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## A Mathematical Model for Load Optimization: Linear Load Curves

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**Abstract:** High energy consumption as well as reducing the known energy resources is the key factors to develop the philosophy of optimum energy consumption. So, with representing different energy optimization methods and ways it is quite possible to reduce the energy consumption (optimum consumption) because the number of energy consumers versus the energy resources and even energy production and conversion facilities is increasing everyday. To get the above goals a simple optimization model (linear model) is discussed in this essay in order to set the level of load optimization management for an acceptable energy distribution according to the costs and expenses. In this model, according to the present costs i.e., demand charges (a) and energy charges (b) and also the given optimization model, we are able to achieve a suitable distribution of the total available energy in accordance with the economic optimization of the total energy which is in turn completely coincided with Iran's present situation and minimum capabilities in energy sector.

**Key words:** Load duration curve, accurate peak shaving, load management, mathematical linear model, numerical analysis models

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### INTRODUCTION

High energy consumption as well as reducing the known energy resources is the key factors to develop the philosophy of optimum energy consumption. So with representing different energy optimization methods and ways it is quite possible to reduce the energy consumption (optimum consumption) because the number of energy consumers versus the energy resources and even energy production and a conversion facility is increasing everyday.

Electrical energy is also involved with the same problem, so optimizing energy production and consumption methods in this field looks quite necessary particularly in domestic section, plants and factories and so on. This can be discussed as a linear or a non-linear optimization issue, but many researchers tend to use the linear assumptions to simplify the issue. Then it is possible to treat the problem as an integer linear planning issue (Zarnikau, 1999).

Reviewing the last year related essays shows an intention to use complement algorithms as a solution for this problem, which has also had some good results (Zarnikau, 1999; Bogdanov and Osman, 2000).

Energy optimization issue has taken into account since the early 1970s considering energy, economy and environment as its three principal factors and since then

industrial countries has vigorously benefited from this issue. Today, these countries deal with the energy consumption management as a new source of energy and believe in that applying this management issue has been the main cause of industrial and economic flourishing as well as developing the culture of reasonable energy consumption. As a result of this a new generation of technology has been developed which is called the clean technology.

It is always questioned that how much of the total energy resources of a country or region is being produced or consumed and this parameter is measured as a variable factor in economic models, so applying the management theories from the electricity producers sounds quite necessary in this regard. The final goal of all such theories is to make a correction in consumption methods as well as merging electrical energy consumption and desirable load curves.

Causes of high lose of energy in Iran as follow:

- Low load ratio which in turn results in a sharp load curve due to a mismatch between producer's consumption (42%) and non-producer's consumption (58%) and not attempting enough in the field of consumption management.
- High energy consumption of some of the power plants especially steam power plants.

- Low maximum power ratio for domestic and commercial applications due to not installing the compensator capacitors in appliances' electric motors.
- Miscellaneous factors.

There is no doubt that in order to reduce the energy loss, it is vital to diminish its causes of increment. With regard to various applications of electrical power and their different effects on economic and welfare developments of the society as well as on technical- operational requirements of the electricity industry, it is absolutely vital to know and categorize the consumers. This categorization when accompanies with the factors affecting the details of each part's consumption can lead to the necessary awareness required by the electricity industry for energy and load management to set the initial backgrounds of consumed energy retrenchment as well as correcting the consumption policies for the benefit of the productive applications using the suitable methods such as setting various costs for different sections and at different consumption periods and time (Talukdar and Gellings, 1987).

Analyzing the consumption pattern of different parts of the country reveals that nearly 40% of the energy is consumed by domestic and commercial applications while 27% is used by the industry sector, however in developing countries it is vice-versa. So to keep the country far away from energy difficulties, employing and performing energy consumption management solutions becomes quite critical. To meet the energy reduction and retrenchment requirements it is necessary to develop optimum usage of energy resources and human resources, using new technology and expanding the technology of decreasing the energy transfer loss.

### LOAD MANAGEMENT

Load management (Bjork, 1989; Vogt and Conner, 1977; Isaksen *et al.*, 1981; Sheen *et al.*, 1994) is the process of scheduling the loads to reduce the electric energy consumption and or the maximum demand. It is basically optimizing the processes/loads to improve the system load factor. Load-management procedures involve changes to equipment and/or consumption patterns on the customer side. There are many methods of load management which can be followed by an industry or a utility, such as load shedding and restoring, load shifting, power wheeling, installing energy-efficient processes and equipment, energy storage devices, co-generation and non-conventional sources of energy and reactive power-control (Cochen and Wang, 1988).

Meeting the peak demand is one of the major problems now facing the electric utilities. With the

existing generating capacity being unmanageable, authorities are forced to implement load shedding in various sectors during most of the seasons. Load shifting will be a better option for most industries. Load shifting basically means scheduling the load in such a way that loads are diverted from peak period to off-peak periods, thereby shaving the peak and filling the valley of the load curve, so improving the load factor.

To encourage load shifting in industries and thereby to reduce peak demand, many utilities have already implemented Time of Use Rates (TOU) or have plans for introducing such rates (Sheen *et al.*, 1994).

