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Modeling and Simulation of Dynamic Systems

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Abstract: This study presents a modeling and simulation of dynamic systems. Modeling and simulation have experienced an amazing development in a last few decades. Today it is available on the desk of all engineers and scientists who needs it. The main focus is on modeling and simulation of thermal dynamic systems. A case study of batch processing in chemical industry is reported in order to demonstrate the ideas of modeling, analytical solution of Ordinary Differential Equations (ODEs) and simulation of thermal dynamic system.

Key words: Modeling, simulation, dynamic system, ordinary differential equation, thermal dynamic system

INTRODUCTION

Modeling and simulation are indispensable when dealing with complex engineering systems. It makes it possible to do essential assessment before systems are built, it can alleviate the need for expensive experiments and it can provide support in all stages of a project from conceptual design, through commissioning and operations.

The real-time simulation of dynamic systems on systolic arrays has emerged as a powerful means for solving several computational problems of practical importance (Vijay and Murthy, 1998).

Computer-based modeling and simulation has been under steady development for about 50 years now. In one form or another, this methodology has become ever more important in an ever broadening arena of applications. Here we overview a part of this wide field, the modeling and simulation of dynamical systems, specifically, systems that can be appropriately modeled by Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs), Differential Algebraic Equations (DAEs) and ODEs interfaced with Discrete-time Algorithms (DTAs). Some aspects of the technical discussion of modeling is, of necessity, be quite sketchy and the more comprehensive presentation of simulation methods focuses primarily on ODE systems, with additional pointers to techniques for dealing with DAEs, PDEs and such systems interfaced with DTAs (Frederick, 1978).

Significant utilization of dynamic modeling and simulation has started at different times and progressed at different rates in various application areas. As a generalization, the early years of modeling and simulation were focused primarily on aerospace and military applications-i.e., high-tech and expensive technological systems, where the engineering analysis and design effort was substantial and the modeling and simulation effort could be "afforded". Then modeling and simulation spread to smaller high-tech and civilian arenas-robotics and transportation systems coming immediately to mind. At the present, it is quite safe to state that modeling and simulation, in one form or another, is applied to almost every area of human endeavor, from agriculture to biology to economics to manufacturing (Åström and Elmqvist, 1998).

Finally, there are numerous driving forces for the rapidly increasing necessity and popularity of modeling and simulation. Five general dominant factors are:

- The continuing need to achieve better process or product performance;
- The increasing complexity of advanced technological systems;
- The growing need for competitive advantage, e.g., efficiency, economy;
- The phenomenal increase in available computer power and decrease in cost;
- The coupling of modeling and simulation with other powerful computer-based methods, e.g., optimization

Models allow the effects of time and space to be scaled, extraction of properties and hence simplification, to retain only those details relevant to the problem.

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The use of models therefore reduces the need for real experimentation and facilitates the achievement of many different purposes at reduced cost, risk and time (Bobrow, 1985).

Thermal systems: Thermal systems are systems in which flow of heat and storage of heat are involved. Their mathematical models are based on the fundamental laws of thermodynamics. Generally thermal systems are distributed and they obey Partial Differential Equations (PDEs) rather than Ordinary Differential Equations (ODEs) (Close *et al*., 2002). Present purpose is to get linear ODEs that are capable of describing the dynamic response to a good approximation

Here is a description of some notations used for variables in thermal systems.

θ: temperature (K)

q: Heat flow rate in J/s, watt

If $q_{in}(t) - q_{out}(t)$ is the net heat flow rate into the body as a function of time then net heat supplied between the time t_0 and t is as:

$$\int_{t_0}^{t} [q_{in}(\lambda) q_{out}(\lambda)] d\lambda \quad (1)$$

We know thermal capacitance 'C' has unit joule per Kelvin (J/K). If a body has a mass 'M' and specific heat 'σ' having units J/(Kg. K) then thermal capacitance is

$$C = M^{\sigma}$$

If we assume that temperature of a body at time t_0 is θ_0 and rise in temperature due to net heat flow into the body is as bellow

$$\frac{1}{C}\int_{t_0}^{t} [q_{in}(\lambda) q_{out}(\lambda)] d\lambda \quad (2)$$

Then the temperature of the body after heat addition is as

$$\theta(t) = \theta_0 \frac{1}{C}\int_{t_0}^{t} [q_{in}(\lambda) q_{out}(\lambda)] d\lambda \quad (3)$$

Differentiating Eq. (5)

$$\dot{\theta}(t) = \theta_0 \frac{1}{C}[q_{in}(t) - q_{out}(t)] \quad (4)$$

This relates the rate of temperature change to the instantaneous net heat flow rate into the body. This is very important equation because we generally select the temperature of the bodies constituting the thermal system as state variable.

