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## An Algebraically Derived Minimal Cost Solution Technique of the EOQ Model Under Conditional Trade Credit

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**Abstract:** In this study the restrictive assumptions of the trade credit independent of the order quantity and the retailer's unit selling price equaled to the purchasing price per unit are relaxed to fit real business situations. This study investigates the retailer's inventory problem under trade credit dependent of the order quantity and the retailer's unit selling price not necessarily equals to the purchasing price per unit within the Economic Order Quantity (EOQ) framework. In addition, we adopt the algebraic procedure to determine the retailer's optimal ordering policy under minimizing the annual total variable cost. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. Then, two theorems are developed to efficiently determine the optimal cycle time and optimal order quantity for the retailer. Finally, numerical examples are given to illustrate these theorems and obtain a lot of managerial insights.

**Key words:** Inventory, EOQ, conditional trade credit, algebraic approach, optimization

### INTRODUCTION

In a real world, the supplier often makes use of the trade credit policy to promote their commodities. Goyal (1985) is frequently cited when the inventory systems under conditions of trade credit are discussed. Goyal (1985) implicitly makes the following assumptions:

- Supplier credit policy offered to the retailer where credit terms are independent of the order quantity. That is, whatever the order quantity is small or large, the retailer can take the benefits of payment delay.
- The unit selling price and the unit purchasing price are assumed to be equal. However, in practice, the unit selling price is not lower than the unit purchasing price in general.
- At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold.

According to the above arguments, this study will adopt the following assumptions to modify the Goyal's (1985) model.

- To encourage retailer to order large quantity, the supplier may give the trade credit period only for large order quantity. In other words, the retailer requires immediate payment for small order quantity. This viewpoint can be found in Chang *et al.* (2003).

- The selling price per unit and the unit purchasing price are not necessarily equal to match the practical situations. This viewpoint can be found in Teng (2002) and Chung *et al.* (2002).
- The retailer needs cash for business transactions. At the end of the credit period, the retailer pays off all units sold and keeps his/her profits for business transactions or other investment use. This viewpoint also can be found in Teng (2002) and Chung *et al.* (2002).

Hence, we want to incorporate the above assumptions (i), (ii) and (iii) to modify the Goyal's model (1985). In addition, in previous all published papers which have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. In recent papers, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. Therefore, we want to adopt the algebraic procedure to investigate the effect of trade credit policy depending on the order quantity and the retailer's unit selling price not necessarily equaled to the purchasing price per unit within the Economic Order Quantity (EOQ) framework. Then, two theorems are developed to

efficiently determine the optimal cycle time and optimal order quantity for the retailer. Finally, numerical examples are given to illustrate these theorems.

**Algebraic model formulation:** Here, we want to develop the inventory model under trade credit to take the order quantity into account. When the order quantity is less than the fixed quantity at which the delay in payments is permitted, the payment for the items must be made immediately. Otherwise, the fixed trade credit period is permitted. The following notation and assumptions will be used to develop our inventory model.

Notation:

- Q = Order quantity
- D = Annual demand
- W = Minimum order quantity at which the delay in payments is permitted
- A = Cost of placing one order
- c = Unit purchasing price
- s = Unit selling price
- h = Unit stock holding cost per year excluding interest charges
- $I_p$  = Interest charges per \$ investment in inventory per year
- $I_e$  = Interest which can be earned per \$ per year
- M = The trade credit period in years
- T = The cycle time in years
- TVC(T) = The annual total variable cost when  $T > 0$
- $T^*$  = The optimal cycle time of TVC(T)
- $Q^*$  = The optimal order quantity =  $DT^*$

**Assumptions**

- Demand rate is known and constant.
- Shortages are not allowed.
- Time period is infinite.
- Replenishments are instantaneous.
- If  $Q < W$ , i.e.,  $T < W/D$ , the delayed payment is not permitted. Otherwise, fixed trade credit period M is permitted. Hence, if  $Q < W$ , pay  $cQ$  when the order is received. If  $Q \geq W$ , pay  $cQ$  M time periods after the order is received.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When  $T \geq M$ , the account is settled at  $T = M$ , the retailer pays off all units sold and keeps his/her profits and starts paying for the higher interest charges on the items in stock. When  $T \leq M$ , the account is settled at  $T = M$  and the retailer does not need to pay any interest charge.
- $s \geq c, I_p \geq I_e$ .

