Journal of Applied Sciences

ISSN 1812-5654

science alert

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Journal of Applied Sciences 6 (3): 710-715, 2006
ISSN 1812-5654

Effect of Sinusoidal Distribution of the Temperature on Laminar Natural Convection in Wavy Rectangular Enclosures

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Abstract: In the present research, a numerical study of the effect of non uniform boundary conditions on the heat transfer by natural convection in rectangular cavities was investigated for the laminar regime. This problem was solved by using the partial differential equations which are the equation of mass, momentum and energy. The aspect ratio of the cavities has also been changed. The tests were performed for different boundary conditions and different Rayleigh numbers while the Prandtl number was kept constant. The configurations with one and three undulations for the hot wall geometry were tested in a laminar regime. For the vertical walls, a non uniform sinusoidal distributions are adopted.The results obtained show that the trend of the local Nusselt number is wavy for all aspect ratios investigated. The mean Nusselt number decreases comparing with the Nusselt number of the rectangular cavity. The sinusoidal function of the temperature distribution in both vertical walls increases the local and the mean Nusselt number comparing with the isotherm walls. Non uniform boundary conditions seem to increase the heat transfer rate. However, increasing aspect ratio results in a decrease of the Nusselt number.

Key words: Natural convection, enclosed cavity, wavy hot wall, non uniform boundary conditions, aspect ratios

INTRODUCTION

Natural convection in rectangular cavity enclosures with differentially heated side walls is an area of practical importance. Amongst its applications are cooling of nuclear reaction, design of solar collector, simulation of fire spread in buildings, etc.

Most of studies in this field are substantially oriented toward the study of rectangular enclosures as by De Vahl Davis[1] Catton[2], Ostrach[3], Yang[4] and Bokhari[5]. A more comprehension of the flow behaviour and the heat transfer in such cavities was needed. The study of aspect ratio has allowed the understanding of the heat transfer behaviour by Arnold[6], Ozoe[7], Holland[8] and Kuyper[9]. Using the vorticity–stream function approach, Wilkes and Churchill[10] applied the ADI method and obtained 2D fluid flow distribution inside the cavity for aspect ratios ranging from 1 to 3. Chu and Churchill[11] used the latter method to describe the solution of heat transfer in a cavity with aspect ratios ranging from 0.4 to 5 and Rayleigh number up to $5x10^4$. Korpela *et al.*[12] presented also natural convection heat transfer in rectangular cavities; they investigated the cavities with aspect ratios up to 40 by determining the boundary between various flow regimes occurring in cavities with rather high aspect ratios.

Yao[13] has studied theoretically the natural convection along a vertical wavy surface. He found the heat transfer rate for a wavy surface smaller than of corresponding flat plate. Saidi *et al.*[14] also presented numerical and experimental results of the flow over and heat transfer from a sinusoidal cavity. They reported that the total heat exchange between the wavy wall of the cavity and the following fluid was reduced by the presence of vortex. Adjlout *et al.*[15] studied the natural convection in an inclined cavity with hot wavy wall; they simulated the heat transfer and the fluid flow with ADI scheme. One of their interesting results was the decrease of average heat transfer with the surface waviness compared with a square cavity. The usual situations analysed refer to enclosures with imposed uniform temperature at the vertical walls. Gilly *et al.*[16] have shown the importance of the non-uniform distribution of the temperature on the hot wall, a more comprehension of the flow behaviour and the heat transfer with such boundary conditions was required.

JianLi *et al.*[17] have treated the problem of the natural convection on vertical flat plate with a sinusoidal

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temperature distribution. In the present study, a numerical investigation of the influence of boundary conditions on free convection in rectangular cavities with wavy wall, differentially heated has been performed. The aspect ratio of the cavities has also been changed. The hot wall is wavy; the non-uniform distribution of temperature is a sinusoidal function as proposed by Gilly *et al.*[16].

Basic equations: The buoyancy driven flow is considered to be bidimensionnal and laminar. The fluid is assumed to be incompressible, with constant physical properties and negligible viscous dissipation. The buoyancy effects upon momentum transfer are taken into account through the Boussinesq approximation. Once above assumptions are employed into the conservation equation of mass, momentum and energy, the following dimensionless variable are introduced as:

$$X = \frac{x}{L} \qquad Y = \frac{y}{L} \tag{1}$$

$$U = \frac{u}{v/L} \qquad V = \frac{v}{v/L} \tag{2}$$

$$P = \frac{p+\rho g\, y}{\rho\,(v/L)^2} \qquad \theta = (T-T_0)/(T_h-T_c) \tag{3}$$

with $T_0 = (T_h + T_c)/2$

the following set of governing equations is obtained:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{5}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Ra}{Pr}\,\theta \tag{6}$$

$$\frac{\partial \theta}{\partial t} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{7}$$

