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A Mode-I Crack Problem for an Infinite Space in Thermoelasticity

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Abstract: A two-dimensional problem for an infinite space weakened by a finite linear opening Mode-I crack is solved. The solid material is assumed to be homogeneous and isotropic. The crack is subjected to prescribed temperature and stress distributions. A rectangular system of Cartesian coordinates is used. The Fourier transform technique is applied to solve the problem. The boundary conditions of the problem are then reduced to a system of two dual integral equations, which are solved analytically. Numerical values for the temperature, stress and displacements are obtained and represented graphically then discussed. All the definite integrals involved were calculated using Romberg technique of numerical integration with the aid of a Fortran program compiled with Visual Fortran v.6.1 on a Pentium-IV pc with processor speed 2.0 GHz.

Key words: Mode-I crack, Cartesian coordinates, Romberg technique

INTRODUCTION

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their many applications in widely diverse fields. First, the high velocities of modern aircrafts give rise to an aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. Secondly, in the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations[1].

In 1950, Danilovskaya was the first to solve an actual problem in the theory of elasticity with non-uniform heat[2] that became known as the theory of uncoupled thermoelasticity, in which the temperature is governed by a parabolic partial differential equation and does not contain any elastic term. Later, the theory of coupled thermoelasticity was then formulated to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature[3]. The heat equations for both theories, however, are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. In 1967, the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body has been introduced by Lord and Shulman[4]. It was then extended to include the anisotropic case[5] where a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law of head conduction. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. Uniqueness of solution for this theory was proved under different conditions by many researcheres[5-8]. The state space approach to this theory was developed for one-dimensional problems[9,10] and for two-dimensional problems[11]. The fundamental solution for this theory was then obtained[12]. Consequently, many problems had been solved concerning magneto-thermoelasticity with thermal relaxation[13], a dynamical problem for an internal penny-shaped crack in an infinite thermoelastic solid[14] and a problem for penny-shaped crack in piezoelectric materials[15].

In recent years, considerable efforts have been devoted to the study of failure and cracks in solids. This is due to the applications of the latter generally in industry and particularly in the fabrication of electronic components. Most of the studies of dynamical crack problems are done using the equations of the coupled or even uncoupled theories of thermoelasticity[16-18]. This is suitable for most situations where long time effects are sought. However, when short time effects are important, as in many practical situations, the full system of generalized thermoelastic equations must be used[4]:

FORMULATION OF THE PROBLEM

In this study, a problem for an infinite homogeneous and isotropic space $-\infty < x < \infty$, $-\infty < y < \infty$, with a crack on the x-axis, $|x| \leq a$, $y = 0$ is considered. The crack surface is subjected to a known temperature and normal stress distributions. There are many types (modes) of crack and

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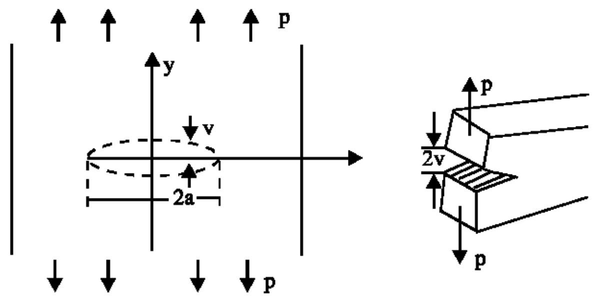

Fig. 1: Displacement of an external Mode I crack

this study will be devoted to Mode-I shown in Fig. 1.
The governing equations have the following form[4]:

$$(\lambda+\mu)\frac{\partial e}{\partial x}+\mu\nabla^2 u-\gamma\frac{\partial T}{\partial x}=0, \tag{1}$$

$$(\lambda+\mu)\frac{\partial e}{\partial y}+\mu\nabla^2 v-\gamma\frac{\partial T}{\partial y}=0, \tag{2}$$

$$\nabla^2 T = 0. \tag{3}$$

In the above equations u, v are the displacement components in the x, y directions, respectively, T is the absolute temperature, λ, μ are Lamé's elastic constants, γ is a material constant. In terms of the elastic constants and the coefficient of linear thermal expansion α, γ is written as $\gamma = (3\lambda + 2\mu)$.
The dilatation e is given by:

$$e = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}. \tag{4}$$

The above equations are supplemented by the constitutive relations giving the components of the stress tensor σ_{ij}, namely[4]:

$$\sigma_{xx} = 2\mu\frac{\partial u}{\partial x} + \lambda e - \gamma(T - T_0), \tag{5}$$

$$\sigma_{yy} = 2\mu\frac{\partial v}{\partial y} + \lambda e - \gamma(T - T_0), \tag{6}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{7}$$

