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The EPQ Model with Shortages and Variable Lead Time

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Abstract: Chang derived The EPQ with shortages and variable lead-time using algebraic method to determine the optimal solution and obtain the minimum total cost of the proposed model. In this note, we will offer a simple algebraic approach to replace his sophisticated algebraic skill.

Key words: Inventory, EPQ, Variable lead-time, Backorders, Algebraic method

INTRODUCTION

The EOQ (Economic Order Quantity) model is widely used by practitioners as a decision-making tool for the control of inventory. However, the assumptions of the EOQ model are rarely met. This has led many researchers to study the EOQ extensively under realistic situations. For minimizing the total relevant costs, in most previous all published papers that have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. But Ronald *et al*. (2004) thought that their algebraic procedure is too sophisticated to be absorbed by ordinary readers. Hence, Ronald *et al*. (2004) derived a procedure to transform a two-variable problem into two steps and then, in each step, they solve a one-variable problem using only the algebraic method without referring to calculus. Recently, Chang *et al*. (2005) rewrote the objective function of Ronald *et al*. (2004) such that the usual skill of completing the square can handle the problem without using their sophisticated method.

Recently, we study the reserch of Chang (2004) that investigated the EPQ model with shortages and variable lead-time using algebraic method. But his method had the same problem as Grubbström and Erdem (1999) and Cárdenas-Barrón (2001). Therefore, in this note, we will offer a simple algebraic approach same as Chang *et al*. (2005) to replace his sophisticated algebraic skill. This method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus as Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) stated.

ALGEBRAIC IMPROVEMENT IN CHANG'S MODEL

We adopt the same notation and assumptions as Chang (2004) in this note. From Eq. 2 of Chang (2004), we know the expected annual total cost, EAC(T, t), can be expressed as:

$$EAC(T,t)=\frac{K}{T}+\frac{hD}{2}[\rho T-2(\mu_L-t)] + \frac{(h+b)D}{2\rho T}[\sigma_L{}^2+(\mu_L-t^2)] \tag{1}$$

Our goal is to find the minimum solution of EAC(T, t) by algebraic approach. Then we rewrite Eq. 1 as:

$$EAC(T,t)=\frac{(h+b)D}{2\rho T}[(\mu_L-t)-\frac{h\rho T}{h+b}]^2+ \frac{hb\rho DT}{2(h+b)}+\frac{K}{T}+\frac{(h+b)D\sigma_L{}^2}{2\rho T} \tag{2}$$

It implies that when T is given, we can set μ_L-t as:

$\mu_L\text{-}t = t(T) = \dfrac{h\rho T}{h+b}$ to get the minimum value of EAC(T, t) as follows:

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$$EAC(T, t(T)) = \frac{hb\rho DT}{2(h+b)} + \frac{K}{T} + \frac{(h+b)D\sigma_L^2}{2\rho T} \tag{3}$$

Then we rewrite Eq. 3 as:

$$EAC(T, t(T)) = \left[\sqrt{\frac{hb\rho DT}{2(h+b)}} - \sqrt{\frac{2\rho K + (h+b)D\sigma_L^2}{2\rho T}}\right]^2 + \sqrt{\frac{2Dhb}{h+b}\left[\rho K + \frac{D(h+b)\sigma_L^2}{2}\right]} \tag{4}$$

Then we can obtain the optimal cycle time:

$$T^* = \sqrt{\frac{2K}{D\rho} + \frac{(h+b)\sigma_L^2}{\rho^2}}\sqrt{\frac{h+b}{hb}} \tag{5}$$

and the optimal reorder time

$$t^* = \mu_L - \frac{h\rho T^*}{h+b} = \mu_L - \sqrt{\frac{h}{b}\left[\frac{2\rho K}{D(h+b)} + \sigma_L^2\right]} \tag{6}$$

Therefore, the minimum value of the expected annual total cost EAC(T*, t*) is

$$EAC(T^*, t^*) = \sqrt{\frac{2Dhb}{h+b}\left[\rho K + \frac{D(h+b)\sigma_L^2}{2}\right]} \tag{7}$$

Equation 5-7, in this note, are the same as Eq. 10, 9 and 8 in Chang (2004), respectively. Present procedure avoids the difficult decomposition, as in Eq. 4 in Chang (2004). We think this method can be easily accepted for ordinary readers and may be used to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

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