There is another factor that is thermal resistance. Therefore, whenever heat flows through the connecting medium from one body to another then heat flow rate can be expressed by the following equation.

$$q(t) = \frac{1}{R}[\theta_1(t) - \theta_2(t)] \quad (5)$$

Where R is a thermal resistance and can be defined as

d = Length of the path

A = Cross-sectional Area

α = Thermal conductivity

Problem description of case study: In chemical Industry the production of chemicals almost always requires control of temperature of liquids contained in vessels. In batch process, a vessel would be filled with liquid, sealed and then heated to a prescribed temperature. In design and processes, it is important to be able to calculate in advance the time required to reach the desired temperature. Such a system has been modeled and analyzed as study case. Heater is placed within a metal jacket that has thermal resistance Rhl. It is assumed that heater and liquid initially are at ambient temperature. The numerical values of the system parameters are as under.

Thermal capacitance of Heater = CH = 20.0×10^3 J/K Thermal capacitance of Liquid = CL = 1.0×10^6 J/K Thermal resistance from heater to liquid = Rhl = 1.0×10^{-3} S.K/J.

Thermal resistance from liquid to ambient = Rla = 5.0×10^{-3} S.K/J

Ambient temperature = θ_a = 300 K

Desired temperature = θ_d = 365 K

At t = 0, heater has been connected to electric supply and we have to determine the response of liquid temperature and also to find time required to reach the desired temperature

System model: Here θ_L and θ_H are the state variables of the thermal system. θ_L and θ_H represent the energy stored in the system. We can write state variable model as

$$\dot{\theta}_H = \frac{1}{C_H}[q_{in}(t) - q_{HL}] \quad (6)$$

$$\dot{\theta}_H = \frac{1}{C_H}[q_{HL} - q_{La}] \quad (7)$$

where

$$q_{HL} = \frac{1}{R_{HL}}[\theta_H - \theta_L]$$

$$q_{La} = \frac{1}{R_{La}}[\theta_L - \theta_a]$$

After substituting q_{HL}, q_{La} and numerical values of the parameters to the state Eq. 6 and 7, the pair of state equations become as

$$\dot{\theta}H = -0.05\,\theta_H + 0.005\,\theta_L + [0.5\times le-4]\,q_{in}(t) \quad (8)$$

$$\dot{\theta}H = -[1.2\times le-3]\theta_L + [le-3]\theta_H + [0.2\times le-3]\theta_a \quad (9)$$

The initial conditions are $\theta_H(0) = \theta_L(0) = \theta_a$
Here we define the relative variables.

$$\hat{\theta}_H = \theta_H - \bar{\theta}_a$$
$$\hat{\theta}_L = \theta_L - \bar{\theta}_a$$
$$\hat{q}_{in}(t) = q_{in}(t)$$

Substituting the relative values to Eqs. 8 and 9

$$\dot{\hat{\theta}}_H = -0.05\hat{\theta}_H + 0.05\hat{\theta}_L + [0.5\times le-4]\hat{q}_{in}(t) \quad (10)$$

$$\dot{\hat{\theta}}_L = -[1.2\times le-3]\hat{\theta}_L + [le-3]\hat{\theta}_H \quad (11)$$

Here initial conditions for incremental values are

$$\hat{\theta}_H(0) = \hat{\theta}_L(0) = 0$$

Here we use the laplace transformation method to solve the ODEs (Gear and Petzold, 1984).

Taking Laplace transformation of above equations

$$(s+0.05)\hat{\Theta}_H(s) = 0.05\hat{\Theta}_L(s) + [0.5\times le-4]\hat{Q}_{in}(s) \quad (12)$$

$$(s+1.2\times le-3)\hat{\Theta}_L(s) = (le-3)\hat{\Theta}_H(s) \quad (13)$$

Combining Eq. 12 and 13 to eliminate

$\hat{\Theta}_H(s)$ and finding H(s) = $\frac{\hat{\Theta}_L(s)}{\hat{Q}_{in}(s)}$

$$H(s) = \frac{0.5\times le-4}{1000s^2 + 51.20s + 0.010}$$
$$= \frac{0.5\times le-7}{(s+0.051)(s+0.000196)} \quad (14)$$

System response

From, $\frac{\hat{\Theta}_L(s)}{\hat{Q}_{in}(s)}$ we can solve for θ_L (t),

$$q_{in}(t) = 1.5\times 1e4 \text{ W} \quad (15)$$

taking Laplace transform of Eq. 15

$$\hat{Q}_{in}(s) = \frac{1.5\times le4}{s} \quad (16)$$

$\hat{\Theta}_L(s)$ from Eq. 14 can be written as

$$\hat{\Theta}_L(s) = \frac{1.5\times le-7}{(s+0.051)(s+0.000196)\hat{Q}_{in}(s)} \quad (17)$$