Hereafter, for simplicity, the following non-dimensional variables will be considered throughout this study.

$$x' = c_1\eta x,\quad y' = c_1\eta y,\quad u' = c_1\eta u,\quad v' = c_1\eta v,$$

$$\tau_0' = c_1^2\eta\tau_0,\quad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu},\quad \theta = \frac{T-T_0}{T_0},$$

where, $\eta = \frac{\rho c_E}{k}$, c_1 is the speed of propagation of isothermal elastic waves given by: $c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$, in which ρ is the density, k is the material's thermal conductivity and c_E is the specific heat capacity.

Using the above non-dimensional variables, and by dropping the primes for convenience, the governing equations and expressions for stress components take the following form:

$$\left(\beta^2-1\right)\frac{\partial e}{\partial x}+\nabla^2 u-b\frac{\partial \theta}{\partial x}=0, \tag{8}$$

$$\left(\beta^2-1\right)\frac{\partial e}{\partial y}+\nabla^2 v-b\frac{\partial \theta}{\partial y}=0, \tag{9}$$

$$\nabla^2\theta = 0, \tag{10}$$

$$\sigma_{xx} = 2\frac{\partial u}{\partial x}+\left(\beta^2-2\right)e-b\theta, \tag{11}$$

$$\sigma_{yy} = 2\frac{\partial v}{\partial y}+\left(\beta^2-2\right)e-b\theta, \tag{12}$$

$$\sigma_{xy} = \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}, \tag{13}$$

where, in terms of Poisson's ratio ν, $\beta^2 = \frac{\lambda+2\mu}{\mu} = \frac{2(1-\nu)}{1-2\nu}$,

and $b = \frac{\gamma T_0}{\mu} = \left(3\beta^2-4\right)\alpha_t$.

It is to be noted that Eq. 4 retains its form.

Eliminating u and v between Eq. 8 and 9 and using Eq. 4, gives

$$\nabla^2 e = 0. \tag{14}$$

The Fourier cosine and sine transforms are defined respectively by the following relations[20]:

$$F_c\left[f(x,y)\right] = \overline{f_c}(q) = \int_0^\infty f(x,y)\cos qx\,dx,$$
$$F_s\left[f(x,y)\right] = \overline{f_s}(q) = \int_0^\infty f(x,y)\sin qx\,dx, \tag{15}$$

with the following relations:

$$F_c\left\{\frac{d}{dx}f(x,y)\right\}=qF_s\{f(x,y)\}-f(0,y),$$
$$F_c\left\{\frac{d^2}{dx^2}f(x,y)\right\}=-q^2F_c\{f(x,y)\}-f'(0,y),$$
$$F_s\left\{\frac{d}{dx}f(x,y)\right\}=-qF_c\{f(x,y)\},$$
$$F_s\left\{\frac{d^2}{dx^2}f(x,y)\right\}=-q^2F_s\{f(x,y)\}+qf(0,y).$$

In addition, their inverses are given by the relation[19,20]:

$$f(x,y)=F_c^{-1}\left[f^*(q,y)\right]=\frac{2}{\pi}\int_0^\infty f^*(q,y)\cos(qx)dq$$
$$f(x,y)=Fs^{-1}\left[f^*(q,y)\right]=\frac{2}{\pi}\int_0^\infty f^*(q,y)\sin(qx)dq$$

Applying the Fourier cosine transform to both sides of Eq. 10, then:

$$\frac{\partial^2\bar{\theta}_c}{\partial y^2}-q^2\bar{\theta}_c-\frac{\partial\theta(0,y)}{\partial x}=0. \tag{16}$$

Since 0 is even in x, it follows that:

$$\frac{\partial\theta}{\partial x}(0,y)=0,$$

Consequently, Eq. 16 takes the form:

$$\frac{\partial^2\bar{\theta}_c}{\partial y^2}-q^2\bar{\theta}_c=0, \tag{17}$$

The solution of the above second order differential equation is bounded as $y\to\pm\infty$ and can be written in the form:

$$\bar{\theta}_c(q,y)=A(q)e^{-q|y|},$$

where, A(q) is a parameter depending on q.