Knowing that $0 \le y \le A$ and $0 \le x \le f(y)$ with $f(y) = [1\text{-amp}+\text{amp}\,(\cos 2\,\pi n y)]$ with n and amp are, respectively, a number of undulations and amplitude as proposed by Adjlout *et al.*[15]. Figure 1 shows the geometry of the enclosure. The study is completed with the definition of the following boundary conditions:

$U = V = 0$ at all the boundaries

$\theta(0,Y) = -0.5 + a\,\mathrm{Sin}\left[\pi\left(\frac{y}{L} - 0.5\right)\right]$ on the cold wall

$\theta(1,Y) = 0.5 + a\,\mathrm{Sin}\left[\pi\left(\frac{y}{L} - 0.5\right)\right]$ on the hot wall

$\frac{\partial \theta}{\partial Y} = 0$ on the adiabatic walls

α is a coefficient, which varies between 0 and 0.4 in the present study.

Numerical procedure: The grid generation calculation is based on the curvilinear coordinate system applied to fluid flow as described by Thompson *et al.*[18]. The Equations governing the flow and the energy are solved using a finite volume method with pressure-correction method as introduced by Patankar[19]. After each line, Gauss Seidel sweep for the momentum, energy and the Poisson equation for the pressure correction was solved directly over the full domain. For the pressure correction, a second order scheme is used with 0.3 under relaxation factor, while the second order upwind scheme is adopted for the momentum and the energy equations with the same value of under relaxation factor 0.7.

RESULTS

The heat transfer rate by convection in an enclosure is obtained from the Nusselt number calculation. On the wavy wall, the local and the mean Nusselt number are expressed respectively:

$$Nu_l = \frac{\partial \theta}{\partial n} \tag{8}$$

$$Nu = \frac{1}{s}\int_0^s \frac{\partial \theta}{\partial n}\, ds \tag{9}$$

Several grids have been tested for the different cavities. Table 1 shows the average Nusselt number for the three grids used for a rectangular cavity (A = 7) with one undulation, $Ra = 10^5$ and $\phi = 90°$. Grid refinement was applied to check the accuracy of the solution. The number of grid cells used in the calculations were chosen such that the variation of the mean Nusselt number is by less than 5%. All grids studied for different cavities are presented in Table 2.

In the present investigation, the tests were performed for the aspect ratios namely 3, 5, 7, 12 and 15.

Figure 2-5 show, respectively the numerical visualisation of the streamlines and isotherms for different aspect ratios and for both undulated cavities. The flow is mainly monocellular for all configurations. The same flow behaviour is observed in the experimental visualisation of Inaba[20], using an uniform boundary conditions. The flow

Table 1: Comparison on Nusselt number at several grids for A= 7, $Ra = 10^5$

Grid	20×141	40×281	60×421
Nu_a	4.221	4.379	4.569

Table 2: Different grids for all aspect ratio, $Ra = 10^5$

A	3	5	7	12	15
Grid	40×121	40×201	40×281	20×241	20×301

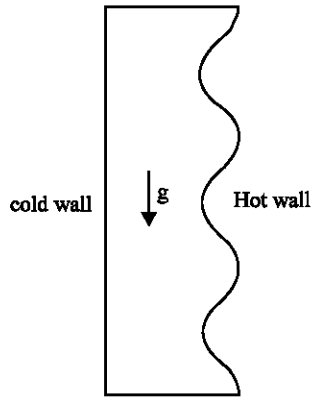

Fig. 1: Physical domain

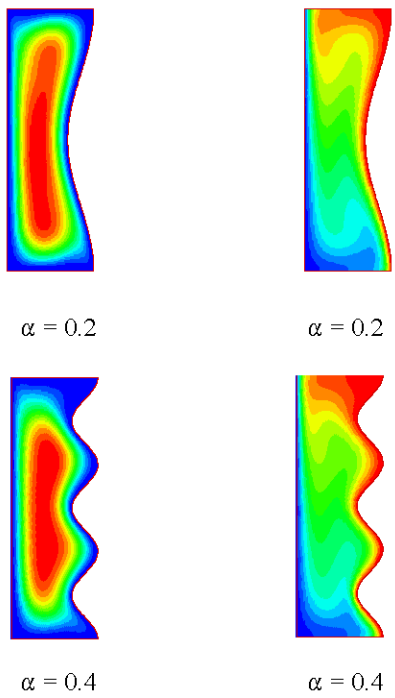

Fig. 2: Streamlines and isotherms for both cavities and all boundry conditions A = 3

features obtained for the case of A=5 and three undulations were also observed for all aspect ratios investigated and the cavity with one undulation. The isotherms for one and three undulations for α = 0.2 and α = 0.4 are presented the results. The central part of the cavity is characterised by stratified flow for both cavities. It result a decrease of the thermal boundary layer thickness. Therefore, an increase in heat transfer rate is obtained. It is noticed that the number of undulations affects the thermal boundary layer thickness. This latter

