



# Journal of Applied Sciences

ISSN 1812-5654

**science**  
alert

**ANSI***net*  
an open access publisher  
<http://ansinet.com>

## Private Charity and Cooperation Vs. Non-cooperation

Mehmet Yazici

Department of International Trade, Çankaya University,  
Oğretmenler Cad, No. 14, 06530 Balgat-Ankara, Turkey

---

**Abstract:** This study analyzes cooperative and non-cooperative outcomes in the context of private charity and compares them on efficiency grounds. Analysis is conducted in a model economy where there are two rich individuals and one poor and the rich care about the well-being of the poor. The results of the analysis suggest that, when making donations to the poor, the rich should get together and act cooperatively instead of behaving independently. Given the fact that there are many donors in reality and therefore it is difficult for them to get together for cooperation, this result implies a role for government to provide a mechanism for cooperation and justifies the support of the governments for charity organizations.

**Key words:** Private charity, pareto efficiency, nash equilibrium, cooperation, noncooperation

---

### INTRODUCTION

When the action of a person affects welfare of another person and vice versa, then there is interdependence among the individuals involved. In such a situation noncooperative outcome where each person behaves independently of others and cooperative outcome where individuals get together and behave in a cooperative way differ in terms of efficiency. For example, in the case of a Cournot duopoly in a given market where each firm treats the output of the other firm as given and maximizes its own profit (non-cooperation), the resulting profit per firm is less than the profit each will get if both get together and act as a cartel (cooperation). This means that non-cooperative outcome of Cournot competition is suboptimal to cooperative outcome of Cartel (Kreps, 1990). It is not, however, always the case that non-cooperative outcome is inferior. For example, we know from the optimal tariff theorem in international trade literature that non-cooperation is better for a large country when the trading partner is small in the sense that it can not influence the terms of trade. In this case it won't be in the best interest of the large country to cooperate.

This research aims to examine non-cooperative and cooperative outcomes in the context of private charity and find out which outcome is superior on the efficiency grounds. For this purpose an economy is considered where there are three individuals, two identical rich and one poor and both rich care about well-being of the poor, which is reflected by the fact that the consumption of the poor enters the utility functions of the rich positively. As a result, each rich has an incentive to make donation to the poor.

Private charity has been analyzed in some studies. For example, Warr (1982) shows that once donors have determined how much to donate voluntarily, additional gains could be achieved from further incremental transfers. No net transfer is achieved, however, unless incremental fiscal redistribution is continued until the point where the private charity has been driven to zero. Another such study is Roberts (1984) who considers a model where private charity and public transfers are determined simultaneously and shows that there is an overprovision of public transfers in the sense that more public transfers take place than altruistic taxpayers prefer. Taking a game-theoretic view, Glazer and Konrad (1996) offers a new motive for giving by treating observable charitable donations as signals of wealth (status-motivated giving) and examines the signaling equilibrium of charitable donations. They find that the resulting equilibrium has such attractive properties as donations increasing proportionately with population size and a rise in the spread between the poorest and the richest leading to increase in donations. Brooks (2002) empirically investigates in a cross-sectional data how the welfare payments affects charitable giving of those receiving payments and finds that charity is negatively associated with welfare receipt. Andreoni and Payne (2003), in addition to the widely accepted hypothesis in the literature that givers, who are also taxpayers, treat their tax-financed donations as an imperfect substitute for their voluntary donations, offers for the crowding-out effect the explanation that charity organizations, upon receiving a grant, will reduce their fund-raising efforts. Employing panel data from arts and social service organizations, they find that government grants to nonprofit organizations are causing significant reductions in their fund-raising efforts.

## MODEL

The model economy used here is adopted from Warr (1982) with the exception that it is assumed in our case that the poor has no income. The economy consists of three individuals, two of whom are rich and one is poor. Two rich individuals are identical and indexed by  $i = 1, 2$ . The poor individual has no income and each rich has an exogenously determined income of  $y$ . The important property of the model is that the consumption of the poor individual enters the utility functions of the rich. Utility functions of the rich and the poor are given by:

$$\begin{aligned} U_i &= U_i(C_i, C_p) \quad i = 1, 2 \\ U_p &= U_p(C_p) \end{aligned}$$

where,  $C_i$ : consumption of rich individual  $i$  and  $C_p$ : consumption of the poor

Since the poor has no income,  $C_p = d_1 + d_2$  where,  $d_i$ : donation made by individual  $i = 1, 2$ .