Putting value of $\hat{Q}_{in}(s)$ from Eq. 16 to 17

$$= \frac{0.75\times le-3}{(s+0.051)(s+0.000196)}$$

We expand it in partial fraction

$$\hat{\Theta}_L(s) = \frac{A}{s} + \frac{B}{s+0.051} + \frac{c}{s+0.000196} \quad (18)$$

A = 75 ; B = 0.289; C = -75.29
Putting the values of A, B and C in Eq. 18

$$\hat{\Theta}_L(s) = \frac{75}{s} + \frac{0.289}{s+0.051} + \frac{75.29}{s+0.000196}$$

Taking inverse Laplace transform

$$\hat{\theta}_L = 75 + 0.289e^{-0.051t} - 75.29e^{-0.00196t}$$

Adding ambient temperature (300 K) to the incremental liquid temperature

$$\theta_L = 375 + 0.289e^{-0.051t} - 75.29e^{-0.00196t}$$

Here $\tau_1 = \frac{1}{0.051} = 19.61$ and $\tau_2 = \frac{1}{0.000196} = 5102$

$$\theta_L = 375 - 75.29^{-t/5102}$$

Now we can determine the time required to reach the desired temperature (365 K)

$$365 = 375 - 75.29^{-t/5102} \quad (19)$$

Solving Eq. 19 the time required to reach the required temperature is 10300 sec or 2.86 h.

If we increase the heater power to double then the time required to reach the required temperature is about 2916 sec or 48.6 min and so on.

Simulation: Now we implement the modeling equations developed for the thermal system to make a block diagram