Due to symmetry, only the case $y > 0$ is to be considered. Accordingly:

$$\bar{\theta}_c(q,y)=A(q)e^{-qy},\qquad y>0, \tag{18}$$

Similarly:

$$\bar{e}=B'(q)e^{-qy},\qquad y>0, \tag{19}$$

where, $B'(q)$ is a parameter depending on q.
Taking the Fourier sine transform for Eq. 8,

$$\left(\beta^2-1\right)\left(-q\bar{e}_c\right)+\frac{\partial^2\bar{u}_s}{\partial y^2}-q^2u_s+qu(0,y)+bq\bar{\theta}_c=0, \tag{20}$$

Since u is odd in y, it follows that u(0, y) = 0, then Eq. 20 takes the form:

$$\frac{\partial^2\bar{u}_s}{\partial y^2}-q^2\bar{u}_s=q\left[\left(\beta^2-1\right)\bar{e}_c-b\bar{\theta}_c\right], \tag{21}$$

Substituting from Eq. 18 and 19 into the right hand side of Eq. 21 we obtain:

$$\frac{\partial^2\bar{u}_s}{\partial y^2}-q^2\bar{u}_s=q\left[\left(\beta^2-1\right)B'-bA\right]e^{-qy}, \tag{22}$$

The solution $\bar{u}_s$ of Eq. 22 has the form:

$$\bar{u}_s=Ce^{-qy}+\frac{\left[bA-\left(\beta^2-1\right)B'\right]}{2}ye^{-qy}, \tag{23}$$

Applying the Fourier cosine transform with respect to the variable x on both sides of Eq. 4 to obtain

$$\bar{e}_c=q\bar{u}_s+\frac{\partial\bar{v}_c}{\partial y}. \tag{24}$$

Substituting from Eq. 19 and 23 into Eq. 24:

$$B\bar{e}^{qy}=q\left[Ce^{-qy}+\frac{bA-\left(\beta^2-1\right)B'}{2}ye^{-qy}\right]+\frac{\partial\bar{v}_c}{\partial y}. \tag{25}$$

Integrating both sides Eq. 25 with respect to y, gives:

$$\bar{v}_c=\left[\frac{bA-\left(\beta^2-1\right)B'}{2q}+C\right]e^{-qy}+\left[\frac{bA-\left(\beta^2-1\right)B'}{2}\right]ye^{-qy}. \tag{26}$$

In order to simplify Eq. 26, the following substitution is to be used:

$$B=\frac{bA-(\beta^2-1)B'}{2},\ \text{or}\ B'=\frac{bA-2B}{\beta^2-1}.$$

Thus
$$\bar{\theta}_c=Ae^{-qy},\quad y>0, \tag{27}$$

$$\bar{e}_c=\frac{bA-2B}{\beta^2-1}e^{-qy},\ y>0, \tag{28}$$

$$\bar{u}_s = Ce^{-qy} + Bye^{-qy}, \qquad y > 0, \tag{29}$$

$$\bar{v}_c = \left[\frac{\left(\beta^2+1\right)B - bA}{q\left(\beta^2-1\right)} + C\right]e^{-qy} + Bye^{-qy}, \qquad y > 0. \tag{30}$$

Appling Fourier cosine transforms to both sides of Eq. 11, 12 and the Fourier sine transform into both sides of Eq. 13:

$$\left(\bar{\sigma}_{xx}\right)_c = 2q\bar{u}_s + \left(\beta^2-2\right)\bar{e}_s - b\bar{\theta}_s, \tag{31}$$

$$\left(\bar{\sigma}_{yy}\right)_c = 2\frac{\partial \bar{v}_c}{\partial y} + \left(\beta^2-2\right)\bar{e}_s - b\bar{\theta}_s, \tag{32}$$

$$\left(\bar{\sigma}_{xy}\right)_s = \frac{\partial \bar{u}_s}{\partial y} + 2q\bar{v}_c. \tag{33}$$