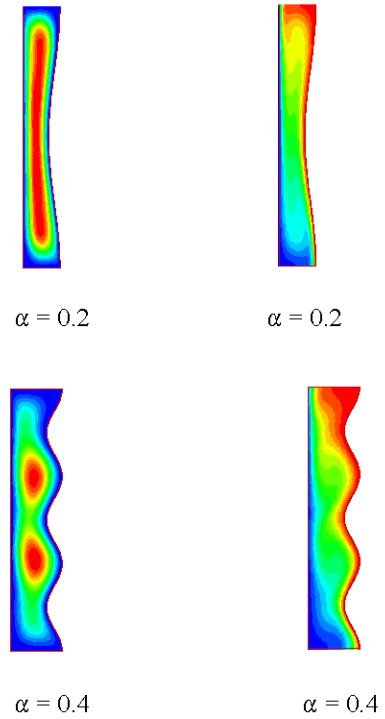

Fig. 3: Streamlines and isotherms for both cavities and all boundry conditions A =5

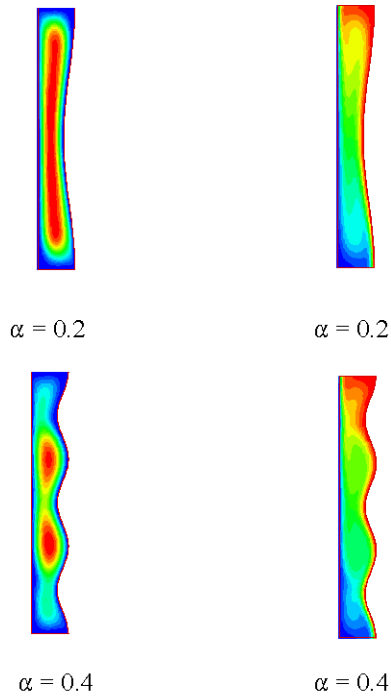

Fig. 4: Streamlines and isotherms for both cavities and all boundry conditions A = 7

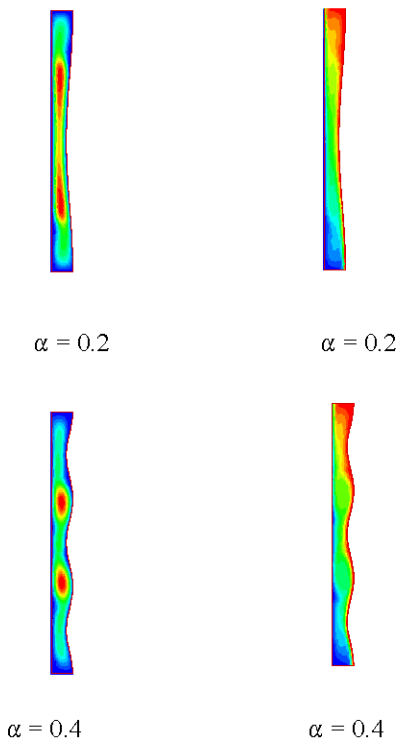

Fig. 5: Streamlines and isotherms for both cavities and all boundry conditions A = 12

seems to increase on the side of the undulated wall and to decrease just before a crest or just after a through.

For the case of the cavity with one undulation, the boundary layer thickness increases with the ascending direction. This result is in agreement with Adjlout *et al.*[15]. Figure 6 shows the local Nusselt number for the cavity with three undulations A=5 and Ra=10^5. It is clea rly seen that an increase in α results in an increase in the local Nusselt number due to thinning of the thermal boundary layer on the wavy wall.

The wavy aspect of the local Nusselt number curves for the cavity with three undulations is well established. Increasing the aspect ratio seems to damp the amplitude of the Nusselt number undulating trend. The growth of the thermal boundary layer becomes monotone

Comparison of the mean Nusselt number for α = 0.2 and 0.4 for different aspect ratios are presented in Fig. 7 and 8. It is clearly seen that the aspect ratio has a great effect on the heat transfer rate for both α investigated and both cavities. The slope of the curves seems to be slightly different and the trend of the curves are linear.