### MATERIALS AND METHODS

#### DEFINING SUGGESTED METHODS AND INDECES

In this method, the objective is to revise the consumption curve of the sample industrial sections and approach it to the desirable curve with evaluating their load curve and applying load management theories. The main purpose here is to reduce the energy consumption in the load peak period of the network (early evening) as well as straightening the curve at other hours of the day.

To get the above goals a simple optimization model (linear model) is discussed in order to set the level of optimal load management for an acceptable energy distribution according to the costs and expenses (Rao, 1984). To formulate this model as well as meeting the model requirements the following assumptions are needed:

1. To get an accurate peak shaving, a correct forecast of the future peak points and also a figure of the load duration curve should be available.
2. Assume that there are no limitations to the heat supplemental capacity.

Assume that the annual energy consumption or annual energy purchase is proportional with the load duration curves shown in Fig. 1.

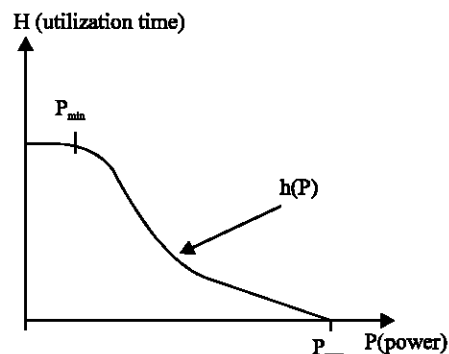


Fig. 1: Sample load continuity curve

The parameters of this model are defined as follows:

- P : Total power variable
- H(p) : Time utilization function regarding load duration curve (LDC)
- P<sub>max</sub> : Maximum peak consumed power for LDC
- P<sub>min</sub> : Minimum peak consumed power for LDC
- h<sub>m</sub> : Number of hours for a specified period (e.g., for a year this will be 8760)

In accordance with the above curve H(p) will be as follows generally:

$$H(P) = \begin{cases} h_m & \text{if } 0 < P < P_{min} \\ h_0(P) & \text{if } P_{min} < P < P_{max} \\ 0 & \text{if } P > P_{max} \end{cases} \quad (1)$$

If assume that the utilization time is h<sub>t</sub>, the result is:

$$P_{max} \cdot h_t = E_t \quad (2)$$

Also:

$$E_t = \int_0^{P_{max}} H(P)dP \quad (3)$$

Which means the total consumed energy or the area under the LDC curve. Merging the Eq. (2) and (3) results the following equation:

$$h_t = \frac{E_t}{P_{max}} = \frac{1}{P_{max}} \int_0^{P_{max}} h(P)dP \quad (4)$$

Analyzing this equation shows that h<sub>t</sub> equals the average of the function H(p). Now assume that (a) is a parameter that represents the energy demand charges and (b) represents the energy costs, for utilization time (h<sub>t</sub>) the interest of the consumed power (k<sub>0</sub>) will be:

$$k_0 = aP_{max} + bE_t = P_{max}(a + bh_t) \quad (5)$$

Also the average price per kWh will be:

$$v(h_t) = k_m = \frac{P_{max}(a + bh_t)}{E_t} = \frac{a + bh_t}{h_t} \quad (6)$$

$$\Rightarrow k = \frac{a}{h_t} + b$$

Regarding the definition of the function (H(p)) according to the present LDC it is needed to estimate this function, which is done using the numerical analysis models. To do this let's consider the curve linear as shown in Fig. 2.

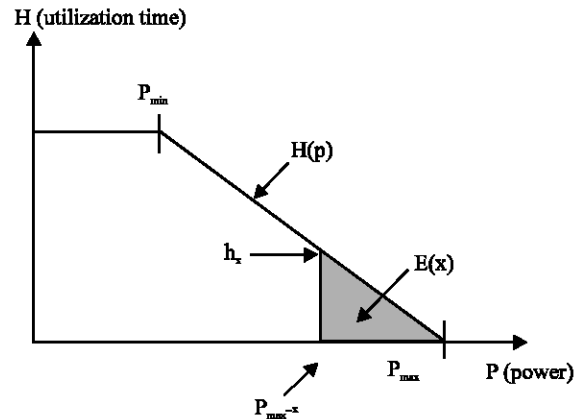


Fig. 2: The curve linear

The related equation for this graph will be:

$$h(x) = \frac{1}{2} xh_x = \alpha x^2 \quad (7)$$

Let's write the equation of the line which can be made using the points (P<sub>min</sub>, h<sub>min</sub>) and (P<sub>max</sub>, h<sub>t</sub>). After equalization between the correspondent parameters the value of (α) will be:

$$\alpha = \frac{h_m}{4E_t \left[ \frac{1}{h_t} - \frac{1}{h_m} \right]} \quad (8)$$

Now the utilization time changes from h<sub>t</sub> to h<sub>t</sub>, which represents the utilization time after optimization as follows:

$$h_t = \frac{(E_t - E(x))}{(P_{max} - x)} \quad (9)$$

If V<sub>x</sub> shows the heat costs, then our chosen linear function will represent the total annual heat cost (for production) as V<sub>x</sub> × E(x).