Substituting from Eq. 27-30 into Eq. 31-33, respectively,

$$\left(\bar{\sigma}_{xx}\right)_c = \left[2qC - \frac{bA + 2\left(\beta^2+1\right)B}{\left(\beta^2-1\right)}\right]e^{-qy} + 2qBye^{-qy}, \tag{34}$$

$$\left(\bar{\sigma}_{yy}\right)_c = \left[\frac{bA - 2\beta^2 B}{\left(\beta^2-1\right)} - 2qC\right]e^{-qy} - 2qBye^{-qy}, \tag{35}$$

$$\left(\bar{\sigma}_{xy}\right)_s = -\left[qC + \frac{\left(\beta^2+1\right)B - bA}{\left(\beta^2-1\right)}\right]e^{-qy} - qBye^{-qy}. \tag{36}$$

Inverting the Fourier transform for Eq. 27-30 and 34-36, to have:

$$\theta(x,y) = \frac{2}{\pi}\int_0^\infty A(q)e^{-qy}\cos(qx)dq, \tag{37}$$

$$e(x,y) = \frac{2}{\pi}\int_0^\infty B(q)e^{-qy}\cos(qx)dq, \tag{38}$$

$$u(x,y) = \frac{2}{\pi}\int_0^\infty (C + By)e^{-qy}\sin(qx)dq, \tag{39}$$

$$v(x,y) = \frac{2}{\pi}\int_0^\infty \left[\frac{\left(\beta^2+1\right)B - bA}{q\left(\beta^2-1\right)} + C + By\right]e^{-qy}\cos(qx)dq, \tag{40}$$

$$\sigma_{xx}(x,y)=\frac{2}{\pi}\int_0^{\infty}\left[2qC-\frac{bA+2\left(\beta^2+1\right)B}{\left(\beta^2-1\right)}+2qBy\right]e^{-qy}\cos(qx)dq, \tag{41}$$

$$\sigma_{yy}(x,y)=\frac{2}{\pi}\int_0^{\infty}\left[\frac{bA-2\beta^2B}{\left(\beta^2-1\right)}-2qC-2qBy\right]e^{-qy}\cos(qx)dq, \tag{42}$$

$$\sigma_{xy}(x,y)=-\frac{2}{\pi}\int_0^{\infty}\left[qC+\frac{\left(\beta^2+1\right)B-bA}{\left(\beta^2-1\right)}+qBy\right]e^{-qy}\sin(qx)dq. \tag{43}$$

Following are the boundary conditions for the heat conduction problem at y = 0,

$$\frac{\partial\theta}{\partial y}=0 \quad , \quad |x|> a, \tag{44a}$$

$$v=0 \quad , \quad |x|> a, \tag{44b}$$

$$\theta=f(x) \quad , \quad |x|< a, \tag{44c}$$

$$\sigma_{yy}= -p(x) \quad , \quad |x|< a, \tag{44d}$$

$$\sigma_{xy}=0 \ , \quad -\infty<x<\infty. \tag{44e}$$

Applying the boundary condition (44e) leads to:

$$C(q)=\frac{bA(q)-2B(q)}{2q\left(\beta^2-1\right)}. \tag{45}$$

Using the above relation, Eq. 39-42 reduce to

$$u(x,y)=\frac{2}{\pi}\int_0^{\infty}\left(\frac{ba-2B}{2q\left(\beta^2-1\right)}+By\right)e^{-qy}\sin(qx)dq, \tag{46}$$

$$v(x,y)=\frac{2}{\pi}\int_0^{\infty}\left[\frac{2\beta^2B-bA}{2q\left(\beta^2-1\right)}+By\right]e^{-qy}\cos(qx)dq, \tag{47}$$

$$\sigma_{xx}(x,y)=-\frac{4}{\pi}\int_0^{\infty}[B-qBy]e^{-qy}\cos(qx)dq, \tag{48}$$

$$\sigma_{yy}(x,y)=-\frac{4}{\pi}\int_0^{\infty}[B+qBy]e^{-qy}\cos(qx)dq. \tag{49}$$

DUAL INTEGRAL EQUATION FORMULATION

Using the boundary conditions (44a) and (44c) together with Eq. 37, it follows:

$$\int_0^\infty A(q)\cos(qx)dq = \frac{\pi f(x)}{2}, \quad x < a, \tag{50}$$

$$\int_0^\infty qA(q)\cos(qx)dq = 0, \quad x > a \tag{51}$$

The boundary conditions Eq. 45b and 45d with Eq. 47 and 49, yield:

$$2\beta^2\int_0^\infty \frac{B(q)}{q}\cos(qx)dq - b\int_0^\infty \frac{A(q)}{q}\cos(qx)dq = 0, \; x > a, \tag{52}$$

$$\int_0^\infty B(q)\cos(qx)dq = \frac{\pi p(x)}{4}, \quad x < a, \tag{53}$$

In Eq. 50-53, the symmetry of the problem has been used to consider x only in the intervals [0,a] and [a,∞].

