we can obtain following results:

$M/\alpha \leq T_1^*$ if and only if

$$-2A + DM^2[c(1-\alpha)I_e + \frac{h+cI_k}{\alpha^2}] \leq 0$$

$W/D \leq T_2^* < M/\alpha$ if and only if

$$-2A + DM^2(cI_e + \frac{h+cI_k}{\alpha^2}) > 0 \quad \text{and}$$

if and only if

$$-2A - cDM^2(I_k - I_e) + D\left(\frac{W}{D}\right)^2[h + cI_k(1+\alpha^2)] \leq 0$$

Furthermore, we let

$$\Delta_6 = -2A - cDM^2(I_k - I_e) + D\left(\frac{W}{D}\right)^2[h + cI_k(1+\alpha^2)] \quad (22)$$

From Eq. 17, 18 and 22, we can easily obtain $\Delta_2>\Delta_1$ and $\Delta_2>\Delta_6$. Summarized above arguments, the optimal cycle time T* can be obtained as follows:

Theorem 2

(A) If $\Delta_1>0$ and $\Delta_6>0$, then TRC (T*) = TRC_2 (W/D). Hence T* is W/D.
(B) If $\Delta_1>0$ and $\Delta_6\leq 0$, then TRC (T*) = TRC_2 (T_2*). Hence T* is T_2*.
(C) If $\Delta_1\leq 0$ and $\Delta_6>0$, then TRC (T*) = TRC_1 (T_1*). Hence T* is T_1*.
(D) If $\Delta_1\leq 0$, $\Delta_2>0$ and $\Delta_6\leq 0$, then TRC (T*) = min{TRC_1 (T_1*), TRC_2 (T_2*)}. Hence T* is T_1* or T_2* associated with the least cost.
(E) If $\Delta_2\leq 0$, then TRC (T*) = TRC_1(T_1*). Hence T* is T_1*.

Case III: Suppose that M/α<W/D: If M/α<W/D, Eq. 1 (a, b, c and d) will be modified as Eq. 16 (a and b). Similar to above Case I and Case II discussions, we can obtain following results:

W/D≤T_1* if and only if

$$-2A + DM^2c(1-\alpha)I_e + D\left(\frac{W}{D}\right)^2(h + cI_k) \leq 0$$

Furthermore, we let

$$\Delta_7 = -2A + DM^2c(1-\alpha)I_e + D\left(\frac{W}{D}\right)^2(h + cI_k) \quad (23)$$

Summarized above arguments, the optimal cycle time T* can be obtained as follows:

Theorem 3

(A) If $\Delta_7>0$, then TRC (T*) = TRC_1(W/D). Hence T* is W/D.
(B) If $\Delta_7\leq 0$, then TRC (T*) = TRC_1 (T_1*). Hence T* is T_1*.

CONCLUSIONS

The assumption in previously published results that the full payments delay is permitted if the retailer ordered a sufficient quantity. We know 100% payments delay is just an extreme case. This article amends the assumption of the full payments delay to partial payments delay when the retailer ordered a sufficient quantity. We adopt the assumption to model the retailer's inventory problem. In addition, we establish three easy-to-use theorems to help the retailer to find the optimal lot-sizing policy.

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