It is assumed that utility functions are continuous, strictly increasing, strictly quasi-concave and everywhere twice differentiable with respect to its arguments.

As we see,  $C_p$  is appearing in the utility functions of both individuals 1 and 2. In other words, each rich is able to derive utility from the total quantity of the consumption of the poor. Therefore,  $C_p$  can be interpreted as a public good for individuals 1 and 2. Quantity appearing ( $C_p$ ) in the utility function of each rich differs from his donation due to the fact that he can enjoy the contribution of the other. From the point of view of the utility function,  $d_1$  and  $d_2$  are perfect substitutes. However,  $d_i$  involves an opportunity cost in terms of  $C_i$  forgone whereas the donation made by the other rich involves no such cost.

## NON-COOPERATIVE EQUILIBRIUM

In determining how much to donate, each rich takes the donation of the other as given. Equilibrium donations  $d_1^n$  and  $d_2^n$  are such that given  $d_2^n$ ,  $d_1^n$  maximizes 1's utility and  $d_2^n$  maximizes 2's utility given that 1's donation is  $d_1^n$ . This is a non-cooperative Nash equilibrium.

The problem of rich  $i$  is given by :

$$\begin{aligned} \max_{C_i, d_i} U_i &= U_i(C_i, C_p) \\ \text{subject to: } C_i + d_i &= y \quad i = 1, 2 \end{aligned}$$

Given  $d_1$  and  $d_2$ , the poor individual gets the utility of  $U(C_p)$  where,  $C_p = d_1 + d_2$ .

The problem of each rich is to determine the levels of  $C_i$  and  $d_i$  in such a way that utility is maximized. This occurs at that combination of  $C_i$  and  $d_i$  where the indifference curve is tangent to the budget line.

To find out how one individual reacts to a change in the donation of the other by adjusting his own donation, suppose that the rich individual under consideration is individual 1 and that individual 2 increases his donation. When  $d_2$  increases, individual 1's budget line shifts out in a parallel way because this is an increase in income paid in kind. It has only income effect due to the fact that it does not cause any change in the price ratio. Given that  $C_1$  and  $C_p$  are normal goods, as a result of increase in  $d_2$ ,  $C_1$  and  $C_p$  go up. However,  $y$  is fixed and equal to  $d_1 + C_1$ . As  $C_1$  increases, it must be the case that  $d_1$  decreases. This indicates that reaction curve is downward sloping. Since  $C_p = (d_1 + d_2)$  is increasing, decrease in  $d_1$  must be less than increase in  $d_2$ . Then absolute value of  $\partial d_2 / \partial d_1$  is greater than 1.

Assuming that the preferences are homothetic, the line connecting the tangency points (expansion path) corresponding to different levels of  $d_2$  will be a straight line. In this case reaction curve will also be a straight line. As a result, given the fact that absolute value of  $\partial d_2 / \partial d_1$  is greater than 1, the resulting Nash equilibrium, determined by the intersection of the reaction curves, will be unique and stable. At this Nash equilibrium, since rich individuals are identical, equilibrium levels of donations will be equal to each other, i.e.,  $d_1^n = d_2^n$ .

This Nash equilibrium is not Pareto efficient, however. The inefficiency of the equilibrium can be shown as follows:

$$\begin{aligned} \text{At equilibrium, } MRS_{C_i, C_p} &= \text{price ratio} = 1 \\ MU_{C_p} / MU_{C_i} &= 1 \Leftrightarrow \\ MU_{C_p} &= MU_{C_i} \quad i = 1, 2 \end{aligned}$$

Now consider a binding contract between individual 1 and 2 so that each rich is supposed to increase his total contribution to the poor by 1 unit. As a result of this arrangement, individual  $i$ 's welfare will be affected as follows:

$$\Delta U_i = MU_{C_i} \Delta C_i + MU_{C_p} \Delta C_p \quad i = 1, 2$$

Since at non-cooperative Nash equilibrium,

$$MU_{C_i} = MU_{C_p}$$

and as a result of binding contract

$$\begin{aligned} \Delta C_i &= -1 \quad \Delta C_p = 2 \Rightarrow \\ \Delta U_i &= -MU_{C_p} + 2MU_{C_p} = MU_{C_p} > 0 \quad i = 1, 2 \end{aligned}$$

Hence everyone is made better off, indicating that non-cooperative Nash equilibrium is not Pareto efficient.