The annual total variable cost consists of the following elements. There are two cases to occur: (1)  $M \geq W/D$  and (2)  $M < W/D$ .

**Case 1:** Suppose that  $M \geq W/D$ .

- Annual ordering cost =  $A/T$
- Annual stock holding cost (excluding interest charges) =  $DTh/2$
- There are three sub-cases to occur in cost of interest charges for the items kept in stock per year.

$$0 < T < W/D \tag{i}$$

In this case, the retailer must pay  $cDT$  when the order is received since the delayed payment is not permitted. Therefore,

Cost of interest charges for the items kept in stock per cycle =  $cI_pDT^2/2$

Cost of interest charges for the items kept in stock per year =  $cI_pDT/2$

$$W/D \leq T \leq M \tag{ii}$$

In this case, the fixed trade credit period M is permitted since  $Q \geq W$ . According to assumption (6), no interest charges are paid for the items kept in stock.

$$M \leq T \tag{iii}$$

In this case, the fixed trade credit period M is permitted since  $Q \geq W$ . According to assumption (6), Cost of interest charges for the items kept in stock per cycle =  $cI_pD(T-M)^2/2$

Cost of interest charges for the items kept in stock per year =  $cI_pD(T-M)^2/2T$

- There are three sub-cases to occur in interest earned per year.

$$0 < T < W/D \tag{i}$$

In this case, no interest earned since the delayed payment is not permitted.

$$W/D \leq T \leq M \tag{ii}$$

In this case, the fixed trade credit period M is permitted since  $Q \geq W$ . According to assumption (6),

Interest earned per cycle =

$$sI_e \frac{DT^2}{2} + DT(M - T) = DTsI_e \left( M - \frac{T}{2} \right)$$

$$\text{Interest earned per year} = DsI_e M - \frac{T}{2}$$

$$M \leq T \tag{iii}$$

In this case, the fixed trade credit period  $M$  is permitted since  $Q \geq W$ . According to assumption (6),

$$\text{Interest earned per cycle} = sI_e \int_0^M Dtdt = \frac{DM^2sI_e}{2}$$

$$\text{Interest earned per year} = \frac{DM^2sI_e}{2T}$$

From the above arguments, the annual total variable cost for the retailer can be expressed as:

$$\text{TVC}(T) = \text{ordering cost} + \text{stock-holding cost} + \text{interest payable} - \text{interest earned}$$

We show that the annual total variable cost,  $\text{TVC}(T)$ , is given by:

$$\text{TVC}_1(T) \quad \text{if} \quad 0 < T < W/D \tag{1a}$$

$$\text{TVC}(T) = \text{TVC}_2(T) \quad \text{if} \quad W/D \leq T \leq M \tag{1b}$$

$$\text{TVC}_3(T) \quad \text{if} \quad M \leq T \tag{1c}$$

Where,

$$\text{TVC}_1(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_pDT}{2}, \tag{2}$$

$$\text{TVC}_2(T) = \frac{A}{T} + \frac{DTh}{2} - DsI_e(M - \frac{T}{2}) \tag{3}$$

and

$$\text{TVC}_3(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_pD(T - M)^2}{2T} - \frac{DM^2sI_e}{2T} \tag{4}$$

Since  $\text{TVC}_1(W/D) > \text{TVC}_2(W/D)$ ,  $\text{TVC}_2(M) = \text{TVC}_3(M)$ ,  $\text{TVC}(T)$  is continuous except  $T = W/D$ .