And if there is a curve [K<sub>v</sub>(E(X))] in the above situation, the total annual cost will be ∫<sub>0</sub><sup>E(x)</sup> V<sub>x</sub>(E)dE.

$$K_v(E(x)) = V_x \cdot E(x) \quad (10)$$

So the total energy including demand and consumption costs will be:

$$K(x) = a(P_{max} - x) + b(E_t - E_x) + K_v(E(x)) \quad (11)$$

It is clear that to get the optimum results the above equation should be optimum, which will be as follows regarding math's formulas:

$$\frac{dK(x)}{x} = 0$$

Summing up what has been discussed so far the result will be:

$$\begin{aligned} \frac{1}{2}xh_x &= \alpha x^2 \\ \Rightarrow h_x &= h(P_{max} - x) = 2\alpha x \end{aligned} \quad (12)$$

Now because of assuming a linear graph, let's replace our assumptions in (\*) and solve the (\*\*) to find the optimum value:

$$\begin{cases} K(x) = a(P_{max} - x) + b(E_t - E_x) + K_v(E(x)) \\ K_v(E(x)) = V_x \cdot E_x \\ \Rightarrow K(x) = aP_{max} + bE_t - ax + (V_x - b)E(x) \end{cases}$$

$$\begin{cases} \frac{dK(x)}{dx} = -a + (V_x - b) \frac{dE(x)}{dx} = 0 \\ \frac{dE(x)}{dx} = h(P_{max} - x) = h_x \end{cases}$$

$$\Rightarrow -a + (V_x - b) \frac{dE(x)}{dx} = 0 \Rightarrow -a + (V_x - b)h_x = 0$$

$$\Rightarrow \begin{cases} a = h_x [V_x - b] \\ h_x = 2\alpha x \end{cases} \Rightarrow a = 2\alpha x [V_x - b]$$

$$\Rightarrow x_{opt} = \frac{a}{2\alpha [V_x - b]} \quad (13)$$

Now with knowing the  $X_{op}$  value it is possible to calculate the optimum value of  $K_{op}$ :

$$\begin{aligned} K_{opt}(x) &= aP_{max} + bE_t - ax_{opt} + (V_x - b)a \left[ \frac{x_{opt}}{2} \right]^2 \\ \Rightarrow K_{opt} &= aP_{max} + bE_t - \frac{a}{2}x_{opt} \end{aligned} \quad (14)$$

So with considering a linear load duration graph and heat unit cost, the retrenchment will be always half of the usual public costs. In other words, if the demand costs, consumption costs, heat unit costs and an approximate load duration curve (according to the previous statistics) are available at the beginning of a new period, it will be possible to calculate the optimum load continuity curve, the optimum peak value and the optimum utilization time, using the mentioned equations, then setting a desirable energy distribution will be quite easy, i.e., electricity distributors should attempt to approach the optimum load continuity concluded from the discussed model in energy distribution (sale).

## NUMERICAL EXAMPLE

The following example shows the same conclusions:  $P_{max} = 134.43\text{MW}$ ,  $E_t = 693.22\text{ GWh}$  results in:

$$\begin{aligned} h_t \frac{E_t}{P_{max}} &= \frac{693.22\text{GWh}}{134.43\text{MW}} = 5157\text{hours/years} \\ V_x &= 5.00\$/\text{KWh} \\ b &= 4.15\$/\text{KWh} \\ \alpha &= 0.0396 \\ h &= 1957 \end{aligned}$$

This amount equals 81 days or 2.7 months. This means that for the given costs and heat unit price for a diesel producer for instance the ultimate retrenchment point is 2.7 months or 22% of the year in which peak load is also reduced by 18% or 24.7 MW and from economic utilization point of view there is a 110.9 M\$ or 7.3% retrenchment. To complete the issue it should be noted that with considering  $X_{opt}$  it is possible to set a price limit to show whether the prices are expensive or not as follows:

$$\text{if } P \geq P_{max} - x_{opt} \Rightarrow \text{the price increases}$$

So both producer and consumer will try to decrease the expenses in this regard.

## CONCLUSION

In this model, according to the present costs i.e., demand charges (a) and energy charges (b) and also the given optimization model, we are able to achieve a suitable distribution of the total available energy in accordance with the economic optimization of the total energy which is in turn completely coincided with Iran's present situation and minimum capabilities in energy sector and with inserting other parameters as a result of studies and researches it will be possible to increase the accuracy and performance of this model. In addition, this paper presented to us that:

### Why peak load management?

- The pressures of load growth have begun to maximize the system during extreme conditions.
- New fish obligations have limited the ability to optimize the value of the Iran system.
- There is a greater emphasis being place on "reliable" economic load reduction as a resource of the future.

**Why the demand exchange?**

- Utilities and consumers are always looking for methods to reduce the cost of electricity. Often, the most expensive period of time for a utility to purchase electricity to meet load is during the coldest or hottest days of the year.
- Finding ways to reduce electrical demands during those times represent another opportunity for us to work with our customers to cut costs and increase system reliability.

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