## COOPERATIVE EQUILIBRIUM

As we have seen above, increase in donation makes everyone better off. This means that the Nash equilibrium level of donation is less than the optimal level i.e., there is an underprovision of the public good,  $C_p$ . Now let's suppose that two rich individuals cooperate in the following way; they get together and decide that one of them will increase the level of his donation and the other will have to donate the same amount. Will this cooperation lead to Pareto efficiency?

Let's suppose that individual 1 will decide on the increase in his donation and individual 2 has to donate the same amount that individual 1 does. If individual 1 increases his donation by one unit, the other has to increase his donation by one unit as well. By giving up one unit of  $C_1$ , he is getting two units of  $C_p$ . The effective price of  $C_p$  in terms of  $C_1$  forgone is  $1/2$ .

At Nash equilibrium,

$$MRS_{C_1, C_p} = MU_{C_p} / MU_{C_1} = 1 > 1/2$$

If he gives up one unit of  $C_1$ , he gets two units of  $C_p$  but to stay as well off as before he needs only one unit of  $C_p$ . This means that by increasing  $C_p$  through increase in donation, he can increase his utility. He will increase the level of donation up to the point at which  $MU_{C_p} / MU_{C_1} = 1/2$  because at this point his utility is maximized.

Since individual 1 and 2 are identical and donations must be equal to each other, it is true that individual 2' utility is also maximized because  $MU_{C_p} / MU_{C_1} = 1/2$ .

When there is a public good,  $C_p$ , Pareto efficiency requires that

$$MRS_{C_1, C_p} + MRS_{C_2, C_p} = MRT_{C, C_p}$$

Where,  $MRT_{C, C_p}$  is marginal rate of transformation between the consumption of either one of the rich individuals and the consumption of the poor individual [Cornes and Sandler (1986)] for a full treatment of public goods.

This is because if one of the rich increases  $C_p$  by one unit, the other rich is also able to consume this one unit increase.

The above condition can be interpreted as

$$MB_{C_p}^1 + MB_{C_p}^2 = MC_{C_p}$$

Where,  $MB_{C_p}^i$  is marginal benefit individual  $i=1, 2$  is getting from  $C_p$ .  $MC_{C_p}$  is marginal cost of  $C_p$ .

Since in our case

$$MRS_{C_1, C_p} = 1/2, MRS_{C_2, C_p} = 1/2 \text{ and}$$

$$MRT_{C, C_p} = 1 \Rightarrow 1/2 + 1/2 = 1$$

This means that Pareto efficiency is achieved and as a result, compared to Nash equilibrium, everyone is made better off.

## AN EXAMPLE

Utility function of each rich has the following form:

$$U_i = C_1^\alpha C_p^{1-\alpha} \text{ where, } 0 < \alpha < 1 \text{ and } i = 1, 2$$

**Nash equilibrium:** Since both rich are identical, we can take only one of them

$$\max_{C_1, d_1} U(C_1, C_p) = C_1^\alpha C_p^{1-\alpha}, \text{ given } d_2$$

subject to:  $C_1 + d_1 = y$  where  $C_p = d_1 + d_2$

$$L = C_1^\alpha (d_1 + d_2)^{1-\alpha} + \lambda [y - C_1 - d_1]$$

First-order conditions are;

$$\begin{aligned} \partial L / \partial C_1 &= \alpha C_1^{\alpha-1} (d_1 + d_2)^{1-\alpha} - \lambda = 0 \\ \partial L / \partial d_1 &= (1-\alpha) C_1^\alpha (d_1 + d_2)^{-\alpha} - \lambda = 0 \\ \partial L / \partial \lambda &= y - C_1 - d_1 = 0 \end{aligned}$$

These conditions imply that

$$\begin{aligned} C_1 &= \alpha (y - d_1) & C_2 &= \alpha (y - d_2) \\ d_1(d_2) &= (1-\alpha)y - \alpha d_2 & d_2(d_1) &= (1-\alpha)y - \alpha d_1 \end{aligned}$$

reaction functions.

Using these reaction functions to solve for Nash equilibrium levels of donations yields

$$d_1^n = d_2^n = d^n = [(1-\alpha)y] / (1+\alpha)$$

Given  $y$ , the higher  $\alpha$ , the lower will be the Nash equilibrium level of donation,  $d^n$ .