Equations (50) and (51) are a set of dual integral equations whose solution gives the unknown parameter A. The following substitution is now taken in order to solve these equations:

$$\int_0^\infty qA(q)\cos(qx)dq = \frac{d}{dx}\left[x\int_x^a \frac{\psi(s)ds}{\sqrt{s^2-x^2}}\right]H(a-x), \tag{54}$$

where, H(a-x) is the Heaviside unit step function.

Inverting the Fourier cosine gives:

$$qA(q) = \frac{2}{\pi}\left\{\int_0^\infty \frac{d}{dx}\left[x\int_x^a \frac{\psi(s)ds}{\sqrt{s^2-x^2}}\right]\cos(qx)dx\right\},$$

Using the integration by parts and the following relations:

$$\lim_{x\to a} x\cos(qx)\int_x^a \frac{\psi(s)ds}{\sqrt{s^2-x^2}} = 0,$$

$$\int_0^s \frac{x\sin(qx)dx}{\sqrt{s^2-x^2}} = \frac{\pi s J_1(qs)}{2}.$$

Consequently,

$$A(q) = \int_0^a s\psi(s)J_1(qs)ds. \tag{55}$$

Substituting from Eq. 55 into Eq. 50 to obtain

$$\int_0^\infty \cos(qx)dq\int_0^a s\psi(s)J_1(qs)ds = \frac{\pi f(x)}{2}, \; x < a,$$

Interchanging the order of the integration in the previous equation and using the following relation:

$$\int_0^\infty J_1(qs)\cos(qx)dq = \begin{cases} \dfrac{1}{s} & , \quad x < s \\ \left(\dfrac{1}{s} - \dfrac{x}{s\sqrt{x^2-s^2}}\right), & x > s \end{cases}$$

Some simplifications are made to get

$$\int_0^x \frac{\psi(s)ds}{\sqrt{x^2-s^2}} = \frac{\pi}{2}\left[\frac{D-f(x)}{x}\right], \quad x < a, \tag{56}$$

where, $D = \frac{2}{\pi}\int_0^a \psi(s)ds.$

Equation 56 is called the Able integral equation and its solution is given by[21]:

$$\psi(s) = -\frac{d}{ds}\int_0^s \frac{f(x)dx}{\sqrt{s^2-x^2}}. \tag{57}$$

Assuming that $f(x) = x^2 - \frac{a^2}{2}$,

Regarding the above equation and Eq. 57, Eq. 55 becomes:

$$A(q) = \frac{-\pi a^2}{2q}J_2(qa). \tag{58}$$

To obtain B (q), the dual integral Eq. 52 and 53 are to be solved. Using Eq. 58 in 52 beside using the following relation[21]

$$\int_0^\infty \frac{J_\mu(at)\cos(bt)}{t}dt = \begin{cases} a^\mu \cos\left(\dfrac{\mu\pi}{2}\right) & , \quad b < a \\ \mu\left(b+\sqrt{b^2-a^2}\right)^\mu & , \quad b > a \end{cases}$$

Equation 52 reduces to

$$\int_0^\infty \frac{B(q)}{q}\cos(qx)dq = 0, \qquad x > a. \tag{59}$$

Considering Eq. 53 and performing the same technique, the system of dual integral equations can be solved to give the solution in the form:

$$B(q) = q\int_0^\infty \phi(s) J_0(qs)\,ds, \qquad (60)$$

where:

$$\phi(s) = \frac{s}{2}\int_0^\infty \frac{p(x)}{\sqrt{s^2 - x^2}}\,dx, \qquad 0 < s < a. \qquad (61)$$

Substituting for p(x) =1 in Eq. 61, Eq. 60 can take the form:

$$B(q) = \frac{\pi a}{4} J_1(qa). \qquad (62)$$

RESULTS AND DISCUSSION

The above evaluations are applied to copper material, whose constants are shown in Table 1.