Table 3 shows the different results for all test performed. It is clearly seen that the mean heat transfer

Table 3: Mean Nusselt number for Ra=10^5, for different α and all aspect ratios

Undulation	1			3		
α	α = 0	α = 0.2	α = 0.4	α = 0	α = 0.2	α = 0.4
A=3	3.956	4.912	6.553	3.482	4.629	5.795
A=5	3.739	4.674	6.233	3.427	4.352	5.677
A=7	3.472	4.379	5.840	3.357	4.191	5.597
A=12	3.187	3.849	5.132	3.092	3.678	4.995
A=15	2.959	3.673	4.941	2.913	3.665	4.840

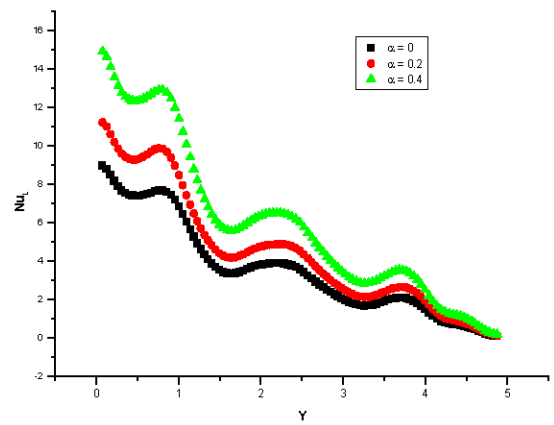

Fig. 6: Local Nusselt number for a cavity with three undulations, A=5, Ra = 10^5 with non uniform wall temperature

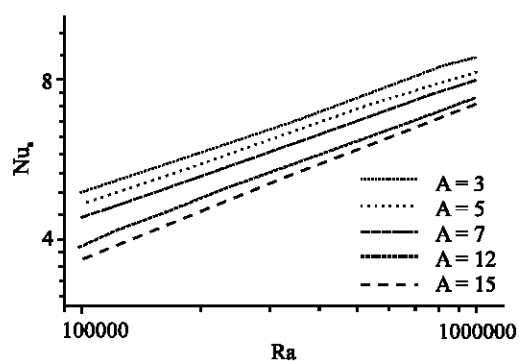

Fig. 7: Comparison of the mean Nusselt number results for α = 0.2, one undulation and different ratios

for the cavity with three undulations is lower than the average Nuselt number for the configuration with one undulation as found by Adjlout *et al.*[15]. The difference between the two configurations heat transfer vanishes with an increase in aspect ratio.

The relative increase of the mean Nusselt number is about (25, 65%), respectively for α = 0.2 , 0.4 and for both configurations compared with all aspect ratios performed at α = 0.

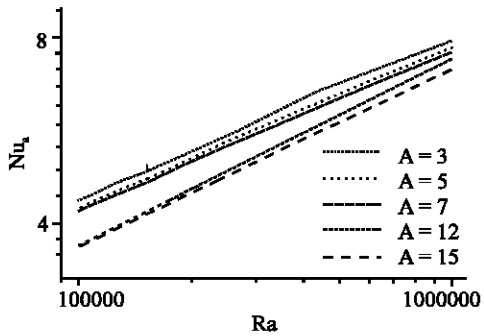

Fig. 8: Comparison of the mean Nusselt number results for α = 0.4, three undulations and different ratios

CONCLUSIONS

The present study deals with the effect of the non uniform boundary conditions on the heat transfer by natural convection in a wavy enclosures. The flow structure remains the same found for the cavity with uniform boundary conditions. The cavity with three undulations seems to reduce more the global heat transfer than the configuration with one undulation. Increasing the aspect ratio affects greatly the difference in the heat transfer between the cavities. The results obtained show that increasing α results in an increase in the global heat transfer for both configurations and all aspect ratios.

Nomenclature

A aspect ratio of enclosure
Cp Specific heat of fluid
g gravitational acceleration
k thermal conductivity
H width of the enclosure
L length of the enclosure
Nu Nusselt number
P,p dimensionless and dimensional pressure
Pr Prandtl number = $\mu Cp/k$

Ra Rayleigh number = $\dfrac{g\,\beta\,(T_h - T_c)\,L^3}{v\,k}$

t time
T, dimensionless and dimensional temperature
U,u dimensionless and dimensional horizontal velocity component
V,v dimensionless and dimensional vertical velocity component
X,x dimensionless and dimensional coordinate along the horizontal direction
Y,y dimensionless and dimensional coordinate along the vertical direction

Greek symbols
β thermal dilatation coefficient
ϕ inclination angle
ν kinematic viscosity
θ dimensionless temperature
ρ density
μ viscosity

Subscripts
a average
L local
h,c hot wall and cold wall

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