The Nash equilibrium levels of the consumption are

$$C_1^n = C_2^n = (2\alpha y) / (1+\alpha), C_p^n = [2y(1-\alpha)] / (1+\alpha)$$

**Cooperative equilibrium:** It must be the case that  $d_1 = d_2 = d$ . This means that  $C_p = 2d$ .

$$\max_{C_1, d} U(C_1, C_p) = C_1^\alpha (2d)^{1-\alpha}$$

subject to:  $C_1 + d = y$

$$L = C_1^\alpha (2d)^{1-\alpha} + \lambda[y - C_1 - d]$$

First-order conditions are:

$$\begin{aligned}\partial L / \partial C_1 &= \alpha C_1^{\alpha-1} (2d)^{1-\alpha} - \lambda = 0 \\ \partial L / \partial d &= (1-\alpha) C_1^\alpha 2^{(1-\alpha)} d^{-\alpha} - \lambda = 0 \\ \partial L / \partial \lambda &= y - C_1 - d = 0\end{aligned}$$

From these conditions, cooperative equilibrium levels of donation are found as

$$d_1^c = d_2^c = d^c = (1-\alpha)y$$

Given  $y$ , the higher  $\alpha$ , the lower will be  $d^c$ .

And cooperative equilibrium levels of the consumption are

$$C_1^c = C_2^c = \alpha y, \quad C_p^c = 2y(1-\alpha)$$

We observe that  $C_1^n = C_2^n > C_1^c = C_2^c$  and  $d_1^n = d_2^n < d_1^c = d_2^c$

$$\Rightarrow C_p^n < C_p^c \Rightarrow U_p^n < U_p^c$$

$$\text{Is } U_i^n < U_i^c ? \quad i = 1, 2$$

$$U_i^n = \frac{[(2\alpha y)/(1+\alpha)]^\alpha [(2y(1-\alpha)/(1+\alpha))^{(1-\alpha)}]}{[2(1-\alpha)y]^{(1-\alpha)}} = U_i^c \Leftrightarrow (2/(1+\alpha)) < 2^{(1-\alpha)}$$

This inequality holds because  $0 < \alpha < 1$ .

This means that under the cooperative equilibrium everybody is made better off than under the Nash equilibrium.

## DISCUSSION AND CONCLUSIONS

The purpose of this research has been to examine the non-cooperative and cooperative outcomes in the context of private charity, compare those outcomes and find out which outcome is superior. A model economy consisting of two rich and one poor individuals where the

consumption of the poor enters positively the utility functions of the rich is used for the analysis. It is shown, first theoretically and then with an example, that noncooperative outcome is not Pareto efficient and the efficiency can be achieved through cooperation. In other words, everybody including donors is better off under cooperation. This implies that when donating to the poor, the rich should behave cooperatively instead of independently of each other so that everyone will be at a higher utility level. In real world, however, there are so many donors, not just two like here. Therefore, it will be extremely difficult for the givers to get together and cooperate. Seeing this difficulty and knowing that cooperative outcome is better than non-cooperative one, government can play a role in the sense that it can provide some mechanisms that will lead to cooperation among donors. It is observed in the analysis that donations are higher under cooperation. Therefore, charity organizations, since in their absence donations would be less, can be regarded as institutions providing a cooperating mechanism.

## REFERENCES

- Andreoni, J. and A.A. Payne, 2003. Do government grants to private charities crowd out giving or fund-raising?. *Am. Econom. Rev.*, 93: 792-812.
- Brooks, A.C., 2002. Welfare receipt and private charity. *Public Budgeting and Finance*, 22: 101-114.
- Comes, R. and T. Sandler, 1986. *The Theory of Externalities, Public Goods and Club Goods*. Cambridge University Press.
- Glazer, A. and K.A. Konrad, 1996. A signaling explanation for charity. *Am. Econom. Rev.*, 86: 1019-1028.
- Kreps, M.K., 1990. *A Course in Microeconomic Theory*. Princeton University Press.
- Roberts, D.R., 1984. A positive model of private charity and public transfers. *J. Polit Econom.*, 92: 136-148.
- Warr, P.G., 1982. Pareto optimal redistribution and private charity. *J. Public Econom.*, 19: 131-138.