A Fortran computer program, compiled with Visual Fortran v.6.1, was utilized. Romberg technique of numerical integration with variable step size was employed to calculate the involved definite integrals. Four values of y, namely y = 0.1, y = 0.2, y = 0.3 and y = 0.4, were substituted in performing the computation.

It should be noted (Fig. 2) that in this problem, the crack's size, x, is unity or is taken to be the unit of length in this problem so that $0 \le x \le 1$, y = 0 represents the plane of the crack that is symmetric with respect to the y–plane. It is clear from the graph that θ has maximum value at the beginning of the crack (x = 0), it begins to fall just near the crack edge (x = 1), where it experiences sharp decreases (with maximum negative gradient at the crack's end). Graph lines for both values of y show different slopes at crack ends according to y-values. In other words, the temperature line for y = 0.1 has the highest gradient when compared with that of y = 0.2, y = 0.3 and y = 0.4 at the edge of the crack. In addition, all lines begin to coincide when the horizontal distance x is beyond the double of the crack size to reach the reference temperature of the solid. These results obey physical reality for the behaviour of copper as a polycrystalline solid.

The horizontal displacement, u, begins with a parabolic decrease then increases again to reach its maximum magnitude just at the crack end. Beyond it, u falls again to try to retain zero at infinity. Despite the

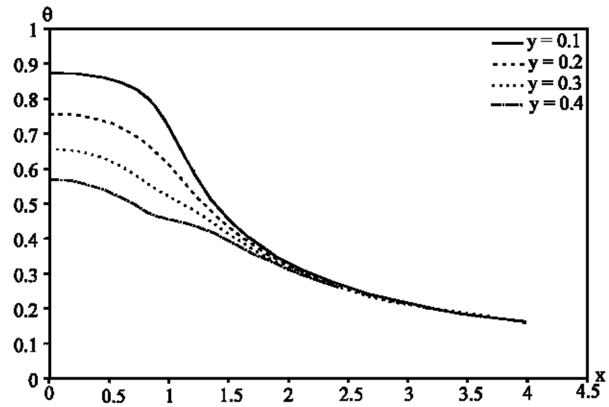

Fig. 2: Temperature distribution

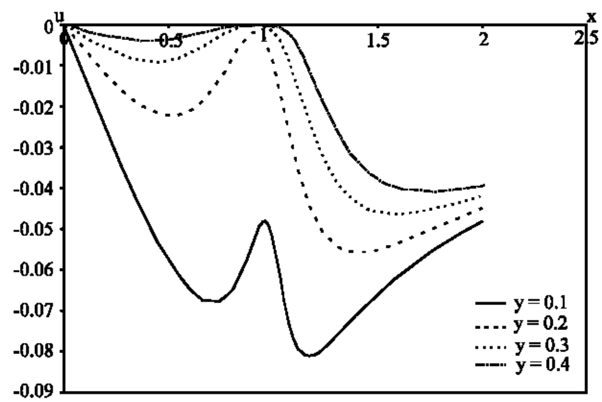

Fig. 3: Horizontal displacement distribution

peaks (for different vertical distances y = 0.1, y = 0.2, y = 0.3 and y = 0.4) occur at equal value of x (x = a = 1), the magnitude of the maximum displacement peak strongly depends on the vertical distance y. It is also clear that the rate of change of u decreases with increasing y as we go farther apart from the crack (Fig. 3). On the other hand, Fig. 4 shows atonable decrease of the vertical displacement, v, at the crack end (x = a = 1) to reach small value beyond x = a = 1 reaching zero at the double of the crack size (state of particles equilibrium). The displacements u and v show different behaviours, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation.

Both of the components show different behaviours; the former tends to increase to maximum just before the end of the crack. Then it falls to a minimum with a highly negative gradient. Afterwards it rises again to a maximum beyond the crack end (about x = 1.25 a). Finally all

Table 1: Thermal and elastic constants for copper.