Then, we can rewrite:

$$\begin{aligned} \text{TVC}_1(T) &= \frac{A}{T} + \frac{DT(h + cI_p)}{2} \\ &= \frac{D(h + cI_p)}{2T} T - \sqrt{\frac{2A}{D(h + cI_p)}} \\ &\quad + \sqrt{2AD(h + cI_p)} \end{aligned} \tag{5}$$

From Eq. 5 the minimum of  $\text{TVC}_1(T)$  is obtained when the quadratic non-negative term, depending on  $T$ , is equal to zero. The optimum value  $T_1^*$  is:

$$T_1^* = \sqrt{\frac{2A}{D(h + cI_p)}} \tag{6}$$

Therefore,

$$\text{TVC}_1(T_1^*) = \sqrt{2AD(h + cI_p)} \tag{7}$$

Similarly, we can derive  $\text{TVC}_2(T)$  without derivatives as follows:

$$\begin{aligned} \text{TVC}_2(T) &= \frac{A}{T} + \frac{DT(h + sI_e)}{2} - DsMI_e \\ &= \frac{D(h + sI_e)}{2T} T - \sqrt{\frac{2A}{D(h + sI_e)}} \\ &\quad + [\sqrt{2AD(h + sI_e)} - DsMI_e] \end{aligned} \tag{8}$$

From Eq. 8 the minimum of  $\text{TVC}_2(T)$  is obtained when the quadratic non-negative term, depending on  $T$ , is equal to zero. The optimum value  $T_2^*$  is

$$T_2^* = \sqrt{\frac{2A}{D(h + sI_e)}} \tag{9}$$

Therefore,

$$\text{TVC}_2(T_2^*) = [\sqrt{2AD(h + sI_e)} - DsMI_e] \tag{10}$$

Likewise, we can derive  $\text{TVC}_3(T)$  algebraically as follows.

$$\begin{aligned} \text{TVC}_3(T) &= \frac{2A + DM^2(cI_p - sI_e)}{2T} + \frac{DT(h + cI_p)}{2} - DcMI_p \\ &= \frac{D(h + cI_p)}{2T} T - \sqrt{\frac{2A + DM^2(cI_p - sI_e)}{D(h + cI_p)}} \\ &\quad + \left\{ \sqrt{D(h + cI_p)[2A + DM^2(cI_p - sI_e)]} - DcMI_p \right\} \end{aligned} \tag{11}$$

From Eq. 11 the minimum of  $\text{TVC}_3(T)$  is obtained when the quadratic non-negative term, depending on  $T$ , is equal to zero. The optimum value  $T_3^*$  is:

$$\begin{aligned} T_3^* &= \sqrt{\frac{2A + DM^2(cI_p - sI_e)}{D(h + cI_p)}} \\ &\quad \text{if } 2A + DM^2(cI_p - sI_e) > 0 \end{aligned} \tag{12}$$

Therefore,

$$TVC_3(T_3^*) = \frac{\sqrt{D(h + cI_p)[2A + DM^2(cI_p - sI_e)]}}{-DcMI_p} \quad (13)$$

**Case 2:** Suppose that  $M < W/D$ .

If  $M < W/D$ , equations 1(a, b, c) will be modified as:

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } 0 < T < W/D \\ TVC_3(T) & \text{if } W/D \leq T. \end{cases}$$

Since  $TVC_1(W/D) > TVC_3(W/D)$ ,  $TVC(T)$  is continuous except  $T = W/D$ .