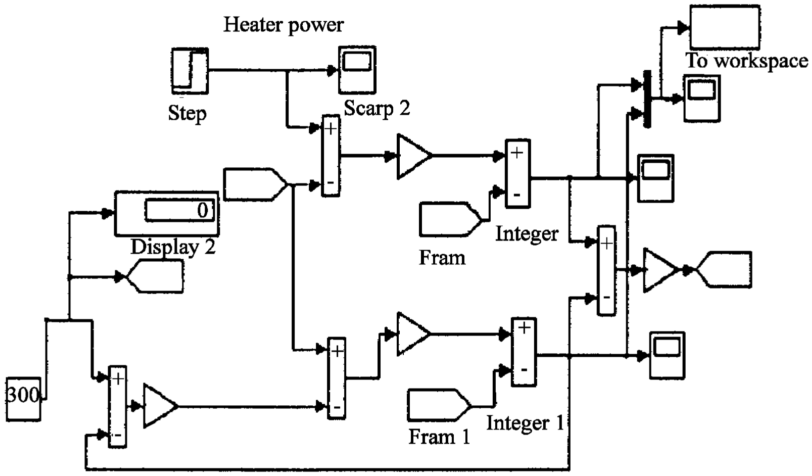

Fig. 1: Block diagram developed in SIMULINK

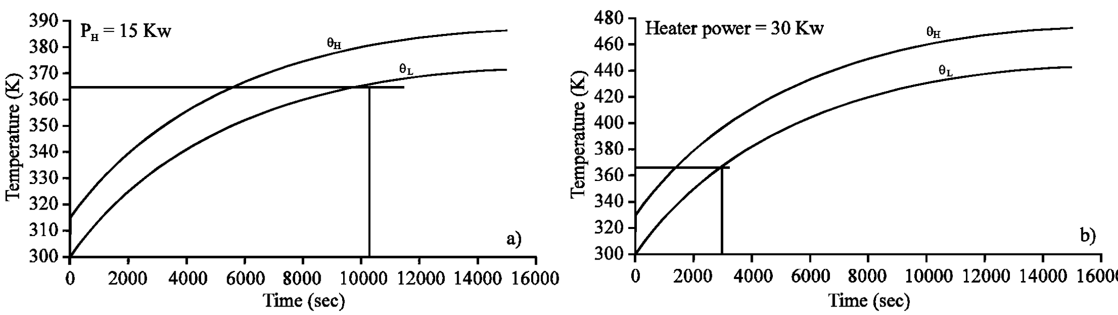

Fig. 2: Graphs of (a) and (b) system using step function as input

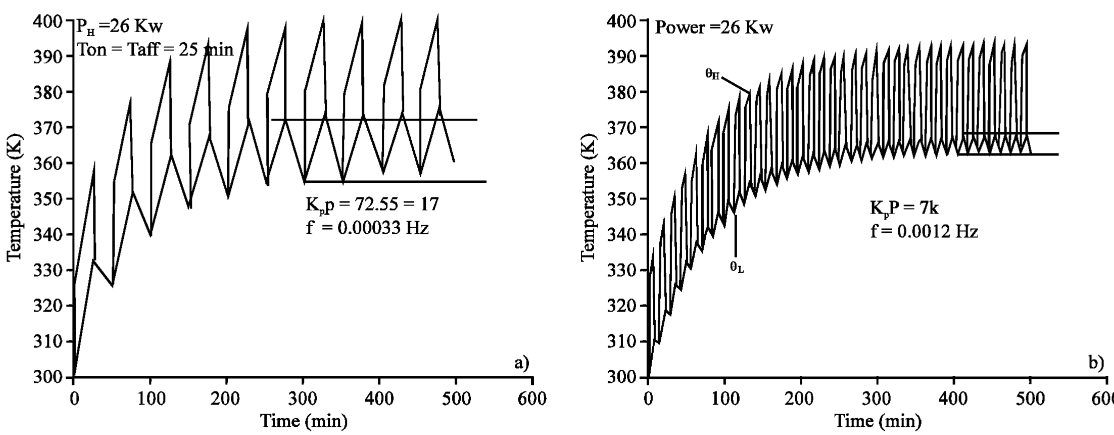

Fig. 3: Graphs (a) and (b) of system using signal generator as input

in MATLAB using SIMULINK Environment (SimuLink User's Guide).

The block diagram obtained by implementing the modeling equations has been shown in Fig. 1.

The analysis of the response of the system using following three different inputs has been conducted:

Step function: Using step function as a constant heating source of 15 and 30 K Joule sec^{-1}, we observed changes in behavior of the heater temperature and in turn liquid temperature as shown in Fig. 2.

From the Fig. 2 we have noted the time required to reach the desired temperature. The rise of liquid

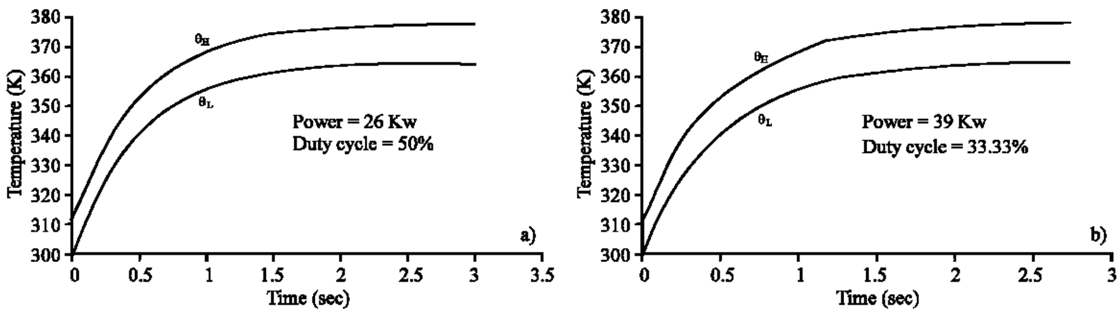

Fig. 4: Graphs (a) and (b) of system using pulse generator as input

temperature has linear relation with that of heating rate.

Signal generator: Here we have considered another situation which is normally demanded by industry to maintain the desired temp. For this purpose we have used signal generator (square wave). Heater power is about 26 KW. The time period for signal is 50 min. Therefore, the frequency is 0.00033 Hz. After implementing data in SIMULINK environment, we have observed that during 4th cycle the liquid temperature reaches to desired temperature (365K). We also see that after 300 min the steady state response achieved but the temperature fluctuation peak to peak is about 17 K. To decrease the fluctuations peak to peak temperature, we have increased the frequency of the signal generator. If we increase the frequency from 0.00033 to 0.0012 Hz then we see from graph that fluctuation range is only about 7 K. The graphs of these two frequencies are shown in Fig. 3.

Pulse generator: We have used Pulse Generator as input source to the system. Heater power is 26 KW and duty cycle is 50%. The out put response of the system has been shown in Fig. 4. We have increased the heater power to 39 KW then we have to decrease the duty cycle to 33.33% to get the same results in first case of using pulse generator.

CONCLUSIONS

The response of the thermal system has been observed using step function, signal generator and pulse generator. The time required to reach the desired liquid temperature by step function has a good agreement with that calculated analytically. It is also observed by graphs that the response of the liquid temperature has linear relationship with heater temperature. By controlling the frequency of the input signal the peak to peak temperature fluctuations (sometime called ripple) in liquid temperature has been decreased.

So this article is an overview of modeling and simulation of dynamic systems especially for thermal systems and a good understanding about how to model, solve and simulate the dynamic thermal systems.

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