$\alpha_t = 1.78\times10^{-5}\ K^{-1}$	$c_E = 383.1\ J\ kg^{-1}\ K^{-1}$	$\eta = 8886.73\ m\ s^{-2}$	$\theta_0 = 1\ K$	$T_0 = 293\ K$
$\mu = 3.86\times10^{10}\ N\ m^{-2}$	$\lambda = 7.76\times10^{10}\ N\ m^{-2}$	$\rho = 8954\ kg\ m^{-3}$	$c_1 = 4.158\times10^3\ m\ s^{-1}$	$\beta^2 = 4$
a = 1	b = 0.042	c = 0.01		

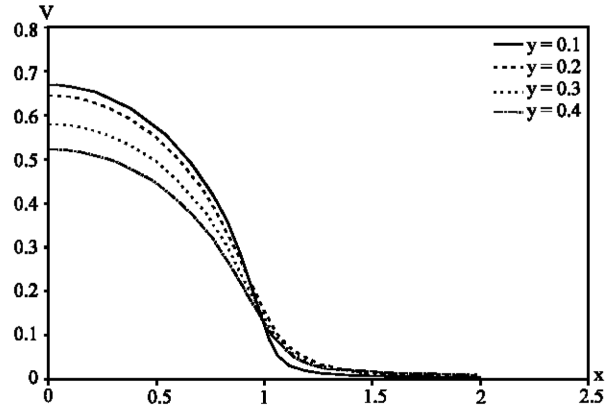

Fig. 4: Vertical displacement distribution

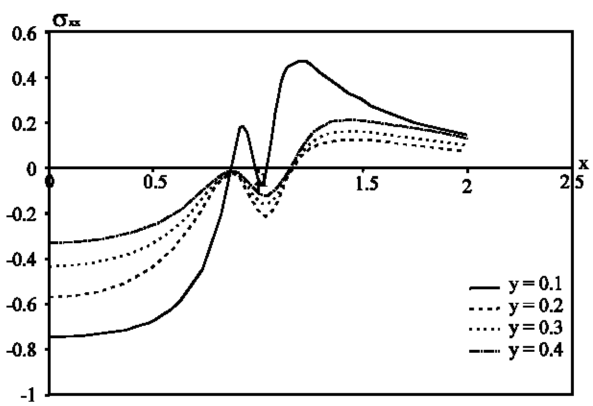

Fig. 5: Horizontal stress distribution

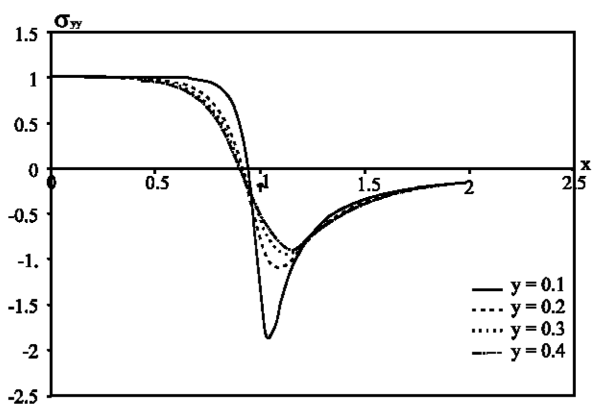

Fig. 6: Vertical stress distribution

horizontal stresses, σ_{xx}, reach coincidence with zero values (Fig. 5). But σ_{yy} had a different behaviour as it retains its maximum strength until reaching the crack end when it falls to a minima then increases again just beyond the crack edge to coincide with other vertical stresses (a different y values) to reach zero after their relaxations at infinity. Variation of y has a serious effect on both magnitudes of mechanical stresses (Fig. 6). These trends obey elastic and thermoelastic properties of the solid under investigation.

CONCLUSIONS

1. Analytical solutions based upon Fourier transforms for thermoelastic problem in solids have been developed and utilized.
2. A linear opening Mode-I crack has been investigated and studied for copper solid.
3. Temperature, radial and axial distributions were estimated at different distances from the crack edge.
4. Horizontal and vertical stress distributions were evaluated as functions of the distance from the crack edge.
5. Crack dimensions are significant to elucidate the mechanical structure of the solid.
6. Cracks are stationary and external stress is demanded to propagate such cracks.
7. It can be concluded that a change of volume is attended by a change of the temperature while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity.
8. The displacements u and v show different behaviours, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation.
9. These behaviours obey elastic and thermoelastic properties of the solid under investigation.

All functions are continuous, especially that of u and v which prove that the crack will not propagate because of the fact that both of the mechanical and thermal stresses are not sufficient to propagate such a crack, and to propagate it, the solid need to be subject to an external stress (tensile, shear,…).

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