**Decision rule of the optimal cycle time T\***

**Case 1:** When  $M \geq W/D$

Equation 6 gives that the optimal value of  $T^*$  for the case when  $0 < T < W/D$  so that  $0 < T_1^* < W/D$ . We substitute Eq. 6 into  $0 < T_1^* < W/D$ , then we can obtain that  $0 < T_1^* < W/D$  if and only if

$$0 < 2A < \frac{W^2}{D}(h + cI_p) \quad (14)$$

Similarly, Eq. 9 gives that the optimal value of  $T^*$  for the case when  $W/D \leq T \leq M$  so that  $W/D \leq T_2^* \leq M$ . We substitute Eq. 9 into  $W/D \leq T_2^* \leq M$ , then we can obtain that  $W/D \leq T_2^* \leq M$  if and only if

$$\frac{W^2}{D}(h + sI_e) \leq 2A \leq DM^2(h + sI_e) \quad (15)$$

Finally, Eq. 12 gives that the optimal value of  $T^*$  for the case when  $T \geq M$  so that  $T_3^* \geq M$ . We substitute Eq. 12 into  $T_3^* \geq M$ , then we can obtain that  $T_3^* \geq M$  if and only if

$$2A \geq DM^2(h + sI_e) \quad (16)$$

Furthermore, we let:

$$\Delta_1 = -2A + \frac{W^2}{D}(h + cI_p), \quad (17)$$

$$\Delta_2 = -2A + \frac{W^2}{D}(h + sI_e) \quad (18)$$

and

$$\Delta_3 = -2A + DM^2(h + sI_e) \quad (19)$$

From Eq. 17, 18 and 19, we can easily obtain  $\Delta_3 \geq \Delta_2$ . In addition, we know  $TVC_1(W/D) > TVC_3(W/D)$ ,  $TVC_2(M) = TVC_3(M)$ ,  $TVC(T)$  is continuous except  $T = W/D$  from Eq. 2, 3 and 4. Then, we can summarize above arguments and obtain following results.

**Theorem 1**

- If  $\Delta_1 > 0$ ,  $\Delta_2 > 0$  and  $\Delta_3 > 0$ , then  $TVC(T^*) = \min\{TVC_1(T_1^*), TVC_2(W/D)\}$ . Hence  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.
- If  $\Delta_1 > 0$ ,  $\Delta_2 \leq 0$  and  $\Delta_3 > 0$ , then  $TVC(T^*) = \min\{TVC_1(T_1^*), TVC_2(T_2^*)\}$ . Hence  $T^*$  is  $T_1^*$  or  $T_2^*$  associated with the least cost.
- If  $\Delta_1 > 0$ ,  $\Delta_2 \leq 0$  and  $\Delta_3 \leq 0$ , then  $TVC(T^*) = \min\{TVC_1(T_1^*), TVC_3(T_3^*)\}$ . Hence  $T^*$  is  $T_1^*$  or  $T_3^*$  associated with the least cost.
- If  $\Delta_1 \leq 0$ ,  $\Delta_2 > 0$  and  $\Delta_3 > 0$ , then  $TVC(T^*) = TVC_2(W/D)$  and  $T^* = W/D$ .
- If  $\Delta_1 \leq 0$ ,  $\Delta_2 \leq 0$  and  $\Delta_3 > 0$ , then  $TVC(T^*) = TVC_2(T_2^*)$  and  $T^* = T_2^*$ .
- If  $\Delta_1 \leq 0$ ,  $\Delta_2 \leq 0$  and  $\Delta_3 \leq 0$ , then  $TVC(T^*) = TVC_3(T_3^*)$  and  $T^* = T_3^*$ .

**Case 2:** When  $M < W/D$

In another condition  $M < W/D$ , equations 1(a, b, c) will be modified as:

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } 0 < T < W/D \\ TVC_3(T) & \text{if } W/D \leq T. \end{cases}$$

Similarly, Eq. 6 gives that the optimal value of  $T^*$  for the case when  $0 < T < W/D$  so that  $0 < T_1^* < W/D$ . We substitute Eq. 6 into  $0 < T_1^* < W/D$ , then we can obtain that  $0 < T_1^* < W/D$  if and only if

$$0 < 2A < \frac{W^2}{D}(h + cI_p) \quad (20)$$

Likewise, Eq. 12 gives that the optimal value of  $T^*$  for the case when  $T \geq W/D$  so that  $T_3^* \geq W/D$ . We substitute Eq. 12 into  $T_3^* \geq W/D$ , then we can obtain that  $T_3^* \geq W/D$  if and only if

$$2A \geq \frac{W^2}{D}(h + cI_p) - DM^2(cI_p - sI_e) \quad (21)$$

Furthermore, we let:

$$\Delta_1 = -2A + \frac{W^2}{D}(h + cI_p) \quad (22)$$

and

**Table 1: The optimal cycle time and optimal order quantity with various values of W and s**

s = \$/unit	W = 300 units/year					W = 450 units/year					W = 600units/year				
	$\Delta_1$	$\Delta_2$	$\Delta_3$	T*	Q*	$\Delta_1$	$\Delta_2$	$\Delta_3$	T*	Q*	$\Delta_1$	$\Delta_4$	T*	Q*	
50	<0	<0	<0	$T_3^* = 0.1026$	513	<0	<0	<0	$T_3^* = 0.1026$	513	>0	>0	W/D = 0.12	600	
75	<0	<0	>0	$T_2^* = 0.09562$	478	<0	<0	>0	$T_2^* = 0.09562$	478	>0	>0	W/D = 0.12	600	
100	<0	<0	>0	$T_3^* = 0.08944$	447	<0	>0	>0	W/D = 0.09	450	>0	>0	W/D = 0.12	600	

Let A = \$200/order, D = 5000 units/year, c = \$30/unit, h = \$5/unit/year,  $I_p = \$0.15/\$/year$ ,  $I_e = \$0.05/\$/year$  and M = 0.1 year

$$\Delta_4 = -2A + \frac{W^2}{D}(h + cI_p) - DM^2(cI_p - sI_e) \quad (23)$$

We know  $TVC_1(W/D) > TVC_3(W/D)$ ,  $TVC(T)$  is continuous except  $T = W/D$  from Eq. (2) and (4). Then, we can summarize above arguments and obtain following results.

**Theorem 2**

- If  $\Delta_1 > 0$  and  $\Delta_4 > 0$ , then  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_3(W/D)\}$ . Hence  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.
- If  $\Delta_1 \leq 0$  and  $\Delta_4 \leq 0$ , then  $TVC(T^*) = TVC_3(T_3^*)$  and  $T^* = T_3^*$ .
- If  $\Delta_1 > 0$  and  $\Delta_4 \leq 0$ , then  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_3(T_3^*)\}$ . Hence  $T^*$  is  $T_1^*$  or  $T_3^*$  associated with the least cost.
- If  $\Delta_1 \leq 0$  and  $\Delta_4 > 0$ , then  $TVC(T^*) = TVC_3(W/D)$  and  $T^* = W/D$ .

**Numerical examples:** To illustrate all results obtained in this research, let us apply the proposed method to efficiently solve the following numerical examples. The optimal cycle time and optimal order quantity are summarized in Table 1.

**CONCLUSIONS**

The purpose of this study adopts the algebraic procedure to investigate the effect of trade credit policy depending on the order quantity and the retailer’s unit selling price not necessarily equaled to the purchasing price per unit within the economic order quantity (EOQ) framework. Using this approach presented in this study, we can find the optimal cycle time and optimal order quantity without using differential calculus. Two ease-to-use theorems help the retailer accurately and quickly determining the optimal inventory policy under minimizing the annual total variable cost.

From the final numerical examples, we can obtain following managerial insights. The retailer will order more quantity to take the benefits of trade credit as possible when the minimum order quantity to obtain the permissible delay is higher. In addition, the retailer will not order too large quantity to pay higher holding cost for the item under the delayed payment is permitted. And last, the retailer will order less quantity to take the benefits of the trade credit more frequently when the larger the differences between the unit selling price and the purchasing price per item.

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